18734: Foundations of Privacy

Secure Two-Party Computation

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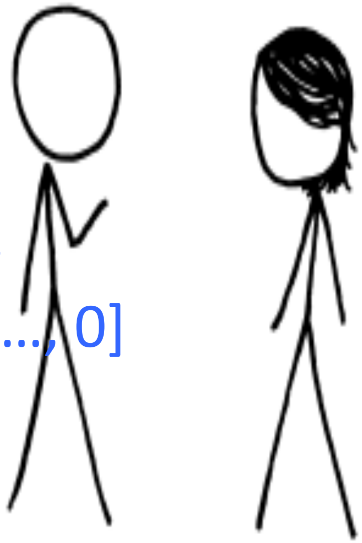
Slide: Evans et al

Secure Two-Party Computation

- Bob's Genome: ACTG…
- Markers (~1000): [0,1, …, 0]
- **Bob**



- Alice's Genome: ACTG…
- Markers (~1000): [0, 0, …, 1]
- **Alice**

$$x = f(g_A, g_B)$$

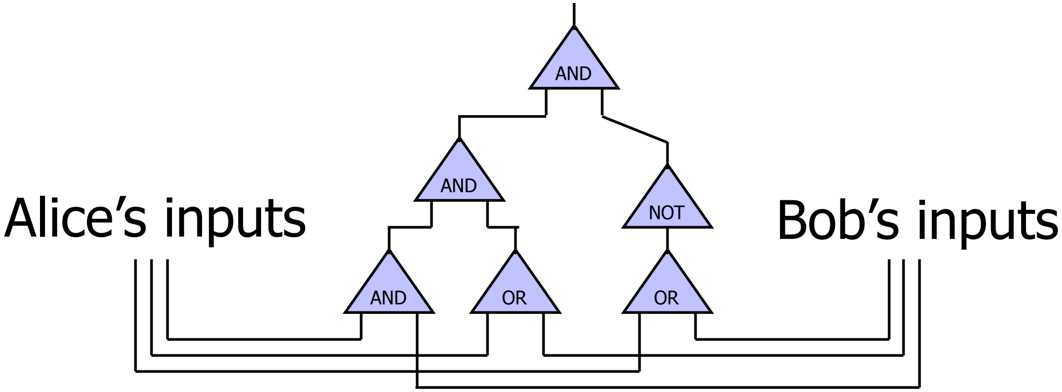
- Can Alice and Bob compute a function of their private data, without exposing anything about their data besides the result?

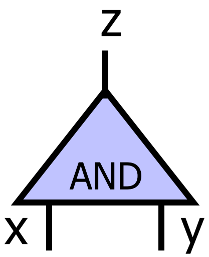
Roadmap

- Yao’s Classic Garbled Circuits
- Recent advances in practical secure two party computations

Yao’s Protocol

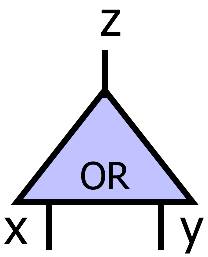
- Compute any function securely
  - … in the semi-honest model
- First, convert the function into a boolean circuit





Truth table:

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

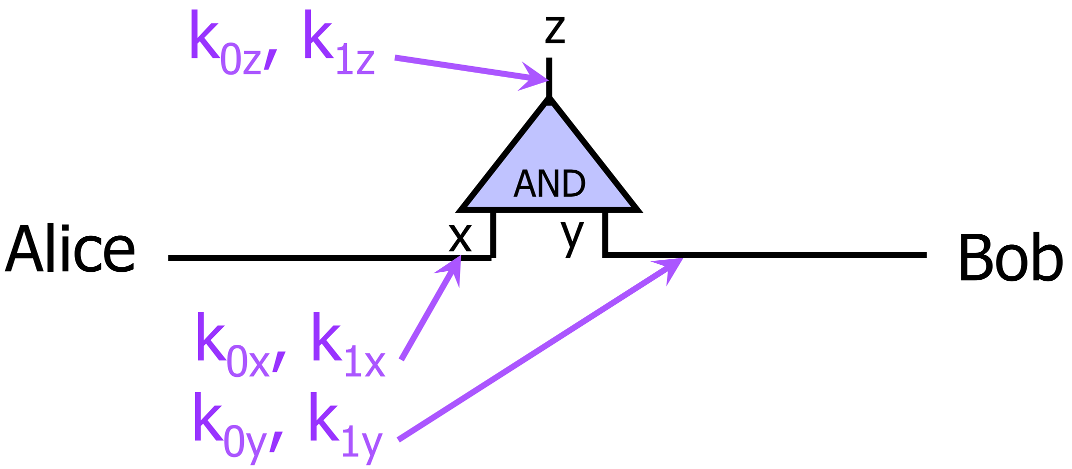


Truth table:

x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

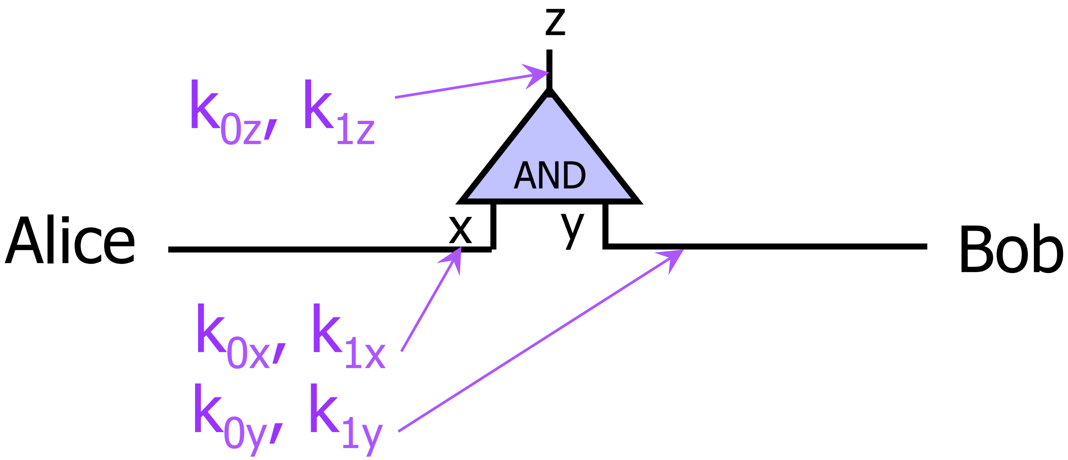
1: Pick Random Keys For Each Wire

- Next, evaluate one gate securely
  - Later, generalize to the entire circuit
- Alice picks two random keys for each wire
  - One key corresponds to “0”, the other to “1”
  - 6 keys in total for a gate with 2 input wires



2: Encrypt Truth Table

- Alice encrypts each row of the truth table by encrypting the output-wire key with the corresponding pair of input-wire keys



Original truth table:

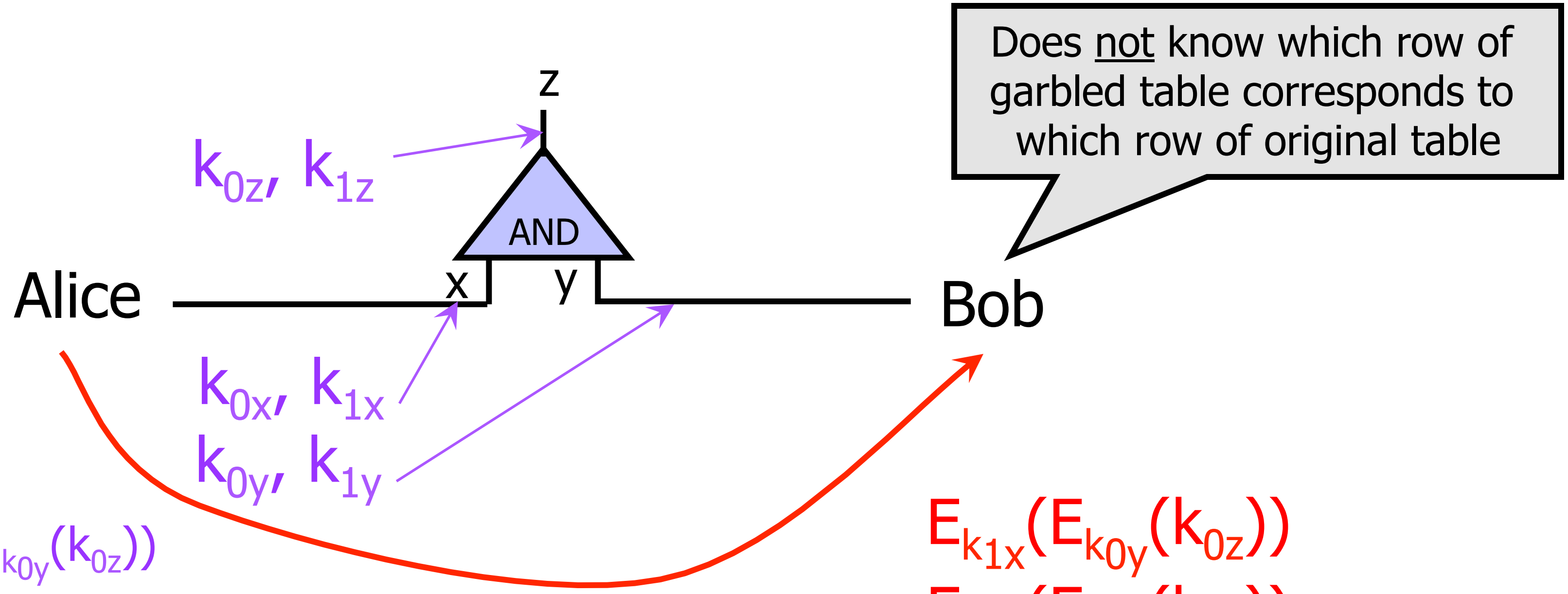
x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

Encrypted truth table:

$$E_{k0x}(E_{k0y}(k_{0z}))$$
$$E_{k0x}(E_{k1y}(k_{0z}))$$
$$E_{k1x}(E_{k0y}(k_{0z}))$$
$$E_{k1x}(E_{k1y}(k_{1z}))$$

3: Send Garbled Truth Table

- Alice randomly permutes (“garbles”) encrypted truth table and sends it to Bob



k_{0z}, k_{1z}

Alice

k_{0x}, k_{1x}

k_{0y}, k_{1y}

Bob

Does not know which row of garbled table corresponds to which row of original table

$E_{k0x}(E_{k0y}(k_{0z}))$

$E_{k0x}(E_{k1y}(k_{0z}))$

$E_{k1x}(E_{k0y}(k_{0z}))$

$E_{k1x}(E_{k1y}(k_{1z}))$

Garbled truth table:

$E_{k1x}(E_{k0y}(k_{0z}))$

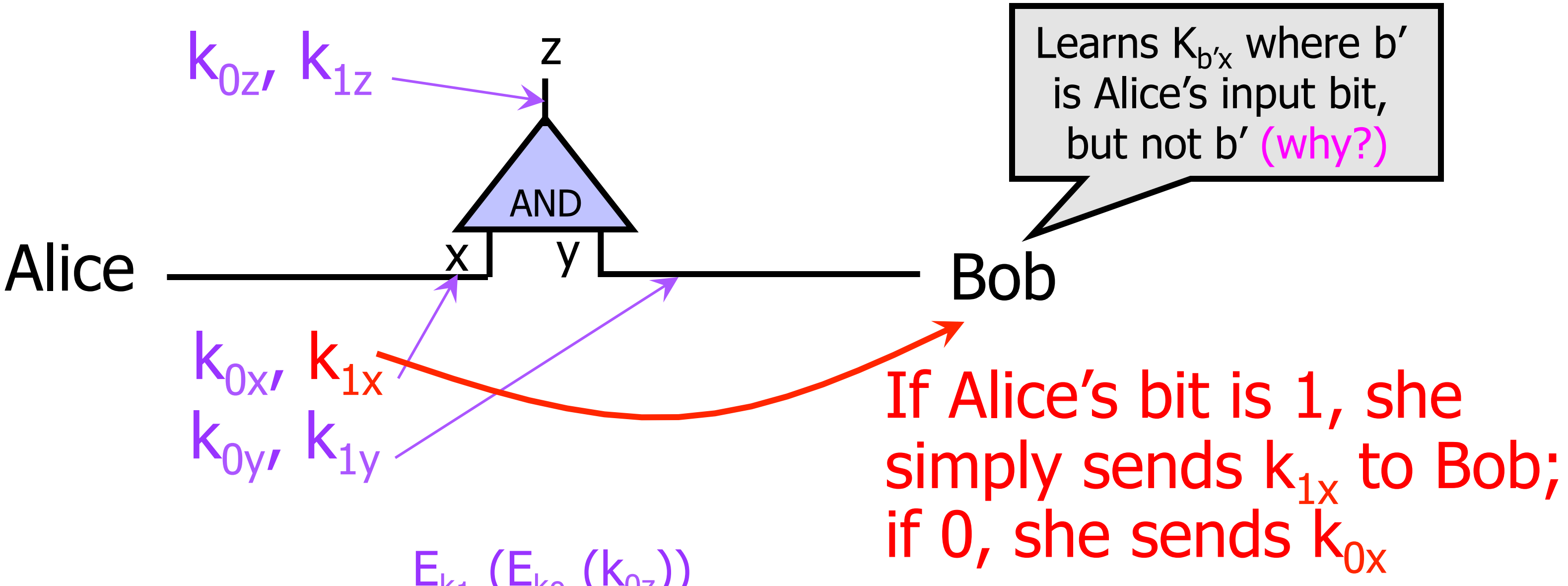
$E_{k0x}(E_{k1y}(k_{0z}))$

$E_{k1x}(E_{k1y}(k_{1z}))$

$E_{k0x}(E_{k0y}(k_{0z}))$

4: Send Keys For Alice’s Inputs

- Alice sends the key corresponding to her input bit
  - Keys are random, so Bob does not learn what this bit is



Learns $K_{b'x}$ where b’ is Alice’s input bit, but not b’ (why?)

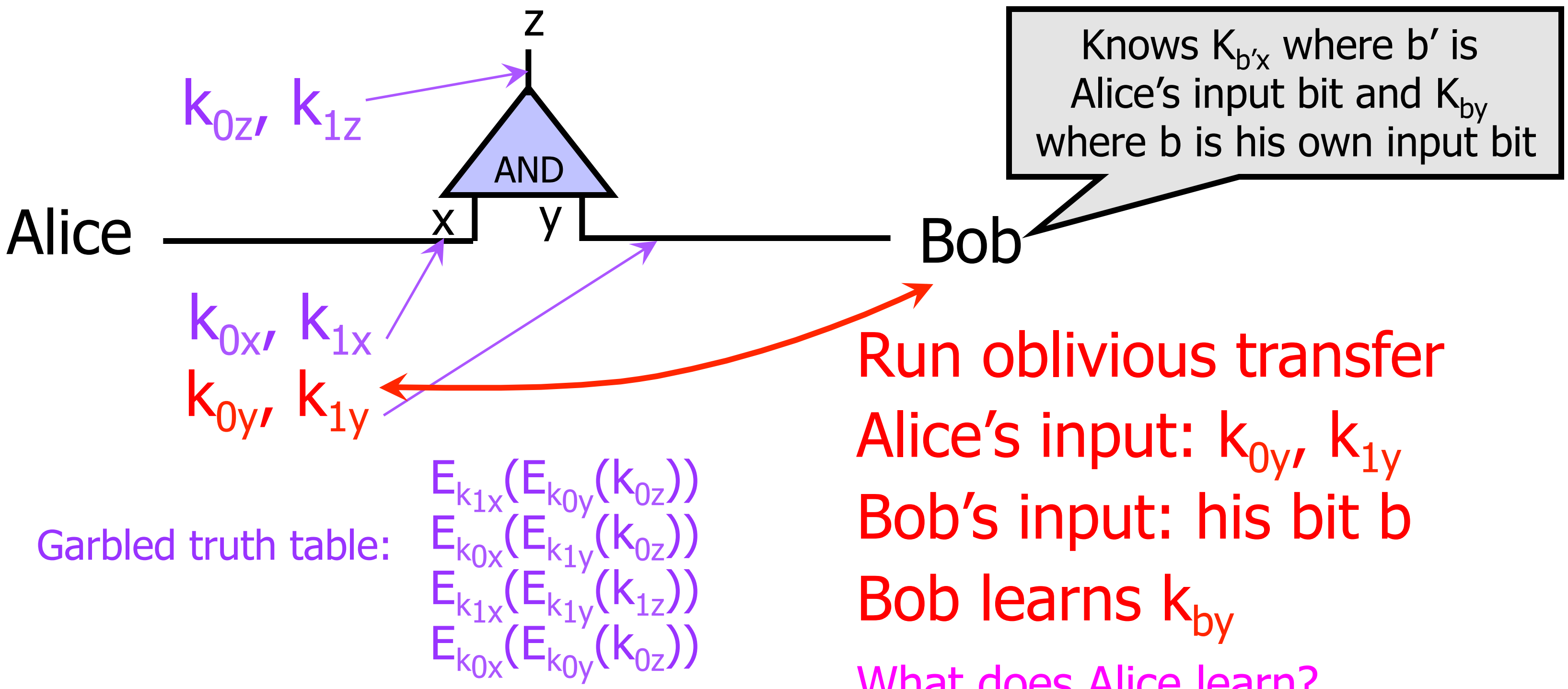
If Alice’s bit is 1, she simply sends k_{1x} to Bob; if 0, she sends k_{0x}

Garbled truth table:

$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$
$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$
$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

5: Use OT on Keys for Bob’s Input

- Alice and Bob run oblivious transfer protocol
  - Alice’s input is the two keys corresponding to Bob’s wire
  - Bob’s input into OT is simply his 1-bit input on that wire



Knows $K_{b'x}$ where b’ is Alice’s input bit and K_{by} where b is his own input bit

Run oblivious transfer

Alice’s input: k_{0y}, k_{1y}

Bob’s input: his bit b

Bob learns k_{by}

Garbled truth table:

$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$

$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$

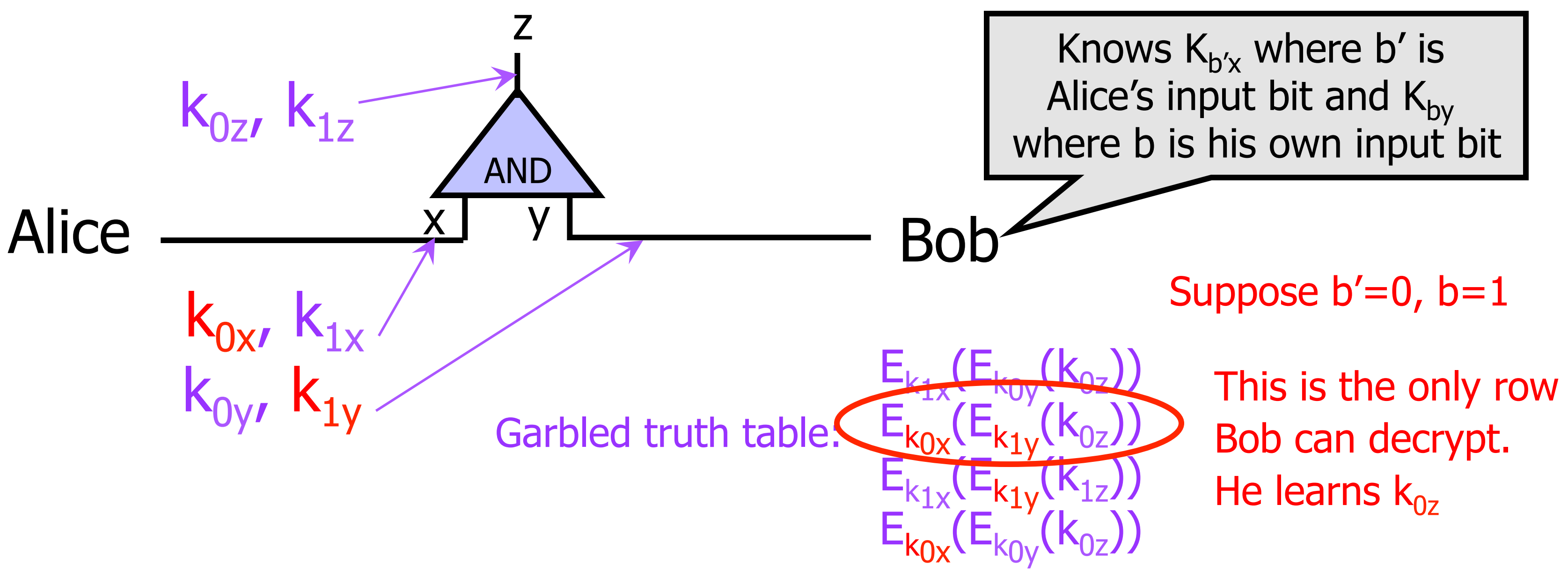
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$

$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

What does Alice learn?

6: Evaluate Garbled Gate

- Using the two keys that he learned, Bob decrypts exactly one of the output-wire keys
  - Bob does not learn if this key corresponds to 0 or 1
    - Why is this important?



Knows $K_{b'x}$ where b′ is Alice's input bit and K_{by} where b is his own input bit

Suppose b′=0, b=1

Garbled truth table:

$E_{k_{1x}}(E_{k_{0y}}(k_{0z}))$

$E_{k_{0x}}(E_{k_{1y}}(k_{0z}))$

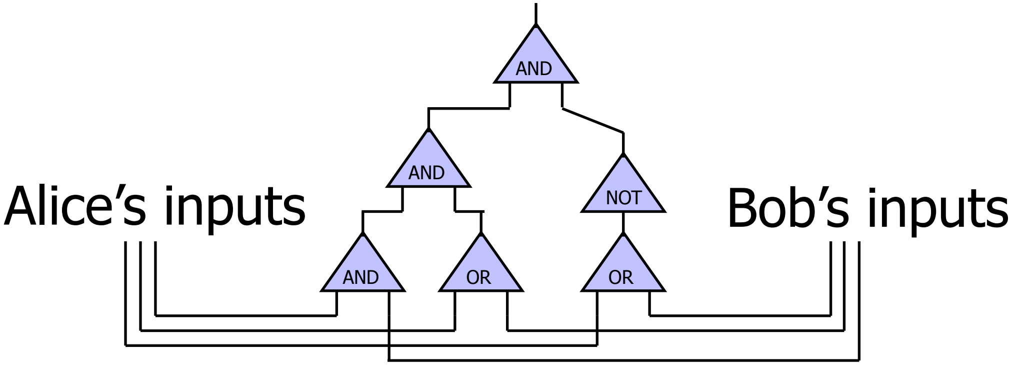
$E_{k_{1x}}(E_{k_{1y}}(k_{1z}))$

$E_{k_{0x}}(E_{k_{0y}}(k_{0z}))$

This is the only row Bob can decrypt. He learns k_{0z}

7: Evaluate Entire Circuit

- In this way, Bob evaluates entire garbled circuit
  - For each wire in the circuit, Bob learns only one key
  - It corresponds to 0 or 1 (Bob does not know which)
    - Therefore, Bob does not learn intermediate values (why?)



- Bob tells Alice the key for the final output wire and she tells him if it corresponds to 0 or 1
  - Bob does not tell her intermediate wire keys (why?)

Brief Discussion of Yao's Protocol

- Function must be converted into a circuit
  - For many functions, circuit will be huge
- If m gates in the circuit and n inputs, then need 4m encryptions and n oblivious transfers
  - Oblivious transfers for all inputs can be done in parallel
- Yao's construction gives a constant-round protocol for secure computation of any function in the semi-honest model
  - Number of rounds does not depend on the number of inputs or the size of the circuit!

Acknowledgments

- Slides 4-12 from Vitaly Shmatikov

Example OT Protocol

Even, Goldreich, Lempel

https://en.wikipedia.org/wiki/Oblivious_transfer