Kolmogorov-Smirnov Test

1 sample version $\quad \chi = \{X_1, \dots, X_n\} \quad X_i \overset{iid}{\sim} G$ (G: distribution)

Want to know: was χ drawn from target distribution F.

Empirical Distribution Function

$$F_n(x) = \frac{1}{n} \sum_{i=1}^{n} I_{(-\infty, x]}(X_i), \quad \text{where } I_{[-\infty, x]}(X) = \begin{cases} 0 & \text{if } x < X \\ 1 & x \geq X \end{cases}$$

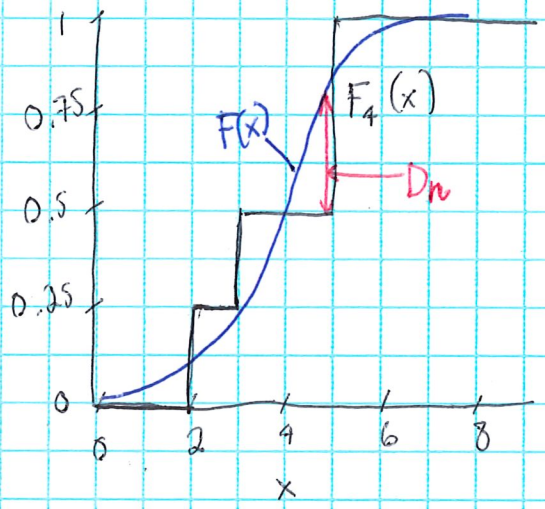
$X_1 = 2$

$X_2 = 3$

$X_3 = 5$

$X_4 = 5$

$n = 4$

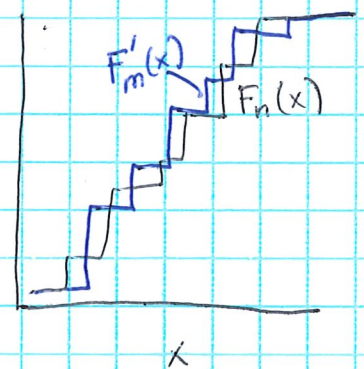


$$D_n = \sup_x |F_n(x) - F(x)|$$

Glivenko-Cantelli Thm - If $X_i \sim F(x)$, then $D_n \xrightarrow{a.s.} 0$

Rule: Reject null hypothesis w/ confidence α if $D_n > \frac{K_\alpha}{\sqrt{n}}$.

2 sample version $\quad \chi = \{X_1, \dots, X_n\} \quad \chi' = \{X'_1, \dots, X'_m\}$



$$D_{n,m} = \sup_x |F_n(x) - F'_m(x)|$$

Rule: Reject null @ confidence α if

$$D_{n,m} > c_\alpha \sqrt{\frac{n+m}{nm}}$$

If $n = m$

$$\sqrt{\frac{2\kappa}{n^2}} = \frac{c}{\sqrt{n}}$$