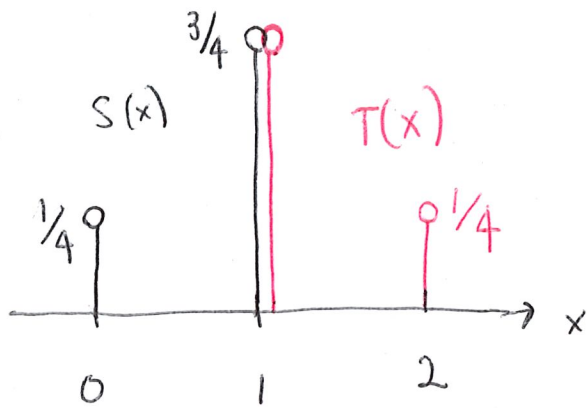


# Example: Earth-Mover's Distance



$$\text{Compute } D_{EM}(S, T) = \min_h \sum_{x, y \in V} h(x, y) d(x, y)$$

$$\text{s.t. } \sum_{y \in V} h(x, y) = S(x) \quad \forall x$$

$$\sum_{y \in V} h(y, x) = T(x) \quad \forall x$$

$$h(x, y) \geq 0$$

$$V = \{0, 1, 2\} \quad d(x, y) = |x - y|$$

Compute  $h(x, y)$

$x \backslash y$	1	2	Sum
0	$m$	$1/4 - m$	$1/4$
1	$q$	$3/4 - q$	$3/4$
Sum	$3/4$	$1/4$	

$$m + q = 3/4 \Rightarrow m = 3/4 - q$$

$$1 - m - q = 1/4 \Rightarrow 1 - 3/4 + q - q = 1/4$$

$$\min_q \left( \frac{3}{4} - q \right) 1 + \underbrace{\left( q - \frac{1}{2} \right) 2}_{2q - 1} + \left( \frac{3}{4} - q \right) \cdot 1 \Leftrightarrow \min_q \frac{1}{2} = \boxed{\frac{1}{2}}$$

$$\text{s.t. } q \geq 0$$

$$\text{s.t. } q \geq 0$$