Problem 1

\[ T_b = 100 \text{ m/sec}. \quad \frac{f_b}{T_b} = \frac{1}{100} = 10 \text{ Kbps}. \]

64 levels = 6 bits \(\Rightarrow\) 6 bits/100 m/sec

Each 16-ary symbol can denote 4 bits

Part (a): Symbol rate = \(\frac{60 \text{ Kbps}}{4 \text{ bits/symbol}}\)

= 15,000 symbols/sec.

Part (b): If signal varies between \(-1\) V and \(+1\) V, then 64-level quantization results in

Quantization step = \(\frac{1}{64} = \frac{1}{16}\) V.

Thus the quantization error can be assumed to be uniformly distributed in the interval \((-\frac{1}{32}, \frac{1}{32})\) V.

\[ \therefore \text{Variance} = \frac{(1/16)^2}{12} = \frac{1}{(12)(256)} = \frac{1}{3072} \text{ Volt}^2 \]
Problem 2

\[ \frac{N_0}{2} = 10^6 \text{ Watts/Hz} \]

\[ H_c(f) \]

-2 \hspace{1cm} 0 \hspace{1cm} 2 \hspace{1cm} \text{D} \hspace{1cm} f \hspace{1cm} (\text{kHz})

**Part (a)**

\[ H(f) = H_c(f) H_r(f) = H_c(f) \text{ Sinc } H_r(f) = 1 \]

For no ISI, we must satisfy Nyquist Condition, i.e.,

\[ \sum_k H_c(f - \frac{k}{T}) \]

must be a constant.

From the following figure, we see that \( \sum_k H_c(f - \frac{k}{T}) \)

is a constant if \( \frac{1}{T} = 2 \text{ kHz} \).

\[ \text{If } \frac{1}{T} \text{ larger than 2 kHz, the sum would not yield a constant.} \]

\[ \therefore \text{ Maximal Symbol Rate with no ISI = 2 kHz} \]

**Part (b)**

\[ H(f) = H_c(f) H_r(f) \]

\[ H_c(f) \]

\[ (1 - \frac{|f|}{2}) \]

-2 \hspace{1cm} 0 \hspace{1cm} 2 \hspace{1cm} \text{D} \hspace{1cm} f \hspace{1cm} (\text{kHz})

\[ H_r(f) \]

\[ \frac{2}{2+|f|} \]

-2 \hspace{1cm} -1 \hspace{1cm} 0 \hspace{1cm} 1 \hspace{1cm} 2 \hspace{1cm} \text{D} \hspace{1cm} f \hspace{1cm} (\text{kHz})

\[ H(f) = H_c(f).H_r(f) \]
For Nyquist condition, $\sum_{k} H(f - \frac{f}{k})$ must be constant.

From the above, we see that $\sum_{k} H(f - \frac{f}{k})$ is a constant if $\frac{f}{f} = 3$ kHz.

:. Maximum symbol rate without ISI = 3 kHz.

Part (e)

Noise goes through the receive filter.

Output noise power spectral density $P_n(f) = \frac{N_0}{2} |H_r(f)|^2$

$$P_n(f) = \begin{cases} \frac{N_0}{2} \left(1 + \frac{|f|}{2}\right)^2 & \text{for } |f| \leq 2 \text{ kHz} \\ 0 & \text{otherwise} \end{cases}$$

Variance $= \int_{-2000}^{2000} P_n(f) df$

$$= 10^{-6} \int_{-2000}^{2000} (1 + \frac{|f|}{2000})^2 df = 2 \times 10^{-6} \int_{0}^{2000} (1 + \frac{f^2}{2000} + \frac{f^2}{12 \times 10^6}) df$$

$$= 2 \times 10^{-6} \left[ f + \frac{f^3}{2000} + \frac{f^3}{12 \times 10^6} \right]_{0}^{2000}$$

$$= 2 \times 10^{-6} \left[ 2000 + 2000 + \frac{8 \times 10^7}{12 \times 10^6} \right]$$

$$= (2)(4667) \times 10^{-6} \text{ watts} = 9.334 \text{ mW}.$$
Part (a)

\[ T_b = \frac{1}{4000} \text{ m/s} = 250 \mu \text{sec} \]

Derived transfer function

\[ H_d(f) = \frac{1}{D = e^{j2\pi ft_b}} \]

\[ = 1 + e^{j2\pi ft_b} = e^{j\pi ft_b} \left( e^{j\pi ft_b} + e^{-j\pi ft_b} \right) \]

\[ = 2e^{j\pi ft_b} \cos(\pi ft_b) \]

\[ H_c(f) = \begin{cases} 
1 - \frac{|f|}{2000} & \text{for } |f| \leq 2000 \\
0 & \text{for } |f| > 2000 
\end{cases} \]

\[ H_r(f) = \frac{H_d(f)}{H_c(f)} ; \]

\[ = \begin{cases} 
\frac{2 \cos (0.25 \pi ft \times 10^{-3})}{1 - \frac{|f|}{2000}} & \text{for } |f| \leq 2000 \\
0 & \text{otherwise} \end{cases} \]
**Problem 3**

All three signals have same energy, i.e.,

\[ \int_0^T \mathbf{\hat{s}}_i(t) \, dt = T \quad i = 1, 2, 3. \]

**Part (a)** Use Gram-Schmidt Procedure to get orthonormal basis set \( \varphi_1(t), \varphi_2(t), \ldots, \varphi_N(t) \).

\[ \varphi_1(t) = \frac{\mathbf{s}_1(t)}{\| \mathbf{s}_1(t) \|} = \frac{1}{\sqrt{T}} \mathbf{s}_1(t) \]

\[ \therefore \mathbf{s}_1(t) = \sqrt{T} \varphi_1(t). \]

\[ \mathbf{\hat{s}}_2(t) = \mathbf{s}_2(t) - \langle \mathbf{\hat{s}}_2, \varphi_1 \rangle \varphi_1(t) \]

\[ \langle \mathbf{\hat{s}}_2, \varphi_1 \rangle = \int_0^{T/2} (\mathbf{\hat{s}}_2) \varphi_1(t) \, dt = \sqrt{T} \]

\[ \therefore \mathbf{\hat{s}}_2(t) = \mathbf{\hat{s}}_2(t) - \sqrt{T} \varphi_1(t) = \varphi_2(t) \]

\[ \varphi_2(t) = \mathbf{\hat{s}}_2(t)/\| \mathbf{\hat{s}}_2(t) \| = \frac{1}{\sqrt{T}} \mathbf{\hat{s}}_2(t) \]

\[ \mathbf{\hat{s}}_3(t) = \mathbf{s}_3(t) - \langle \mathbf{\hat{s}}_3, \varphi_1 \rangle \varphi_1(t) - \langle \mathbf{\hat{s}}_3, \varphi_2 \rangle \varphi_2(t) \]

\[ = \mathbf{s}_3(t) + \sqrt{T} \varphi_1(t) - \sqrt{T} \varphi_2(t) = 0 \]

\[ \therefore \text{need only two orthonormal functions}, \quad i.e., \ N = 2. \]

\[ \mathbf{\hat{s}}_1(t) = \sqrt{T} \varphi_1(t); \quad \mathbf{\hat{s}}_2(t) = \frac{1}{\sqrt{T}} \mathbf{\hat{s}}_2(t) + \sqrt{T} \varphi_2(t) \]

\[ \mathbf{\hat{s}}_3(t) = -\sqrt{T} \mathbf{\hat{s}}_1(t) + \sqrt{T} \mathbf{\hat{s}}_2(t) \]
Part (b)

Coordinates are:

\[ \begin{align*}
\mathbf{x}_1 & : (\sqrt{T}, 0) \\
\mathbf{x}_2 & : (\sqrt{T}, \sqrt{T} / 2) \\
\mathbf{x}_3 & : (\sqrt{T}, -\sqrt{T} / 2)
\end{align*} \]

All 3 signals are at the same distance (namely \( \sqrt{T} \)) from the origin indicating that they have the same energy.

Part (c) Since all 3 signals have same energy and are all equally likely, minimum Pe decision boundaries are obtained by drawing perpendicular bisectors as follows. Since their 3 points are on a circle of radius \( \sqrt{T} \) centered at origin, all bisectors go through the origin.

\[ \begin{align*}
\tan^{-1} \left( \frac{1}{1+\sqrt{T}} \right) & = 0.3927 \text{ radian} \\
\tan^{-1} \left( \frac{1}{\sqrt{T}} \right) & = 22.5^\circ
\end{align*} \]
Part (d) First, we need to form $z_i = \int_0^T r(t) \psi_i(t) \, dt$, $i=1,2$.

But since $\psi_1(t)$ and $\psi_2(t)$ are pulses of height $\sqrt{2}$ and durations from $0$ to $\frac{T}{2}$ and $\frac{T}{2}$ to $T$, respectively, we need an integrator that resets to $0$ after every $T/2$ seconds.

If $\tan^{-1}(\frac{2_2}{2_1}) = \phi$ is an angle between $-112.5^\circ$ to $+22.5^\circ$, choose $h_1(t)$.

Between $22.5^\circ$ to $90^\circ$, choose $h_2(t)$.

Between $90^\circ$ to $-112.5^\circ$, choose $h_3(t)$.