1) Consider a thin film sample of material with its surface normal to the z-direction and infinite in extent in the x and y-directions. When a field are applied in the x-direction are the following M(H) loop is observed.

Consider now that a spatially uniform field is applied to the sample and the surrounding region sufficient to saturate it, as shown below.

a) Give the value of M, H, and B at points P and Q for these cases.

b) What if the applied field is reduced to half of its value in a). Again give the values of M, H and B at points P and Q.

Make sure to specify units, and work in the SI system.
2) Consider a cylindrically magnetized core with the magnetization:

\[
\mathbf{M} = M_0 \frac{r_1}{r} \left[ \Theta(r - r_1) - \Theta(r - r_2) \right] \hat{\mathbf{\theta}}
\]

where \( \hat{\mathbf{\theta}} \) is a unit vector in the circumferential direction (everywhere perpendicular to the radial direction), and \( r \) is the coordinate in the radial direction. Note that the magnetic material extends from the inner radius of the core, \( r_1 \) to the outer radius, \( r_2 \).

a) Calculate the “magnetic” current density, \( j_{\text{mag}} \) everywhere in space, by taking a curl of the appropriate quantity in cylindrical coordinates.

b) Using the result in a) find the “magnetic” current, \( i_{\text{mag}} \), within the line contour at radius, \( R \) (shown as a dashed line in the diagram) by integrating over the appropriate area.

c) Combine the result in b) with amperes law and the definition of \( B \) to show the following result for this geometry (which is also true in the general case):

\[
\oint_C \mathbf{B} \cdot d\mathbf{\sigma} = \mu_0 \left( N i_{\text{free}} + i_{\text{mag}} \right)
\]

where the line contour, \( C \), over which the integration is done is, as above, the dashed line at radius, \( R \), and \( d\mathbf{\sigma} \) is a differential element of the line contour, \( C \).