Distributed MPC for Efficient Coordination of Storage and Renewable Energy Sources Across Control Areas

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***Abstract*—In electric power systems, multiple entities are responsible for ensuring an economic and reliable way of delivering power from producers to consumers. With the increase of variable renewable generation it is becoming increasingly important to take advantage of the individual entities' (and their areas') capabilities for balancing variability. Hence, in this paper, we employ and extend the approximate Newton directions method to optimally coordinate control areas leveraging storage available in one area to balance variable resources in another area with only minimal information exchange among the areas. The problem to be decomposed is a model predictive control problem including generation constraints, energy storage constraints, and AC power flow constraints. Singularity issues encountered when formulating the respective Newton–Raphson steps due to intertemporal constraints are addressed and extensions to the original decomposition method are proposed to improve the convergence rate and required communication of the method.**

***Index Terms*—Distributed optimization, model predictive control, storage, AC optimal power flow, approximate Newton direction method.**

Nomenclature

N	optimization horizon
N_B	number of buses in the system
P_{G_i}	active power output of generator at bus i
a_i, b_i, c_i	cost parameters of generator at bus i
P_{W_i}	active power output of wind generator at bus i
P_{L_i}	active power load at bus i
P_{I_i}	power injected into storage at bus i
P_{O_i}	power drawn from storage at bus i
P_{ij}	active power flowing on line ij
Ω_i	set of buses connected to bus i
Q_{G_i}	reactive power output of generator at bus i
Q_{L_i}	reactive power load at bus i
Q_{ij}	reactive power flowing on line ij
E_i	energy level in storage at bus i
T	time between two consecutive timesteps
η_{c_i}	charging efficiency of storage at bus i
η_{d_i}	discharging efficiency of storage at bus i
$\overline{V}_i$	maximum voltage magnitude at bus i
$\underline{V}_i$	minimum voltage magnitude at bus i
$\overline{P}_{G_i}$	maximum power output of generator at bus i
$\Delta\overline{P}_{G_i}$	maximum ramp rate of generator at bus i
$\underline{E}_i$	lower energy limit of storage at bus i
$\overline{E}_i$	energy capacity of storage at bus i
$\overline{P}_{E_i}$	maximum dis/charging rate of storage at bus i

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I. Introduction

As ELECTRIC power systems span entire continents, the control responsibility for these large systems is shared among multiple entities. Each of these entities is responsible for a specific geographic area called control area. The coupling of the control areas via tie lines allows for exchanging power across their boundaries but also leads to the need to coordinate the actions in the areas. Traditionally, this is being done by agreeing on a tie line flow, e.g., based on market mechanisms, and then optimize the schedule of generation within the areas to balance supply and demand. This leads to an overall suboptimal usage of the available resources because the optimization is limited to the localized areas.

As long as resources are distributed throughout the system in a fairly homogenous way in terms of their capabilities and costs, the suboptimality may be acceptable. However, once the resources in one area have considerably different characteristics compared to the resources in the neighboring area, substantial improvements in terms of providing reliable and cost effective electric power supply may be achieved. In this paper, we particularly consider the situation in which one area has significant amounts of variable renewable generation resources and the other area has significant amounts of storage. In that case, it is beneficial for both areas to improve the coordination such that the storage is being used to balance the variability of the renewable resources.

Generally, this means that the control areas should be optimized jointly in a single centralized optimization problem requiring that the areas share their system information either with the other entities or a centralized entity overseeing all areas. Another option is to use decomposition techniques to decompose the centralized problem into subproblems each associated with a particular control area. The result is an iterative process where each area solves the problem assigned to it and then provides some information about the solution at the buses located at the boundary of the area to the other areas. Specifically, the voltage magnitude and angle as well as the Lagrange multiplier corresponding to the power balance equation at the boundary buses are exchanged between areas. Based on the information received the areas update their solution and keep exchanging until convergence has been achieved. The final solution should be equal to the solution obtained if the problem would be solved by a centralized entity across all areas.

Given that the intention is to optimize the usage of storage and that how a storage can be used in the future timesteps heavily depends on how it is used at the current timestep, we formulate a multi-step AC Optimal Power Flow problem and implement a receding horizon. Consequently, the resulting problem is a Model Predictive Control (MPC) problem [1]. MPC, or look-ahead optimization, has been shown to benefit the operation of power systems significantly [2]. We decompose this problem using the Optimality Decomposition and Approximate Newton Directions method [3] where each subproblem is associated with a particular control areas. In order to improve the convergence rate of the algorithm, two approaches are proposed: the first is derived from the Jacobi method for solving a linear system of equations and the second adds an additional term to the update which better reflects the impact of one area on the other. The consequence is that the number of times that the areas need to communicate to converge toward the overall optimum is reduced.

Hence, the outline of the paper is as follows: Section II provides an overview of some related work. In Section III, the MPC problem formulation for the centralized problem is given. This problem is then decomposed in Section IV where the proposed modifications to the decomposition algorithm are presented as well. Cases when the Jacobian matrix becomes singular due to the intertemporal constraints from storage are discussed and resolved in Section V. Simulation results are shown for all three distributed methods in Section VI, and Section VII concludes the paper.

II. Related Work

The distributed AC Optimal Power Flow problem has been addressed in the literature by a variety of approaches, most of them derived from the Augmented Lagrangian method [4]–[6]. The Approximate Newton Directions method is used to decompose the AC OPF problem in [7], and is also used in [8] for decentralized control of power flow devices across overlapping control areas. Fully decentralized optimization on the nodal level for the AC Optimal Power Flow problem is discussed in [9] on a 4 and 6-bus network.

Here, the distributed problem is extended across optimization timesteps to optimize over a prediction horizon using Model Predictive Control in order to determine the optimal use of the storage device and optimal generation settings. Geographical regions are decomposed and a small amount of information is communicated between areas without the need for a centralized controller, coordinating storage in one area with renewable energy in an adjacent area. MPC has previously been applied to power systems for energy storage control. In [10]–[13], centralized MPC is used on relatively small scale systems to optimally control a battery to reduce the effect of fluctuations in the power supply due to renewable generation. Rolling horizon control has been applied to the unit commitment problem in [14]–[16]. A Model Predictive Control AC OPF problem was solved in a distributed manner in [17] using AND on the IEEE-14 bus system. In [18], an MPC problem including DC power flow constraints is solved in a distributed way using a proximal message passing method. Distributed MPC is implemented for another purpose in [19] for Automatic Generation Control, and in [20] for the mitigation of cascading failures in a power system.

In this paper, distributed MPC is used to coordinate renewable generation in one area with storage in another area. Two extensions are derived for the original AND method which for a variety of cases significantly improve the rate of convergence of the optimization. A proof of concept is given for the IEEE-57 and IEEE-118 bus test systems. Generally, solving the AC OPF problem for each step in an entire prediction horizon results in a very large nonlinear optimization problem; however, the straightforward decomposition of the problems using AND makes the optimization of the individual subproblems easily parallelizable. When this method is applied to the multi-area OPF problem as in [7] and [17], the variables exchanged between the areas corresponds to the voltage magnitudes and angles at buses connected across areas, as well as the Lagrange multipliers at these connected buses and lines. There is no need for areas to share information with other areas that is not related to their physically connected buses, and the problem converges to the centralized solution provided the convergence criteria is met [3].

III. Problem Formulation

The problem that we address in this paper is a Model Predictive Control problem to minimize the overall cost of supplying the load by optimally using the available energy storage. Hence, it is a multi-step optimal power flow problem which includes inter-temporal constraints on energy storages and the AC power flow constraints. The overall problem formulation is therefore given by

$$\min_{P_G} \sum_{k=1}^{N} \left(\sum_{i=1}^{N_B} a_i P_{G_i}^2 + b_i P_{G_i} + c_i \right) \tag{1}$$

$$s.t.\ \ P_{G_i}(k) + P_{W_i}(k) - P_{L_i}(k) \tag{2}$$

$$- P_{I_i}(k) + P_{O_i}(k) - \sum_{j \in \Omega_i} P_{ij}(k) = 0, \tag{3}$$

$$Q_{G_i}(k) - Q_{L_i}(k) - \sum_{j \in \Omega_i} Q_{ij}(k) = 0, \tag{4}$$

$$E_i(k+1) = E_i(k) + \eta_c T P_{I_i}(k) - \frac{T}{\eta_d} P_{O_i}(k), \quad (5)$$
$$\underline{E}_i \leq E_i(k+1) \leq \overline{E}_i, \quad (6)$$
$$0 \leq P_{I_i}(k) \leq \overline{P}_{E_i}, \quad (7)$$
$$0 \leq P_{O_i}(k) \leq \overline{P}_{E_i}, \quad (8)$$
$$0 \leq P_{G_i}(k) \leq \overline{P}_{G_i}, \quad (9)$$
$$\underline{V}_i \leq V_i(k) \leq \overline{V}_i, \quad (10)$$
$$|P_{G_i}(k+1) - P_{G_i}(k)| \leq \Delta\overline{P}_{G_i}, \quad (11)$$

for $k = \{0, \ldots, N-1\}$ and $i = \{1, \ldots, N_B\}$.

Whenever there is no generator connected to bus i, it is assumed that equations (2) – (4) are reduced to not include the generation output variable. The same holds for the wind generator output, the loads and the storage variables. Equalities and inequalities (5) – (9) are only included if there is a generator or a storage connected to bus i. Equations (2) – (4) represent the power flow equations in the system where the flows P_{ij} and Q_{ij} are functions of the voltage magnitudes and angles at the ends of the lines and the line parameters. For the generator buses, the voltage magnitude at the respective buses are set to fixed values and for the slack bus additionally the voltage angle is set to zero. Equation (10) represents a constraint on the upper and lower level of the voltage magnitude at each bus. If there is a generator at bus i, equation (11) represents the ramping limit of that generator.

As Model Predictive Control is used here, once the problem (1) – (11) is solved for timestep t, the solution for the first step $k = 0$ is applied. Then, the optimization horizon is shifted by T and the problem is solved for the new time horizon.

IV. Distributed MPC

The resulting optimization problem (1) – (9) corresponds to the centralized problem including multiple control areas. Decomposing the problem allows each control area to solve the optimization problem associated with its own part of the system while optimally coordinating with its neighboring areas. It is assumed that these control areas are separate and do not coordinate in any other means other than exchanging the tie-line variables. In the situation considered here, energy storage is located in one of these areas and renewable generation in the other.

The focus of this paper is on vertically integrated utilities where the goal is to minimize overall generation cost to supply the load. Even if the optimization takes place for one timestep, or over a horizon, or over a rolling horizon, the approach can be used to optimally coordinate neighboring control areas. The original method has been presented for the purpose of optimizing for a single timestep [7]. Expanding it to multiple timesteps also allows for an optimal integration of storage devices. Hence, the main focus in this paper is on enabling optimal coordination across areas and particularly the coordination of variable renewable generation with storage. However, decomposition of the considered problem generally allows for parallelized computations and therefore for solving large scale optimization problems which otherwise could not be solved or not solved within a reasonable amount of time. As the prediction horizon N increases, the considered MPC problem may lead to such a large scale problem and despite the potentially existing centralized coordinator could require a distributed solution process.

In this section, we first show how the Unlimited Point Algorithm [21] is used to handle the inequality constraints in the problem formulation, then we provide the decomposed problem formulation using the Approximate Newton Direction and finally describe the proposed modifications to the algorithm which lead to an improved convergence speed.

A. Unlimited Point Method

There are various ways to handle inequality constraints in an optimization problem. In this paper, we use the Unlimited Point method [21], however, the derivations provided beyond this subsection stay the same even if another method is used to incorporate inequality constraints into a Newton-Raphson update (such as Interior Point). In the Unlimited Point method, the inequality constraints in the general optimization problem

$$\min_{x} \quad f(x) \quad (12)$$
$$s.t. \quad g(x) = 0 \quad (13)$$
$$h(x) \leq 0 \quad (14)$$

are transformed into equality constraints according to

$$h_n(x) + s_n^2 = 0 \quad (15)$$

for inequality n and where s_n is a slack variable. Squaring the slack variable ensures that the original inequality constraint is fulfilled. The first order optimality conditions are then formulated as

$$\frac{\partial f}{\partial x} + \lambda^T \cdot \frac{\partial g}{\partial x} + \mu^{2^T} \cdot \frac{\partial h}{\partial x} = 0 \quad (16)$$
$$g(x) = 0 \quad (17)$$
$$h(x) + s^2 = 0 \quad (18)$$
$$diag(\mu) \cdot s = 0 \quad (19)$$

Hence, similar to the slack variables also the Lagrange Multipliers are replaced with squared variables to ensure that these Lagrange Multipliers take values which are greater than zero without having to explicitly include such non-negativity constraints.

The Unlimited Point formulation is applied to (1) – (11) in this paper. The next step now is to apply the Approximate Newton Direction method [3] to the resulting first order optimality conditions in order to be able to solve these in a distributed way.

B. Application of Approximate Newton Directions Method

Generally, an optimization problem can be solved by applying the Newton-Raphson method to the first order optimality conditions of the problem. Such an update is given by

$$x^{(p+1)} = x^{(p)} + \alpha \cdot \Delta x^{(p)} = x^{(p)} - \alpha \cdot \left(J_{tot}^{(p)}\right)^{-1} \cdot d^{(p)}, \quad (20)$$

where p is the iteration counter. In our case, the right hand side vector $d^{(p)}$ includes the first order optimality conditions (16) – (19) for problem (1) – (11) ordered according to the subproblems and evaluated at $x^{(p)}$, and the update vector is given by $\Delta x^{(p)}$. The parameter α is used to control the step size to avoid divergence due to overshooting. The Jacobian matrix $J_{tot}^{(p)}$ is also evaluated at $x^{(p)}$ and is given by

$$J_{tot}^{(p)} = \begin{pmatrix} J_{1,1}^{(p)} & J_{1,2}^{(p)} & \cdots & & & & \cdots & J_{1,M}^{(p)} \\ J_{2,1}^{(p)} & \ddots & \ddots & \vdots & & & & \vdots \\ \vdots & \ddots & \ddots & J_{q-1,q}^{(p)} & & & & \\ & \cdots & J_{q,q-1}^{(p)} & J_{q,q}^{(p)} & J_{q,q+1}^{(p)} & \cdots & & \\ & & & J_{q+1,q}^{(p)} & \ddots & \ddots & & \vdots \\ \vdots & & & \vdots & \ddots & \ddots & & J_{M-1,M}^{(p)} \\ J_{M,1}^{(p)} & \cdots & & & & \cdots & J_{M,M-1}^{(p)} & J_{M,M}^{(p)} \end{pmatrix}. \tag{21}$$

The block element $J_{l,j}^{(p)}$ corresponds to the Jacobian matrix of the first order optimality conditions associated with the constraints in area l with respect to the variables associated with area j.

In the Approximate Newton Directions method, the decomposition into M subproblems is achieved by setting the off-diagonal block matrices $J_{l,j}^{(p)},\ l \neq j$, equal to zero. These off-diagonal matrices are generally sparse because the only non-zero elements arise from coupling constraints, i.e., constraints which couple the variables of area l with the variables of area j. The resulting Newton-Raphson update can then be solved in a distributed way, i.e.,

$$x_q^{(p+1)} = x_q^{(p)} + \alpha \cdot \Delta x_q^{(p)} = x_q^{(p)} - \alpha \cdot \left(J_{q,q}^{(p)}\right)^{-1} \cdot d_q^{(p)}, \tag{22}$$

for $q = 1, \ldots, M$. Hence,

$$d^{(p)} = \left[d_1^{(p)}, \ldots, d_M^{(p)}\right]^T, \tag{23}$$

$$x^{(p)} = \left[x_1^{(p)}, \ldots, x_M^{(p)}\right]^T \tag{24}$$

$$\Delta x^{(p)} = \left[\Delta x_1^{(p)}, \ldots, \Delta x_M^{(p)}\right]^T. \tag{25}$$

In the considered problem, the optimization problem is decomposed according to geographical areas, i.e., the variables in $x_q^{(p)}$ correspond to the variables associated with buses in area q. As the considered problem spans multiple timesteps, this variable vector includes copies of all the variables within that area for all timesteps in the optimization horizon, i.e., $P_{G_i}(0), \ldots, P_{G_i}(N-1)$.

The advantage of this method is that instead of solving each subproblem to optimality before exchanging information with the other subproblems, data can be exchanged after each Newton-Raphson iteration. And unlike other Lagrangian-based decomposition methods such as Lagrangian Relaxation and Augmented Lagrangian, there is no need for a centralized entity or tuning of parameters to update the Lagrange multipliers; subproblems simply exchange data directly with their neighbors and the updates for the multipliers come directly from the other subproblems.

C. Modifications of the AND Method

In this paper, we consider two adjustments to the Approximate Newton method, both with the intention to reduce the gap between the distributed variable update and the centralized update, thus improving the convergence rate in some cases.

1) Jacobi Update: The first modification is derived from the Jacobi method for solving a linear system of equations [22]. Instead of setting the off-diagonal block matrices in the Jacobian matrix to zero, the information from the previous iteration $p-1$ is used to update the variables at iteration p. The variable update for each subproblem $p = 1, \ldots, M$ is now equal to

$$J_{q,q}^{(p)} \cdot \Delta x_p^{(p)} = -d_q^{(p)} - \sum_{m=1, m\neq q}^{M} (J_{m,q}^{(p-1)} \cdot \Delta x_m^{(p-1)}). \tag{26}$$

Even with these additional terms in the update it is not necessary to exchange the full update vectors $\Delta x_m^{(p-1)}$ among the areas. Area m can, without additional information exchange, evaluate $J_{m,q}^{(p-1)}$ at iteration $p-1$ and then compute the multiplication with the update vector $\Delta x_m^{(p-1)}$. As $J_{m,q}^{(p-1)}$ is very sparse, the multiplication with $\Delta x_m^{(p-1)}$ results in a sparse vector and only the non-sparse elements need to be shared.

The issue with this update is that it basically builds upon the assumption that $\Delta x_m^{(p-1)}$ and $\Delta x_m^{(p)}$ will be similar, which does not necessarily have to be the case and may therefore only result in improved performance in certain cases.

2) Additional Term in Right Hand Vector: The second approach is a bit more involved in its derivation. Hence, we use a two area example to present the main idea. The centralized update is given by

$$\begin{bmatrix} J_{11}^{(p)} & J_{12}^{(p)} \\ J_{21}^{(p)} & J_{22}^{(p)} \end{bmatrix} \cdot \begin{bmatrix} \Delta x_1^{(p)} \\ \Delta x_2^{(p)} \end{bmatrix} = -\begin{bmatrix} d_1^{(p)} \\ d_2^{(p)} \end{bmatrix} \tag{27}$$

By reordering the terms in the rows, the following formulas for the updates result,

$$\Delta x_1^{(p)} = -J_{11}^{(p)^{-1}} \cdot d_1^{(p)} - J_{11}^{(p)^{-1}} J_{12}^{(p)} \cdot \Delta x_2^{(p)}, \tag{28}$$

$$\Delta x_2^{(p)} = -J_{22}^{(p)^{-1}} \cdot d_2^{(p)} - J_{22}^{(p)^{-1}} J_{21}^{(p)} \cdot \Delta x_1^{(p)}. \tag{29}$$

By substituting (29) into (28) and vice versa, the updates can be written as a function of the matrices and the right hand side vectors, i.e., (for simplification, we do not indicate the iteration counter in these equations)

$$\Delta x_1 = \left(J_{11} - J_{12} J_{22}^{-1} J_{21}\right)^{-1} \cdot \left(-d_1 + J_{12} J_{22}^{-1} \cdot d_2\right), \tag{30}$$

$$\Delta x_2 = \left(J_{22} - J_{21} J_{11}^{-1} J_{12}\right)^{-1} \cdot \left(-d_2 + J_{21} J_{11}^{-1} \cdot d_1\right). \tag{31}$$

This update corresponds to the exact update, i.e., the update that is obtained if Newton Raphson is applied to the first order optimality conditions of the centralized problem. As can be seen, even for two areas, a fairly complicated update results if this is to be done in a distributed way. Consequently, we propose to simplify this update to

$$\begin{aligned} \Delta x_1^{(p)} &= J_{11}^{(p)^{-1}} \cdot \left(-d_1^{(p)} + J_{12}^{(p)} J_{22}^{(p)^{-1}} \cdot d_2^{(p)}\right) \\ &= J_{11}^{(p)^{-1}} \cdot \left(-d_1^{(p)} + \hat{d}_{12}^{(p)}\right) \end{aligned} \tag{32}$$

$$\begin{aligned} \Delta x_2^{(p)} &= J_{22}^{(p)^{-1}} \cdot \left(-d_2^{(p)} + J_{21}^{(p)} J_{11}^{(p)^{-1}} \cdot d_1^{(p)}\right) \\ &= J_{22}^{(p)^{-1}} \cdot \left(-d_2^{(p)} + \hat{d}_{21}^{(p)}\right) \end{aligned} \tag{33}$$

This update is significantly less computationally intense than (30) – (31). Area 1 can compute $\hat{d}_{21}^{(p)}$ without additional knowledge from area 2 and then provide area 2 with the non-zero entries in this vector. The computation of $\hat{d}_{21}^{(p)}$ involves the inverse of $J_{11}^{(p)}$. However, that inverse is needed for the update of $\Delta x_1^{(p)}$ anyway. Consequently, area 1 can reuse the inverse for the computation of $\hat{d}_{21}^{(p)}$. Generally, there should only be very few terms in the vectors $\hat{d}_1^{(p)}$ and $\hat{d}_2^{(p)}$ which are non zero, namely the ones which correspond to first order optimality constraints which include variables from both subproblems. Hence, only a limited amount of additional information needs to be exchanged among the subproblems (not the entire additional vector) to carry out (32) and (33).

We now generalize the update for the case with multiple areas. Hence, we propose the following update

$$\Delta x_q^{(p)} = J_{q,q}^{(p)^{-1}} \cdot \left(-d_q^{(p)} + \sum_{m=1, m\neq q}^{M} J_{q,m}^{(p)} J_{m,m}^{(p)^{-1}} \cdot d_m^{(p)} \right)$$
$$= J_{q,q}^{(p)^{-1}} \cdot \left(-d_q^{(p)} + \sum_{m=1, m\neq q}^{M} \hat{d}_{q,m}^{(p)} \right) \quad (34)$$

With the communication of these few extra terms, the updates carried out locally for the areas is closer to the centralized update. Hence, it can be expected that the number of iterations until convergence is reached is reduced compared to the original method or the method with the modification based on the Jacobi update.

V. Singularity Issues

One major challenge encountered when including inter-temporal constraints such as the constraint on energy storage (5) in combination with the inequality constraints (6) – (8) on the variables of the storage into a multi-timestep optimization problem is that in some cases, the Jacobian matrix of the first order optimality conditions may become singular at the optimal solution. This may cause the Jacobian to be ill-conditioned as it approaches the optimal solution, increasing the number of iterations to optimality or in some cases causing the optimization to diverge. In this section, we discuss when such singularity occurs by analyzing the structure of the Jacobian and a solution to the problem is presented.

A. Causes of Singularities

The formulation given in (1) – (11) will result in a singular Jacobian when the gradients of storage constraints (5) – (8) are simultaneously binding. An inequality constraint is called "binding" or "active" at the optimal solution if its corresponding slack variable is zero (all equality constraints are considered active). This is due to the Linear Independence Constraint Qualification (LICQ), which states that at the optimal solution, the gradients of all binding constraints must be linearly independent or there exists no unique solution for the Lagrange Multipliers [23].

The following analysis of the structure of the Jacobian matrix is shown for the Jacobian of the first order optimality conditions without the Unlimited Point modification because the issue is encountered independent of if Unlimited Point, Interior Point, or another method which uses the Newton-Raphson Jacobian is used. This is because the singularity is due to the linear dependence of the gradients of binding constraints and has nothing to do with the way how inequality constraints are handled.

The Jacobian has the following structure:

$$\begin{bmatrix} \nabla_{xx}^2 \mathcal{L}(x, z, \lambda, \mu) & \nabla g(x)^T & \nabla h(x)^T & 0 \\ \nabla g(x) & 0 & 0 & 0 \\ \nabla h(x) & 0 & 0 & I \\ 0 & 0 & diag\{s\} & diag\{\mu\} \end{bmatrix} \quad (35)$$

The rows of the Jacobian which become singular when LICQ is not satisfied are:

$$\begin{bmatrix} \nabla g(x) & 0 & 0 & 0 \\ \nabla h(x) & 0 & 0 & I \\ 0 & 0 & diag\{s\} & diag\{\mu\} \end{bmatrix}. \quad (36)$$

When an inequality constraint $h_n(x)$ is binding, $s_n = 0$ and $\mu_n \neq 0$. If a set of binding constraints $\Delta g(x)$ and $\Delta h(x)$ are linearly dependent, (36) will have dependent rows and thus the entire Jacobian (35) will be singular.

B. Singularities From Storage Constraints

Although the aforementioned Jacobian singularity can occur with many various sets of constraints, it may particularly occur in multi-step optimization problem formulations which include inter-temporal constraints such as generator ramp limits and constraints on storage devices [24]. With respect to storage, the singularity occurs when it is optimal for the energy storage system to keep its energy level at a minimum or maximum for multiple timesteps. This can be seen by considering the following variable vector:

$$x = [E_i(k) \quad P_{I_i}(k) \quad P_{O_i}(k) \quad E_i(k+1)], \quad (37)$$

If the optimal solution for these variables is $x^* = [\underline{E}_i \ \ 0 \ \ 0 \ \ \underline{E}_i]$, i.e., the storage is empty for two consecutive timesteps (and thus there is no charging/discharging during these times), the matrix of the gradients of the binding constraints (5) – (8) is given by

$$\begin{bmatrix} 0 & \cdots & 0 & -1 & -T\eta_c & \frac{T}{\eta_d} & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & 0 & -1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & -1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & -1 & 0 & 0 & \cdots & 0 \end{bmatrix} \quad (38)$$

which, upon inspection, has linearly dependent rows. Hence, the LICQ is not fulfilled, and the Jacobian matrix is singular. Similar effects can be observed for ramping constraints but as discussed in [24], situations in which singularity is caused by the ramping constraints are rather rare. The reader is therefore referred to [24] for a more extensive discussion on singularity caused by ramping constraints.

C. Modification of Storage Model

Multiple solution techniques are presented in [24] to avoid the singularity problem. The singularity of the matrix at the optimal solution indicates that there are multiple solutions that satisfy the first order optimality conditions and result in the same objective function value. Consequently, a Moore-Penrose Pseudoinverse may be used to solve the underdetermined set of equations. While the pseudoinverse will be able to solve the system, it requires the extra step of decomposing the Jacobian matrix using Singular Value Decomposition. The time required to perform this operation for every Newton-Raphson iteration can be costly.

Another method that is proposed in [24] is to detect when the storage constraints are close to being binding, and then to remove these constraints from the set of constraints. However, this method runs the risk of prematurely removing constraints and variables before they are actually at the optimal solution, resulting in the original first order optimality conditions not being satisfied. If the constraints are removed too late, the Jacobian may already be too ill-conditioned for the iterations to continue.

The method that is adopted here is to incorporate storage standby losses into the model. A constant, small ϵ can be included to represent a loss in the current energy level over time. For example, this could represent charge leakage from a battery or inertia losses from a flywheel. This does not only prevent the Jacobian matrix to become singular but also more accurately models the behavior of a storage device. This value ϵ is subtracted from the energy balance equation at each timestep, hence modifying (5) to

$$E_i(k+1) = E_i(k) + \eta_c T P_{I_i}(k) - \frac{T}{\eta_d} P_{O_i}(k) - \epsilon. \quad (39)$$

This loss prevents all of the storage constraints from being simultaneously binding. If the storage is at a minimum at time k, it must store energy (have a nonzero $P_{I_i}(k)$) to avoid dipping below the minimum value at time $k+1$. If the storage is full at time k, it cannot be full at $k+1$ unless $P_{O_i}(k)$ is nonzero. Thus, the standby loss prevents all of the storage variables from simultaneously being at their limits, and the Jacobian singularity is avoided. This of course only makes sense if the lower limit $\underline{E}_i$ is not equal to zero because an empty storage cannot lose any energy. However, it is reasonable to set $\underline{E}_i \neq 0$ to avoid very deep discharging of the storage.

VI. Simulation Results

In this section, simulation results are provided as a proof of concept for the IEEE-57 and IEEE-118 bus system for varying horizons. Results of the distributed MPC and comparisons between the conventional and modified AND methods are shown.

A. IEEE-57 Bus Simulation Setup

The IEEE-57 bus system is decomposed into two geographical regions as shown in Figure 1. Two wind generators are placed at buses 17 and 43, and a storage device with roundtrip efficiency of $\eta_c \cdot \eta_d = 0.95\%$, standby loss of $0.005 p.u.\cdot$10-minutes and maximum capacity of $0.5 p.u.\cdot$10-minutes is placed at bus 7. Simulations were run for a 24-hour period with prediction horizons of $N = 1, 3, 6$ and 9 where $T = 10min$, hence, the horizon length correspond to no horizon, 30-minute, 60-minute, and 90-minute horizons. Historical data for the wind and load was used from the Bonneville Power Administration and for the particular simulation presented here the load and wind curve as given in Fig. 2 are used. There is a 27% level of wind energy penetration in the system by energy. The cost functions and maximum output limits for the generators were obtained from the IEEE-57 bus specifications in MATPOWER [25]. Here, the storage is operated at the 10-minute scale to balance out the intra-hourly fluctuations in the power supply caused by variations in the wind and load. Storage could also be used with this method on an hourly scale for longer term load shifting applications. For this test system, we neglect generator ramping and voltage constraints but later include them for the simulations in the IEEE-118 bus test system.

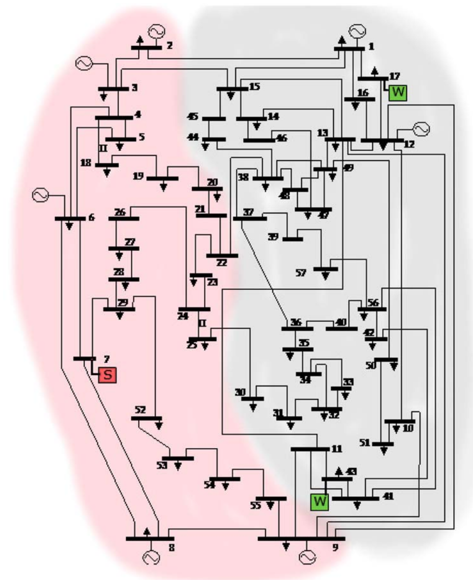

Fig. 1. IEEE-57 bus system decomposed into two areas.

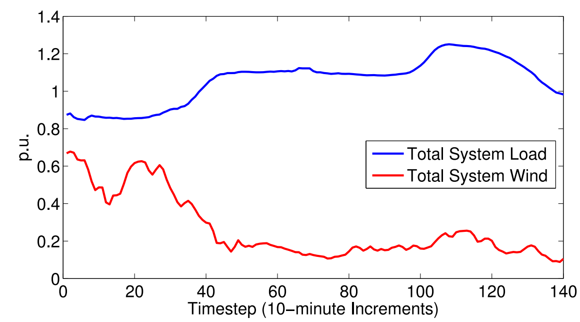

Fig. 2. 24-Hour input data with 10-minute intervals.

The results were compared with the solution of the centralized problem achieved by the SNOPT solver in the commercial optimization package TOMLAB and found to be within $1e^{-3}$ of the solution. The algorithm is considered to have converged

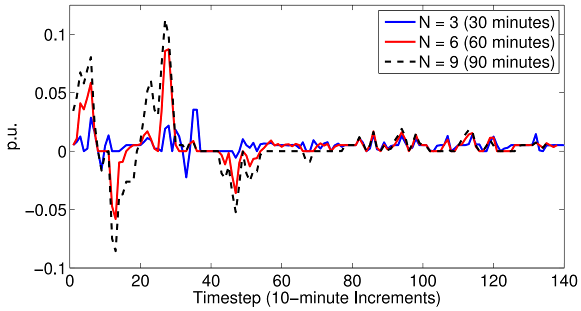

Fig. 3. Optimal power input (positive) and output (negative) from storage.

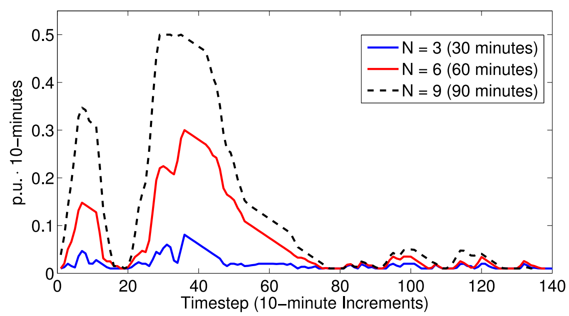

Fig. 4. Optimal state of charge of storage device.

once the maximum absolute value over all elements in the vector of the first order optimality conditions d is less than $1e^{-4}$. It is important to note that in these simulations, because this is a proof of concept simulation, the predictions for the wind are assumed to be perfect; i.e., there is no forecast error, and a longer horizon always results in a lowered objective function value. This may not necessarily be the case if prediction errors are considered. With prediction errors the results depend on the level of the prediction error.

B. Effect of Optimization Horizon

Figure 3 shows the power charged/discharged from the storage device over the 24-hour period, and Figure 4 shows the state of charge of the storage over the 24-hour period. The longer the horizon, the better is the utilization of the energy storage as longer term variations in net load can be predicted and be accounted for. The effect is that less ramping is need from the generators as can be seen in Fig. 5, where the total generation output from dispatchable generators is shown for the horizon $N = 9$. It should be noted that as AC power flow is used, the overproduction in generation is mostly due to AC power flow losses.

The total amount of generator ramping summed over all individual generators was measured for each horizon $N = 1$ (no MPC), 3, 6, and 9 and the total generation costs were calculated. As indicated by the results shown in Table I, with the use of MPC, the overall required amount of generator ramping decreases. Without the use of storage, generators must adjust their output more frequently to account for the fluctuations in the power supply introduced by the wind. The reduction in overall generation cost for the considered day and compared for the different horizon lengths is quite low. However, that measure heavily depends on the particular load and wind

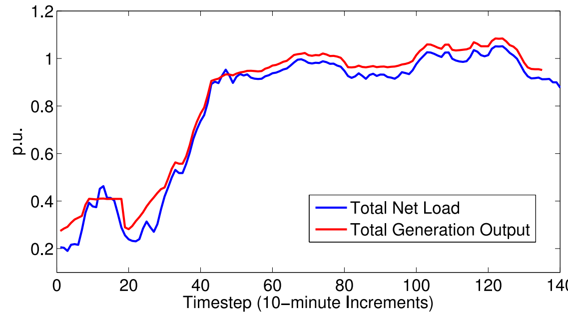

Fig. 5. Optimal generation levels for N = 9.

TABLE I
REQUIRED GENERATOR RAMPING AND TOTAL GENERATION COST

N	1	3	6	9
Total Generator Ramping (p.u.)	2.29	1.88	1.73	1.53
Total Generation Cost ($)	587,235	587,003	586,716	586,568

curves for the considered day as well as the composition of the generators, i.e., which generators become the marginal generators and which reach their capacity limit. Also note that this does not say anything about the difference in cost if coordination is used and if it is not. The comparison solely is focused on the different lengths in horizon. As this paper focuses on the method of how to coordinate the areas, a full economic analysis is beyond the scope of this paper. However, it can be expected that the greater the differences in cost parameters and the higher the fluctuations in net load are, i.e., the higher the penetration of variable renewable generation, the higher the benefit of longer horizons.

C. Comparison of Convergence Rates

1) Requirement for Convergence: In order for the AND method to converge to the optimal solution x^* of the problem described in (1) – (9), the following must hold true at the optimal solution [3]:

$$\rho\left(I - \hat{J}_{dec} \cdot \hat{J}_{tot}\right) < 1 \tag{40}$$

where ρ indicates the spectral radius. Matrix $\hat{J}_{tot}$ is the Jacobian matrix (21) evaluated at the optimal solution and $\hat{J}_{dec}$ is the Jacobian matrix with off-diagonal elements set to zero again at the optimal solution. If the condition on the spectral radius is not fulfilled, the optimization may be unable to converge to the optimal solution. In these cases, a preconditioned conjugate gradient method such as the generalized minimal residual method (GMRES) [22] may be used to improve the convergence. In the system decomposition used in this paper, the spectral radius was calculated to be around 0.88, fulfilling the convergence criteria. This value was not found to change dramatically depending on the horizon length or point in the simulation for the considered case.

2) Comparison of Distributed Methods: In Table II, the minimum, maximum, and median number of iterations to convergence for each method for horizons $N = 1, 3, 6$ and 9 is shown. Figure 6 shows the rate of convergence at simulation

TABLE II
NUMBER OF ITERATIONS TO CONVERGENCE

Method	N	Minimum Iterations to Convergence	Median Iterations to Convergence	Maximum Iterations to Convergence
Original AND	1	356	511	704
Jacobi Update	1	360	364	642
Additional Term	1	337	363	500
Original AND	3	353	380	947
Jacobi Update	3	354	411	773
Additional Term	3	353	377	637
Original AND	6	369	373	837
Jacobi Update	6	351	374	822
Additional Term	6	351	371	808
Original AND	9	561	614	909
Jacobi Update	9	515	608	909
Additional Term	9	376	409	1053

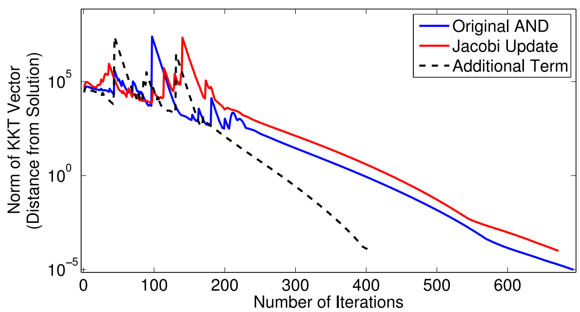

Fig. 6. Convergence rates for N=9.

timestep $t = 2$ for each of the three distributed optimization methods for $N = 9$. To ensure better convergence properties at the cost of a higher number of iterations, the damping parameter α on the Newton-Raphson step was initially chosen to be 0.25. In the rare cases where the method still continued to diverge, a higher damping of 0.1 was chosen for the iterations which leads to a few outliers in terms of iteration numbers. To reduce the number of iterations, an adaptive approach for setting the damping parameter could be used. For the sake of comparison, the same damping factors have been used throughout the iteration process for a particular timestep.

As the results indicate, the Jacobi update method only leads to significant improvements for $N = 1$, whereas the method with the additional term leads to significant reduction for horizons of $N = 1$ and $N = 9$ and stays roughly the same for the other horizons. The conclusion that can be drawn is that an analysis should be done for the particular considered problem to determine whether or not it is useful to add in the additional term.

D. IEEE-118 Bus Test System

To demonstrate the effect of scaling these methods to a larger system, the IEEE-118 bus test case was decomposed into two areas as shown in Figure 7. For this test case, we now do include generator ramping and bus voltage constraints. The cost function and maximum output capacities for the generators were taken from the IEEE-118 bus case in MATPOWER. The minimum and maximum voltage magnitude $\underline{V}_i$ and $\overline{V}_i$ are set to $0.9p.u.$ and $1.1p.u.$, respectively. Generator ramping limits $\Delta\overline{P}_{G_i}$ are set to $\overline{P}_{G_i}$. The wind and load data were

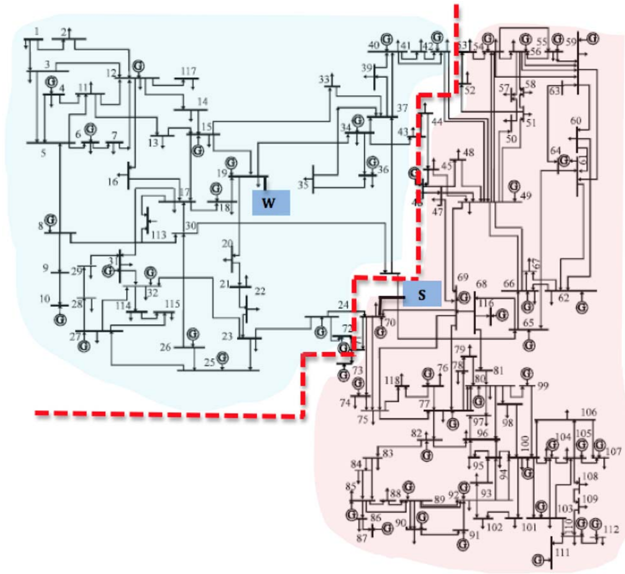

Fig. 7. IEEE-118 bus system decomposed into two areas.

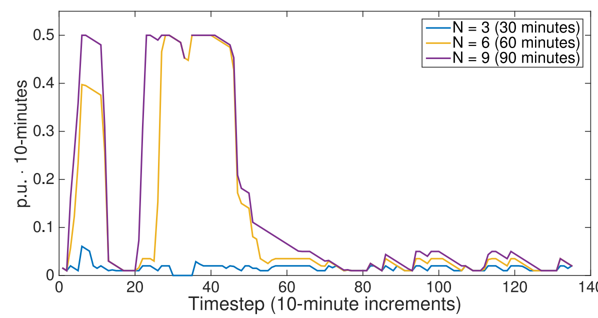

Fig. 8. Optimal state of charge of storage device.

obtained from the Bonneville Power Administration, and the storage parameters were chosen to be the same as in the 57-bus case. Wind generation is located in area 1 at bus 19, and the storage is located in area 2 at bus 70.

Similarly to the IEEE-57 bus system, the storage is utilized more during simulations with longer horizons, as seen in Figure 8. This is again due to the fact that the longer the horizon, the more the long-term variations in net load can be accounted for.

However, in contrast to the 57-bus system, the Newton-Raphson damping parameter α was only stepped down to 0.5 in these simulations, resulting in fewer iterations overall. As distributed algorithms usually perform better in systems where the coupling among areas is reduced, it can be expected that larger systems generally benefit from a stronger internal coupling and weaker inter-area coupling, i.e., the number of buses per area increases which overall should benefit the performance of the algorithm, as seen in Table III. The original AND method failed to converge for timestep 36 for N=3 and timestep 122 for N=6, and these timesteps are not included in the calculations. However, the other two extensions of the AND method were able to converge for all timesteps.

TABLE III
NUMBER OF ITERATIONS TO CONVERGENCE

Method	**N**	**Minimum Iterations to Convergence**	**Median Iterations to Convergence**	**Maximum Iterations to Convergence**
Original AND	3	84	87	213
Jacobi Update	3	44	46	73
Additional Term	3	61	64	91
Original AND	6	86	92	421
Jacobi Update	6	44	47	88
Additional Term	6	62	68	105
Original AND	9	87	93	165
Jacobi Update	9	44	48	89
Additional Term	9	63	70	118

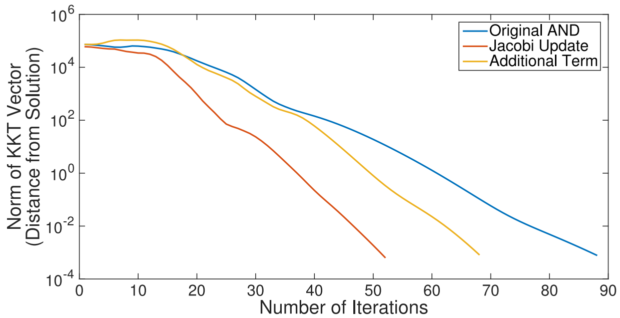

Fig. 9. Convergence rates for N=9 at timestep 2.

The results from Table III indicate that in general, the Jacobi method resulted in the least number of iterations for this system. And overall, the iterations from all methods indicate an improvement over the 57-bus system, perhaps due to the decrease in coupling between subproblems. A comparison of the convergence rates for each of the three methods for $N = 9$ at timestep 2 is shown in Figure 9.

E. Simulation With Four Areas

In order to compare convergence rates for a greater number of subproblems, the 118-bus test system was decomposed into four areas as seen in Figure 10. Simulations were performed for the original AND method as well as the Additional Term method and shown in Table IV. The Additional Term method shows a significant improvement over the original method in the four-area case and performs more robustly in terms of the maximum number of iterations, when compared with the two-area results seen in Table III. In general, it can be seen that the number of iterations increases as the number of areas is increased. This is due to the fact that the variable exchange required in the four-area case is much greater, requiring more variables to iterate to optimality between areas rather than within a single area.

Further case studies can be found in [26] for non-MPC power flow simulations where AND and the Additional Term method are used including line constraints on a synthetic system with 12 118-bus systems. More specifically, if the tie lines get congested, the convergence speed will decrease with the above two methods, which can be resolved by devising a new partition of the system. However, if the inner lines within areas are congested, the distributed methods will hardly be affected.

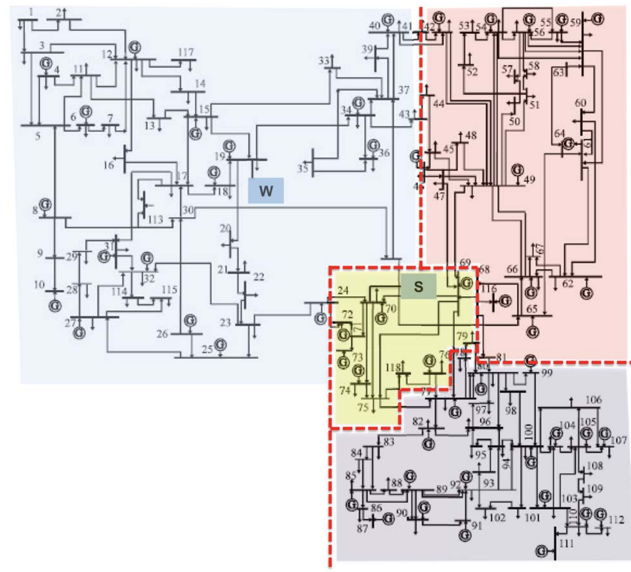

Fig. 10. IEEE-118 bus system decomposed into four areas.

TABLE IV
NUMBER OF ITERATIONS TO CONVERGENCE FOR FOUR AREAS

Method	**N**	**Minimum Iterations to Convergence**	**Median Iterations to Convergence**	**Maximum Iterations to Convergence**
Original AND	3	191	222	313
Additional Term	3	123	138	155
Original AND	6	199	231	346
Additional Term	6	124	145	166
Original AND	9	205	243	611
Additional Term	9	125	150	206

VII. CONCLUSION

In this paper, a distributed Model Predictive Control problem was solved to coordinate resources across areas by only exchanging the variables corresponding to the tie-lines between control areas. Two methods were developed that extended the Approximate Newton Directions method for non-linear optimization decomposition; one method was based on the Jacobi method, and the other method was derived directly from AND and utilized a few additional terms in the variable update. With all of these distributed methods, areas only have to exchange a limited amount of information to achieve the same solution as the centralized optimization problem, maximizing the amount of social welfare in the system.

The convergence rates of each of the distributed approaches is shown in the simulation results, indicating that it depends on the horizon and most likely also the considered problem if the extensions of the AND method improve upon the rate of convergence of the original method. For the 57-bus system, a considerable improvement can be seen for the longer time horizon and the second approach in which the off diagonal elements are approximated by additionally exchanged information. However, the results from the 118-bus system indicate that the Jacobi-based method is the most beneficial.

The optimal generation values and state of charge of the storage device is shown for various optimization horizons in the MPC problem, showing a utilization of the storage for the purpose of reducing the fluctuations in the power supply introduced by an increase in renewable energy penetration. Overall, the results look promising for predictive control Optimal Power Flow problems across areas that do not fully communicate their system data, achieving the centralized solution in a more efficient, distributed way, and allowing for a more effective utilization of energy resources across areas.

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