Towers of Hanoi
18578

Team C – ¡Los Murcielagos!
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Abstract

This report details the development and final implementation of the Towers of Hanoi solver machine. It includes written descriptions of the major subsystems, algorithms and functionality, as well as mechanical and electrical diagrams and schematics and flowcharts of the software. In addition, there is a lengthy discussion of the problems encountered and their solutions, as well as where improvement could have been made. A number of photographs of the final system are included in the appendix at the end.
Description

The overall task for this project was to build a machine that physically solves a given Towers of Hanoi puzzle set, according to the standard rules for the game: move one disc at a time, only placing a disc on top of a larger one. The goal is to transfer a stack of discs from one of three pegs to another, using the third peg for intermediate moves, with any peg arrangement.

For this project, our systems had to meet additional constraints for size; 2’x2’x2’ limitation, and for speed; a 3 time limit. We were given a refundable budget of $200, with an extra $100 non-refundable to spend. Each group was given a set of identical (within tolerances) pegs and discs that we were not allowed to modify in any way. Although the set included a wooden base to hold the three pegs, we were not required to use it; we could mount the pegs any way we wanted. On top of those basic requirements, we had to come up with a “coolness factor.” Ours was a “manual mode” where the user can control the machine through a computer serial connection.

Our system consists of three major components: the turntable, the elevator and the claw. The three pegs are mounted to the turntable, equally spaced. The elevator is outside the perimeter of the turntable, and it lifts the claw up and down so that it can manipulate the discs. Controlling everything is a PIC 16F877 microprocessor. Each of the subsystems and the software will be elaborated upon in the following sections.

Most of the system is enclosed in a 2’x2’x1’ box made of 1/8” clear acrylic sheeting, which we made from AutoCAD drawings using an automated laser-cutter. There is a platform roughly two-thirds of the way up where the turntable sits. The elevator fixture sits on the floor, and extends through the platform and above the top of the box to the 2’ maximum height. The electronics are below the platform, out of the way of all moving parts.

Desired Functionality

As outlined above, we have three main subsystems: the turntable, the elevator and the claw. In this section, the functionality of each will be described in detail. These descriptions are augmented by diagrams and schematics in the following sections.

The basic functionality of our system is straightforward. For each move, six things happen: the turntable rotates the source peg into position under the claw, the elevator lowers the claw, the claw closes under the top disc, the elevator lifts the claw (now holding the disc) above the peg onto the guide rod, the turntable rotates the destination peg into position and the claw releases the disc.

Turntable

In the center of our system is the turntable. The three pegs are mounted to it 120 degrees apart, and it rotates the correct peg into position under the claw for each move. It is actuated by a motor at the center, connected through an aluminum hub that fits onto the output shaft of the motor and is attached to the turntable by a screw. To ensure smooth rotation, the turntable sits on a ball bearing track around the edge. We had mixed results with the ball bearing track; it supports the
weight well, and the rotation is mostly very smooth, but there are certain areas where it has significantly more friction, and that leads to sporadic problems.

Alignment of the turntable is controlled by an infrared detector corresponding to each peg, and one infrared source fixed underneath the turntable. This method has advantages and disadvantages; it provides an absolute way of determining when the peg is in the correct alignment, but it is more difficult to correct for problems like overshoot compared to, for example, encoder-based control. When the detector is aligned with the source, the turntable is definitely aligned correctly (assuming that the sensor was physically mounted in the correct place), problems like gear backlash or skipping will not throw the sensor off like they would an encoder. It is also very simple to write the software for the detector/emitter method, as opposed to a complicated PID algorithm. The difficulties we had with this method are detailed in a later section, but despite the problems we had the sensors worked very well for ensuring that the turntable got to the correct position.

Elevator
The purpose of the elevator is to raise the claw up and down once the correct peg has been rotated into place by the turntable. It is actuated by a motor attached to a lead screw, on which a threaded hub is fitted. The hub is linked to a platform that slides up and down on two guide rods; this is where the claw is mounted.

To determine when the elevator has raised high enough or lowered down enough, we use limit switches. One is mounted on the frame that holds the lead screw, and the other is attached to the claw. It hangs out over the claw and it is activated by the top disc on the peg. The limit switch is mounted at the precise height so that when it is activated the gripper is aligned with the bottom of the top disc. The gripper design accounts for the varying thicknesses of the different sized discs.

Claw
The claw is how our system interacts with the discs. It has two thin rectangular pieces of aluminum that slide under each side of the disc when it closes. Since the top and bottom of each disc is rounded, the claw can slide into place very easily even if it is not at the perfect height. This allows for the use of a simple limit switch to detect the top disc as described in the Elevator section. Once the claw has closed under the top disc, it is raised by the elevator until the disc is off the peg. To make sure that the disc stays aligned, we mounted a tapered wooden stick above the claw such that it is aligned with the peg, this way the disc is lifted off the peg and onto the guide rod to keep it in position so it can be dropped right onto the destination peg.

We bought the claw as a unit, and modified it to fit our specific needs. A servo is used to open and close the claw; this was included with the claw kit when we bought it. We went through one major design change; originally we were going to use the fingers that came with the claw to grab the discs, but there were a number of problems associated with this, so we cut off the fingers and attached the thin aluminum pieces on instead. The revisions are elaborated on later.
Coolness Factor – Manual Mode
Our system also implements a special “manual” mode, which we deem to be one of our coolness factors. Once the machine aligns itself to Peg 1, the code prompts the user (via HyperTerminal) to pick an operation mode: manual or automatic. If the user selects manual mode, a menu displays that lists three options: (T)urntable, (E)levator, and (G)ripper. Depending on what key the user enters, a different sub-menu appears. If the user enters ‘T’, the following options appear: Align to Peg 1, Align to Peg 2, Align to Peg 3, and ‘Q’uit to main menu. If the user selects an alignment peg, the machine will call the corresponding “move to” function. If the user enters the Elevator sub-menu, he may either raise or lower the elevator. The elevator will automatically stop moving once a limit switch engages. Finally, if the user selects the Gripper option, the possible choices are open and close.

The manual mode does not keep track of the user’s moves and will not attempt to correct/stop any illegal moves.

Since the manual mode calls the same functions used in the automatic mode, it turned out to be exceptionally useful for debugging. It allowed us to quickly check the alignment of the pegs as well as other issues with the three subsystems. Not only was it useful, this mode turned out to be much more fun (and cool) than originally expected.

Block Diagram
The block diagram above shows the high level algorithm of our system. Starting with the user input in yellow, and proceeding into the main algorithm loop in blue where the PIC cycles through the array of moves computed ahead of time (green) until the game is completed.

The block diagram above shows the connections between the PIC, the H-bridge motor driver board and the motors themselves. Only one of the PIC’s PWM outputs is needed since only one motor will be on at a time.

**Mechanical Schematics**

The figure above shows an isometric view of our system. It is not entirely accurate or complete in terms of the final project, but the major subsystems are represented. The turntable is clearly visible in the center of the box, with the three pegs attached to it. In one corner is the claw, attached to a simplified version of the lead screw driven elevator. Not shown is the turntable motor in the center, or the guide rod above the claw. This AutoCAD drawing is just an overview of the general subsystem breakdown.
This figure shows a cross section of the turntable and the ball bearing track. Although not to scale, it gives an accurate representation of the different layers of acrylic and how the ball bearing track is constructed. Each layer of acrylic is 1/8” thick, and the ball bearings are 3/16” in diameter.

The above diagram shows the gripper in the fully-raised position, with the upper limit switch (Switch 1) activated. Switch 2 has already been activated by the disc, which has now been picked up off the peg. For clarity, the structure supporting the guide rod is not shown in this picture.

**Electrical Schematic**

Aside from the motors and switches, the only electrical components in our system are the infrared sensors and the emitter. The circuits for the motors and switches are so simple that the block diagram view is adequate, but the sensors and how they interface with the PIC’s analog inputs are somewhat more complicated.
The figure above shows the schematic for an IR emitter/detector pair, and how where the PIC analog input is read from (voltage across the resistor in series with the IR detector). Three of the detector circuits are built on the circuit board area of the PIC board, with the detectors attached with long wires (the resistors are on the board). The single emitter is wired directly between +5V and ground, with the resistor attached at the LED.

Software Flowchart

Main Flowchart
The software overview flowchart above gives a high-level description of the code. After being released from reset, the program sits in an idle state until the user hits a key (all inputs are done through HyperTerminal via the serial line). Once the user hits a key, the machine initializes and aligns to peg 1. The user MUST have previously aligned the turntable to be anywhere between Pegs 1 and 3 (we chose specifically at Peg 2 for simplicity and repeatability).

The machine rotates the table clockwise until it reaches the IR sensor corresponding to Peg 1 and stops (Flowchart A below describes this in more detail). Once it aligns itself, the code prompts the user and asks if he would like to enter "Manual" mode or "Automatic" mode. Manual mode is part of our "coolness" factor which lets the user order the individual subsystems to actuate. This proved to be considerably more fun than originally expected and also came to be quite useful in debugging. Automatic mode puts the code into the autonomous mode where it will
solve the Towers of Hanoi puzzle independently.

If automatic mode is chosen, the user is prompted for the number of disks, start peg, and end peg. Once the user enters the information, he is asked to push the Ultra Red Switch (URS) to begin operation. This is the last action the user is required to take, unless he wants to halt operation at any point (at which he can turn off the URS).

Manual mode presents the user with a menu for actuating each individual subsystem. The options are: (T) Turntable, (E) Elevator, (G) Gripper. Each option also has its own sub-menu that enumerates the available motions. The blocks with (*) next to them call the methods from Flowchart B (below).

Flowchart A
The Flowchart above expands on the initial alignment to Peg 1. Once the table has been manually aligned to Peg 2, the system begins taking readings from the IR sensors using the ADC input. The PIC then sets the PWM duty cycle to 95% and commands Motor E (Elevator) to raise the arm. Once the upper limit switch is reached, Motor E is commanded to stop. Next, the PIC sets the PWM duty cycle down to 60% and drives Motor T (Turntable) clockwise until the IR 1 sensor reaches the threshold reading. Motor T stops once this point is reached. However, since overshoot always occurs, the PIC then reverses Motor E at a slower PWM duty cycle (50%), until the IR sensor triggers again. At this point, the PIC has successfully aligned the turntable to Peg 1.
Flowchart B goes into the details of how each move occurs. As a predecessor to the cycle, the code calls the corresponding function (dependent on number of disks) that fills the two arrays which hold the moves. There are two parallel arrays, once that holds each move's source peg and another which holds the each move's destination peg. Once the two arrays are generated, the movement can begin. The first step is to align to the source peg. The correct direction is calculated based on the current Peg position. The simple algorithm described in the block allows us to support the multiple peg configuration requirement. Once the direction is chosen, the PIC drives the turntable motor until it reaches the corresponding IR sensor. We encountered quite a bit of overshoot (which varied depending on which peg we were coming from), so we had to reverse afterwards. We reverse at a slightly lower speed and when the IR sensor re-engages, we pulse brake in the opposite direction for a fraction of a second (50 us).

The PIC then lowers the elevator until the disk limit switch touches the topmost disk. At this point the claw closes (by setting the duty cycle of the servo to 20%) and the claw arms close beneath the topmost disk. The PIC then commands the elevator to raise via Motor E's PWM signal until the upper limit switch engages. The movement path is then repeated towards the destination peg (this uses the same function call from before but with different parameters). Once
the alignment is complete, the claw opens (servo duty cycle set to 85%) and the disk (usually) falls into place on the corresponding peg. The "currentPeg" variable is updated after every 'moveToPeg' call. Also, the number of current moves increments after the "open claw" command drops the disk. If this number reaches the total number of moves required to solve the puzzle, the PIC orders all motors to stop and a "Finished" message displays in the HyperTerminal window.

**Parts List**

<table>
<thead>
<tr>
<th>Q</th>
<th>Brand</th>
<th>Model</th>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>-</td>
<td>-</td>
<td>1/8&quot; Acrylic sheets used for walls, supports, and turntable mechanism</td>
<td>$20 (Estimate)</td>
</tr>
<tr>
<td>200</td>
<td>-</td>
<td>-</td>
<td>3/16” Ball Bearings used underneath the turntable to facilitate rotation</td>
<td>$8</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>Aluminum motor mount was assembled from scraps scavenged from the lab</td>
<td>$5 (Estimate)</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>Wooden IR emitter mount was assembled from scraps found in the lab</td>
<td>$5 (Estimate)</td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>Aluminum elevator assembly was assembled from scraps found in the lab (this includes an ACME lead screw and stainless steel guide rods)</td>
<td>$30 (Estimate)</td>
</tr>
<tr>
<td>1</td>
<td>Ultra</td>
<td>WIN-300PS</td>
<td>ATX Switching Power Supply – 300W DC output.</td>
<td>$35</td>
</tr>
<tr>
<td>1</td>
<td>Jameco</td>
<td>155862</td>
<td>DC gearhead motor used for our turntable. 60:1 Gear Ratio. 4.5V to 12V DC operating range.</td>
<td>$23</td>
</tr>
<tr>
<td>1</td>
<td>Maxon</td>
<td>47-040-038-00-19-107</td>
<td>24VDC precision gear motor. The output speed of the 6:1 ratio spur gear head is about 660 RPM at 24V DC.</td>
<td>$30</td>
</tr>
<tr>
<td>3</td>
<td>Ligitek</td>
<td>LPT2023</td>
<td>NPN Silicon Phototransistor LED Lamps (IR Detectors) used in conjunction with the IR emitter for alignment of the turntable</td>
<td>$0.30/each</td>
</tr>
<tr>
<td>1</td>
<td>Ligitek</td>
<td>LIR204X</td>
<td>Infrared Emitting Diode used to align the turntable.</td>
<td>$0.30</td>
</tr>
<tr>
<td>1</td>
<td>Budget Robotics</td>
<td>“Big Gripper”</td>
<td>Big Gripper measures 6” long, 2 3/4” wide, and 1 1/4” thick (with servo the total thickness is 2”). Weight with servo is 3.7 ounces. Lifting capacity is 8-12 ounces, but is conservatively limited by the wrist mechanism you use, if any. Full-open to full-close of the fingers requires approximately 90 degrees of servo rotation.</td>
<td>$25</td>
</tr>
<tr>
<td>2</td>
<td>-</td>
<td>-</td>
<td>Limit switches acquired from Tech Electronics to set bounds for our elevator assembly. The upper one was placed at the appropriate “top limit” height on the bottom one was placed at a height where it would allow the gripper to lift any type of disk.</td>
<td>$4 (Estimate)</td>
</tr>
<tr>
<td>1</td>
<td>Radioshack</td>
<td>275-601</td>
<td>20A 12VDC toggle switch with safety cover. 3-prong SPST. Used to start and stop our automatic mode.</td>
<td>$5</td>
</tr>
</tbody>
</table>
Performance Evaluation

Our system met all of the specifications with the exception of the time limit. Depending on the peg configuration chosen, a four disc game took between 3:40 and 3:50 minutes. There are two big factors that slowed us down.

The first is that the lead screw we used had a pretty fine thread. We could have corrected this problem either by using a different lead screw, one with coarser threading, or increasing the voltage to the lead screw motor. The motor is rated for up to 24V, but because we had both motors running off the same H-bridge, we only had about 16V available since the turntable motor was rated lower. We could have used a separate H-bridge board for each motor and run the lead screw motor at its maximum 24V to increase the speed.

Although our turntable is fairly fast, it overshoots the correct position every time, and it goes back slowly to correct for this. This accounts for a little bit more lost time that could be improved by using a slower, higher torque motor to eliminate the overshoot and/or refining the ball bearing track to eliminate more of the resistance so that a lower PWM could be used to slow down the motor.

Apart from the speed, our system works well. The turntable alignment is very good and consistent, and the claw picks up the all the discs very easily.

Experimental Results

In the course of our testing, we did not really make any quantitative measurements, apart from timing the system on several runs. As stated earlier, the final times were between 3:40 and 3:50. In a typical run of the final system, there may be one or two small failures, where a bump is needed. The turntable may get stuck or the alignment might be off just a little bit, but nothing major.

Problems and Solutions

Over the course of the semester, we ran into quite a few problems, most of which we were able to solve. The first major problem was with the turntable. Originally we wanted to use a ring gear on the inside of the turntable but that turned to be impractical for two reasons. The first is that it was almost impossible to find a source for buying ring gears, and when we finally found one it was outrageously expensive. We tried to cut teeth into the turntable piece with the laser-cutter and we found that there was too much friction between the outside edge of the turntable piece and the inside edge of the platform; it could barely turn at all. We noticed this problem even in our mockup, but we mistakenly thought that the ball-bearing track would help alleviate it. To fix this problem, we just attached the turntable directly to the motor shaft. This resulted in the turntable being significantly faster than we expected, even once we switched to a motor with a higher gear ratio, and that caused more problems.
Since the turntable ended up being so fast, we had significant overshoot problems. We tried to fix this by lowering the PWM duty cycle to the turntable motor, but it had to be nearly 100% to be able to overcome the friction in some areas of the ball bearing track. We reduced the problem by replacing the original motor with one that had twice the gear ratio – half the speed and twice the torque, but we found that even with that overshoot was still a problem. To correct it, we implemented a backtracking routine in the software, so that the turntable turns in the opposite direction as slow as possible until it is aligned again, and then a hard brake is applied. This was very effective and nearly always got the peg aligned well enough.

As mentioned briefly in several places already, the ball bearing track had some issues. Due to inconsistencies in the track width and maybe the platform being slightly warped, some areas of the track had binding problems that caused significant resistance to rotation. We tried to fix this by applying first silicone-based light oil and, after that failed, heavier lithium grease. Neither one worked; we never found a real solution to this problem.

There were two major problems we ran into with the claw. The first was that the PWM output of the 16-series PIC is not capable of having a slow enough period for the servo. We tried to make the built-in PWM functionality work for a long time, but it was not possible. Instead, we used one of the timers and a digital output pin to make our own PWM generator. This worked perfectly, and with some trial and error we found good positions for the servo to open and close the claw.

The other problem we had with the claw was that in its original configuration, it was not able to pick up the discs very well. This was mainly because it could not open wide enough, which we fixed by sanding down the physical stops on the finger pieces. Our original plan was to grab the discs by the edges, but this proved difficult because of the shape of the grippers. We would also have had to precisely control how much to close the claw for each disc, since they are different sizes. Just clamping down on each of the discs probably would have turned up the servo from stalling it so much. Our redesigned claw has thin sheets of aluminum that slide under the disc and lift it off the peg. This makes picking the discs up more consistent and much easier on the servo, and takes the disc of the disc out of the equation entirely.

Conclusions

The main thing that we would do differently is to spend a lot more time planning than we did, mainly by designing the whole thing in AutoCAD or SolidWorks. If we had thought out the whole project earlier, we would have seen a lot of the problems we ended up having later in the project. One of the first things we would have realized was that the whole thing could have been significantly smaller than it was. This would have helped just by making things more manageable and decreasing the weight of the turntable allowing it to turn more easily.

It would have made the project a lot easier if we had built everything such that we could replace parts more easily, the turntable motor especially. We really needed to replace the motor we were using with a much slower one, but it was nearly impossible due to the poorly-thought-out way that we had built the structure supporting it. This goes hand in hand with another change we would make; choosing the right components the first time around. We went through a number of
motors for both the turntable and the elevator without really testing them or doing any kind of scientific analysis to determine which would be best.

Building our turntable with more attention to detail and tolerances would probably have gone a long way towards eliminating the binding problems we had with the ball bearing track, which would have helped reduce other problems, as mentioned earlier.

A lot of the changes listed above would be made easier if we had taken some time to learn a little about machining, either through the Mechanical Engineering class offered or the Robotics Club. That was one thing we had a lot of trouble with, building things. We mostly relied on finding appropriate scraps around the lab. It worked pretty well, most of the time; we were luck to find the major pieces of our elevator in the lab built already. But it would have been nice to machine our own couplers, for example.

One change that would have required a lot of work, but probably really help would be to add an encoder to our turntable motor. Using both the infrared sensor/emitter method and an encoder would have allowed us to exploit the advantages of both systems to have a very accurate and robust control system.

Overall, our design was relatively easy to implement; we did not have any complex sensor systems and software algorithms, and the mechanical design was also straightforward. It would have been significantly easier if we had some machining experience and capabilities, though, which may have left us more time to expand on some of the other areas like the turntable control system. We ran into a lot of problems late in the project, though, and these were very time consuming to fix.

**Recommendations for Future Work**

If we were to start over, our design would not change all that much. We would incorporate the ideas discussed in the previous section, and especially make it smaller if this was going to be some kind of commercial product. We would also change the software so that it can handle larger numbers of discs; we would implement an iterative solution to handle at least up to six discs, or make a lookup table and use different sections of it for few numbers of discs than the maximum.

**Citations and References**

http://www.ece.ubc.ca/~elec474/reports/mar02/group4.pdf - information servo period and duty cycle
Appendix – Additional Pictures

Rendering from an animation to illustrate the infrared sensor/detector system

A close up of the claw and the guide rod assembly
The lead screw

A close-up of a peg aligned with the guide rod
The Ultra Red Switch (URS)

The ball bearing track, an IR detector and the IR emitter

Overhead view of the entire system