Some Useful Magnetic Units, Equations, and Definitions (mksa)

\[ B = \mu_0 (H + M) \]  \hspace{1cm} \text{Eq. 1a}

- \( B \) \text{ Magnetic flux density} (Wb/m}^2\), webers per square meter), equal to the flux per unit area \( \Phi / A \).
- \( \mu_0 \) \text{ Permeability of free space} (constant, \( 4\pi \times 10^{-7} \) H/m, henries per meter).
- \( H \) \text{ Magnetic field} (A/m, amperes per meter), produced by currents in conductors and equivalent currents in magnetic materials.
- \( M \) \text{ Magnetization} (A/m) or \text{magnetic moment per unit volume} (m/V).

Eq. 1a is sometimes written in one of the following forms
\[ MHB = 0 \]  \hspace{1cm} \text{Eq. 1b}
\[ MHB = 0 \]  \hspace{1cm} \text{Eq. 1c}
\[ MHB = H + M \]  \hspace{1cm} \text{Eq. 1d}

where \( H \) and \( M \) adopt Wb/m}^2\ units in the absence of \( \mu_0 \).

Magnetization can only occur in a material. Therefore, in free space \( M = 0 \) and Eq. 1a reduces to \( B = \mu_0 H \).

The magnetic field generated by a solenoid is given by
\[ H = \frac{n \cdot i}{L} \]  \hspace{1cm} \text{Eq. 2}

- \( n \) \text{ Number of turns} (dimensionless)
- \( i \) \text{ Current} (A)
- \( L \) \text{ Length} (m)

Starting with one of Maxwell’s equations:
\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \]  \hspace{1cm} \text{Eq. 3a}

we can find
\[ \iint_A (\nabla \times \vec{E}) \cdot d\vec{A} = -\frac{\partial}{\partial t} \int_A \vec{B} \cdot d\vec{A} \]  \hspace{1cm} \text{Eq. 3b}
\[ \oint_C \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int_A \vec{B} \cdot d\vec{A} \]

Substituting \( \Phi = \int_A \vec{B} \cdot d\vec{A} \) and \( V = \oint_C \vec{E} \cdot d\vec{l} \) yields \text{Faraday’s Law}:
\[ V = -\frac{\partial \Phi}{\partial t} \]  \hspace{1cm} \text{Eq. 3c}

Physically, this means that the voltage produced in a coil of wire is equal to the time derivative of the magnetic flux (without any proportionality constants!). Note: if the flux is not changing, then no voltage will be produced \textit{regardless of the strength of the field}. On the other hand, if the coil is in motion then the voltage produced will be proportional to both the field strength and the velocity.

Starting with another one of Maxwell’s equations:
\[ \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \]  \hspace{1cm} \text{Eq. 4a}

assuming that there are no time-varying electronic fields (i.e., \( \frac{\partial \vec{D}}{\partial t} \to 0 \)) and utilizing the Stokes theorem, we see
\[ \int_{A} (\nabla \times \vec{H}) \cdot d\vec{A} = \int_{A} \vec{J} \cdot d\vec{A} \]  
\[ \oint_{c} \vec{H} \cdot d\vec{l} = I_{total} \]  
Eq. 4b

also known as **Ampere’s Law**. We can use this to derive the field inside a solenoid (Eq. 2).

The remaining two Maxwell’s equations are

\[ \nabla \cdot \vec{D} = \rho \text{ or } \int_{V} D_{v} dV = q \]  
Eq. 5a

and

\[ \nabla \cdot \vec{B} = 0 \text{ or } \oint_{s} B_{v} dS = 0 \]  
Eq. 5b

The former, also known as **Gauss’ Law**, simply states that the electric flux through a closed surface is equal to the enclosed charge; this law is rarely used in magnetostatics. Eq. 5b, on the other hand, implies that there are no magnetic monopoles – magnetic fields are always the result of a dipole.

Magnetic instruments and researchers often use a different set of units, including cgssa and English linear measures; for example, it is not uncommon to hear researchers in the DSSC talk about 10 Gbit/m² areal density and 3000 Oe media coercivity. Most scientists and engineers, however, use the mkssa system. A short table is presented below, derived from Cullity¹ and additional notes written in the margins:

### Magnetism

<table>
<thead>
<tr>
<th>cgssa units</th>
<th>mkssa units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B = H + 4\pi M )</td>
<td>( B = \mu_0 (H + M) )</td>
</tr>
<tr>
<td>( B ) in gauss (G)</td>
<td>( B ) in webers/meter² (Wb) = tesla (T)</td>
</tr>
<tr>
<td>( H ) in oersteds (Oe)</td>
<td>( H ) in amperes/meter (A/m)</td>
</tr>
<tr>
<td>( M ) in emu/cm³</td>
<td>( M ) in amperes/meter</td>
</tr>
<tr>
<td>( \mu_0 ) (vacuum) = 1</td>
<td>( \mu_0 ) (vacuum) = ( 4\pi \times 10^{-7} ) weber/(ampere meter)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>cgssa to mkssa</th>
<th>mkssa to cgssa</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 G = ( 10^{-4} ) Wb/m²</td>
<td>1 Wb/m² = ( 10^6 ) G</td>
</tr>
<tr>
<td>1 Oe = 79.6 A/m</td>
<td>1 A/m = ( 12.57 \times 10^{-3} ) Oe</td>
</tr>
<tr>
<td>1 emu/cm³ = ( 12.57 \times 10^{-4} ) Wb/m²</td>
<td>1 Wb/m² = 796 emu/cm³</td>
</tr>
</tbody>
</table>

**mkssa only**

1 Wb = 1 V s  
1 Wb/A = 1 H (inductance)

### Length

| 1 angstrom (Å) = \( 10^{-10} \) m = \( 10^{-8} \) cm |
| 1 in = 0.0254 m = 2.54 cm |
| 1 mil = \( 10^{-3} \) in = 25.4 µm |
| 1 µin = \( 10^{-6} \) in = 25.4 nm |

### Energy

| 1 erg = \( 10^{-7} \) J |
| 1 eV = \( 1.602 \times 10^{-19} \) J |

1 J = \( 10^7 \) erg  
1 J = \( 6.25 \times 10^{18} \) eV

The Biot-Savart law is useful for determining the field generated by various coil shapes:

\[
\vec{H} = \frac{\int \vec{I}dl \times \hat{r}}{4\pi r^2}
\]

Eq. 6

This can also be used to calculate the field generated by a solenoid.

Applied field energy density: \( E_M = -\mu_0 M \cdot H = -\mu_0 M \cdot H \cdot \cos \theta \)  
Eq. 7a

Anisotropy energy density: \( E_K = K_a \sin^2 \theta \)  
Eq. 7b

Demagnetization energy density: \( E_d = -\frac{1}{2} \mu_0 M \cdot H_d = \frac{1}{2} \mu_0 M \cdot H_d \)  
Eq. 7c