Problem 1 (6.1 in Sklar)

\[ g_1(x) = x + x^2 \]
\[ g_2(x) = 1 + x \]
\[ g_3(x) = 1 + x + x^2 \]

All adders are binary adders.

State Diagram

---

input bit 1

---

input bit 0
Problem 2 (6.3 in Sklar)

\[ K = 3 \quad q_1(x) = 1 + x + x^2 \]
\[ q_2(x) = x + x^2 \]

rate = \( \frac{1}{2} \)

--- input bit 0  
--- input bit 1

Tree Diagram

Trellis Diagram
Problem 3 (6.8 in Sklar)

Using the results from Problem 3:

**Modified state diagram**

State equations

\[
\begin{align*}
X_b &= DX_a + DX_c \\
X_c &= D^2 X_b + X_d \\
X_d &= DX_b + DX_e \\
X_e &= D^2 X_c
\end{align*}
\]

Transfer function \( T(D) = \frac{X_e}{X_a} = \frac{D^2 X_c}{X_a} = \frac{D^4 X_b + X_d}{\frac{D^4}{1-D} X_b} \)

\[
\begin{align*}
X_b &= DX_b + DX_c \\
\Rightarrow X_b &= \frac{D}{1-D} X_b \\
X_c &= D^2 X_b + X_d \\
\Rightarrow X_c &= \frac{1}{D} X_b - X_c \\
X_a &= \frac{D}{1-D} X_b - \left[ D^2 X_b + \frac{D}{1-D} X_b \right]
\end{align*}
\]

\[
T(D) = \frac{X_e}{X_a} = \frac{D^4 X_b + X_d}{\frac{D^4}{1-D} X_b} = \frac{D^4 - D^5 + D^3}{1-D - (D^3 - D^3)} = \frac{D^5 - D^6 + D^4}{1-D - D^3 + D^4 - D^2 + D^4 + D^5 - D^6 + \ldots}
\]

So, \( s_f = 4 \).
Problem 4 (6.10 in Sklar)

Figure 6.3 encoder (K=3, rate=\( \frac{1}{2} \))

Received \( Z = (1100001011 \text{ rest } 0) \)

Use the Trellis diagram in Fig 6.7

\[ Z \]

\[ a=00 \]

\[ b=10 \]

\[ c=01 \]

\[ d=11 \]

Decoded sequence 1 0 1 0 0 0

b) If \( m = 10100 \) was the message, then the encoded sequence \( U = 1110001011 \).

Since \( Z = 1100001011 \text{ rest } 0 \),

This bit is in error.
Problem 5 (6.12 in Sklar)

\[
\frac{E_b}{N_0} = 6 \text{dB} \quad \text{coherent BPSK} \quad \text{rate} = \frac{1}{2} \quad \frac{E_c}{N_0} = \frac{r}{2} \frac{E_b}{N_0}
\]

From Section 6.4.1, for the Figure 6.3 encoder we have

\[
\left. \frac{dT(D, N)}{dN} \right|_{N=1} = \frac{D^5}{(1-D)^2}
\]

By equation 6.21,

\[
P_b = Q\left( \sqrt{2 \cdot \frac{D}{d} \frac{E_c}{N_0}} \right) \cdot \left( \frac{dE_c}{N_0} \right) \frac{dT(D, N)}{dN} \left|_{N=1} \right. = e^{-1}\frac{E_c}{N_0}
\]

From Section 6.4.1, \(df = 5\).

\[
\frac{E_c}{N_0} = \frac{1}{2}(6 \text{dB}) = 3 \text{dB} \approx 2.
\]

\[
P_b = Q\left( \sqrt{2 \cdot 5 \cdot 2} \right) \cdot (5 \cdot 2) \cdot e^{-1}\frac{E_c}{N_0} \cdot (1-2\cdot e^{-2})^2
\]

\[
= Q\left( \sqrt{50} \right) \cdot e^{-10} \cdot (8.0535 \times 10^{-5})
\]

\[
= Q(4.47) \times 1.88
\]

\[
\approx 0.00447 \times 1.88 = 7.6 \times 10^{-6}
\]

b) Uncoded:

BPSK: \(P_b = Q\left( \sqrt{2 \frac{E_b}{N_0}} \right) = Q\left( \sqrt{2 \cdot 3.98} \right) = Q(2.82)\).

From Table B.1, \(Q(2.82) = 2.4 \times 10^{-3}\)

Performance Improvement = \(\frac{2.4 \times 10^{-3}}{7.6 \times 10^{-6}} = 315.8\)
Problem 6 (6.17 in Sklar)

rate = $\frac{2}{3}$ state contents are rightmost
$k = 2$  $K-1 = 1$ 2-bit registers
$K = 2$

Since 2 input bits come in at a time, there are 4 outgoing branches from each state. We show the input in parentheses.

Tree Diagram

Trellis Diagram

State Diagram