Problem 1 (Prob. 2.8 from Sklar)

a) From equation 2.20,
\[
\left( \frac{S}{N} \right)_q = 3L^2 \geq 30 \text{dB}
\]

\[
10 \log_{10} (3L^2) \geq 30 \text{dB} \iff 3L^2 \geq 1000
\]

\[L = 18.257, \text{ so } \boxed{L=19} \text{ levels are needed.}
\]

Minimum bits per sample:

\[L = \log_2 L = \log_2 19 = 4.2848, \text{ so } 5 \text{ bits/sample needed.}
\]

b) Time to transmit a bit \(T_b = \frac{T_s}{2} = \frac{1}{2f_s} = \frac{1}{5(8000)} = 2.5 \text{ microsec}
\]

Bandwidth \(W = \frac{1}{T_b} = \frac{1}{25 \text{ micro}} = 40 \text{ kHz}\)

**Bandwidth** \(W = 40 \text{ KHz}\).
Problem 2 (Prob. 2.11 from Sklar)

\[ x(t) = \cos(2\pi f_0 t) \]

\[ x(f) = \frac{1}{2} [ \delta(f-f_0) + \delta(f+f_0) ] \]

\[ f_s : \text{Sampling frequency} \]

\[ x_s(f) = f_s \sum_{n=-\infty}^{\infty} \delta(f-nf_s) \quad f_s = \frac{3}{2} f_0 \]

FT of sampled signal

\[ f_s - f_0 = \frac{3}{2} f_0 - f_0 = \frac{f_0}{2} \]

If we use an ideal LPF with gain \( \frac{1}{f_s} \) in the range \(-\frac{f_s}{2}, \frac{f_s}{2}\), reconstructed signal will be \( \cos(2\pi (f_0/2) t) \), half the original frequency.
Problem 3

Using equation 2.41, let \( a_1 = 1 \) and \( a_2 = -1 \).

a) \( P_B = Q \left( \frac{a_1 - a_2}{\sigma_0} \right) = Q \left( \frac{1 - (-1)}{2(0.05)} \right) = Q(1, 0) \). From Table B.1, \( P_B = 0.1587 \).

b) \( P_B = Q \left( \frac{1 - (-1)}{2x(0.01)} \right) = Q(100) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx \approx 0 \)

For large \( x \), \( Q(x) \approx \frac{1}{x \sqrt{2\pi}} e^{-\frac{x^2}{2}} \) \hspace{1cm} \text{Eq. (2.43)}

\[ P_B = Q(100) \approx \frac{1}{100 \sqrt{2\pi}} \exp(-5000) \]

Problem 4

(proposed 2.15 from Sklar)

\( P_B = Q \left( \sqrt{\frac{2A^2 - 1}{N_0}} \right) \)

\( A^2 = 1 \) for bipolar (+1V, -1V).

\( N_0 \) \( \frac{T}{2} \) = two-sided noise PSD = 10^{-2}

See Eq. 2.67

Find \( x \) such that \( Q(x) \leq P_B = 10^{-3} \).

From Table B.1, \( Q(x) \leq 10^{-3} \) \( \Rightarrow x = 3.08, 3.09, 3.10 \).

Since we want highest bitrate, choose smallest \( x = 3.08 \).

\( \sqrt{\frac{T}{10^{-3}}} = 3.08 \) \( \Rightarrow T = 9.486 \times 10^{-3} \)

bit rate = \( \frac{T}{5} = 105.414 \text{ bits/sec} \)
18-550 HW#2 Solutions

Problem 5 (Problem 2.20 from Sklar)

Want bandwidth $W = \frac{1}{2T} = \frac{R_s}{2} \text{ Hz}$, $R_s$ # symbols/sec

# levels $L = \frac{1}{2p}$, $p = 0.001$ $\Rightarrow$ $L = 500$ levels

# bits/PCM word $L = \log_2 L$ $\Rightarrow$ $L = 9$ bits/PCM word

bit rate $R_b = \frac{9 \text{ bits}}{\text{sample}} \cdot \frac{8000 \text{ samples}}{\text{sec}} = 72000 \text{ bits/sec}$

$R_s = \frac{R_b}{\log_2 M} = \frac{72000}{\log_2 32} = 14,100$ symbols/sec

So, $W = \frac{R_s}{2} = 7,100 \text{ Hz}$