Accurate Passivity-Enforced Macromodeling for RF Circuits via Iterative Zero/Pole Update Based on Measurement Data

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ABSTRACT

Passive macromodeling for RF circuit blocks is a critical task to facilitate efficient system-level simulation for large-scale RF systems (e.g., wireless transceivers). In this paper we propose a novel algorithm to find the optimal macromodel that minimizes the modeling error based on measurement data, while simultaneously guaranteeing passivity. The key idea is to attack the passive macromodeling problem by solving a sequence of convex semi-definite programming (SDP) problems. As such, the proposed method can iteratively find the optimal poles and zeros for macromodeling. Our experimental results with several commercial RF circuit examples demonstrate that the proposed macromodeling method reduces the modeling error by 1.31-2.74× over other conventional approaches.

1. INTRODUCTION

To efficiently design radio frequency (RF) and microwave system (e.g., wireless transceiver), simple-yet-accurate macromodels are required to quickly simulate the entire system and evaluate its system-level performance (e.g., bit error rate). Macromodels of passive RF circuit blocks (e.g., packages, interconnects, passive filters, passive switches, etc.) are often extracted from their network parameters (i.e., *S*-, *Y*- or *Z*-parameters) measured over a discrete set of frequencies. Given the measurement data, various methods (e.g., vector fitting [1]) have been developed in the literature to compactly express the network parameters as a set of rational functions. These rational functions can then be incorporated into most RF simulation tools (e.g., Agilent ADS, Cadence SpectreRF, etc.) for time- and frequency-domain analyses.

Most conventional macromodeling methods can accurately capture the in-band frequency response for which the measurement data is available for the network parameters of interest. These methods, however, often fail to preserve the important properties (e.g., passivity) of RF circuits over a wide frequency range, because the out-of-band frequency response may not be explicitly provided as the input to these macromodeling tools. In the worst case, if a non-passive macromodel is used to model a passive circuit, it may result in unstable transient response when the entire RF system is simulated in time domain at system level.

To address this issue, significant efforts have been made over the past decade to generate passive macromodels for RF circuits. A comprehensive survey of this topic can be found in [2]. Most passive macromodeling methods take a two-phase approach. In the first phase, an initial macromodel is extracted by using the conventional approaches (e.g., vector fitting [1]). Next, in the second phase, the initial macromodel, which is not necessarily passive, is further perturbed so that the resulting macromodel preserves passivity.

The existing passive macromodeling methods can be classified into three broad categories: (i) semi-definite programming (SDP), (ii) Hamiltonian eigenvalue perturbation, and (iii) iterative gain reduction. The SDP methods enforce passivity by adding a set of linear matrix inequality (LMI) constraints based on the positive real lemma (PRL) or bounded real lemma (BRL) [3]-[4]. The Hamiltonian eigenvalue perturbation approaches move the eigenvalues associated with a macromodel away from the imaginary axis, because any eigenvalue located on the imaginary axis implies passivity violation [5]-[7]. Finally, the iterative gain reduction techniques reduce the energy gain of a macromodel via small perturbation of its zeros based on local optimization [8]-[12]. More recently, several new macromodeling algorithms have been further developed to update the system poles to preserve passivity [13]-[15].

While the aforementioned passive macromodeling methods have been successfully applied to many practical applications, they suffer from various limitations. For example, several conventional macromodeling methods attempt to minimize the amount of perturbation when updating the zeros and/or poles to preserve passivity. In this case, the modeling error is not explicitly minimized and an inaccurate macromodel may be generated, especially if the initial macromodel extracted from the first phase heavily violates the passivity constraint and, therefore, cannot be converted to a passive macromodel with small perturbation. Other conventional macromodeling methods directly minimize the modeling error. However, they only update the zeros without changing the poles. Since the poles are not optimized, the resulting macromodel may not be optimal either.

In this paper, we propose a novel passivity-enforced macromodeling algorithm that iteratively updates both zeros and poles while explicitly minimizing the modeling error at each iteration step. The key idea is to formulate and solve a sequence of SDP problems where the modeling error is set as the cost function and the BRL is set as the constraint to guarantee passivity. In addition, a number of heuristic techniques (e.g., step size control) are developed to further improve the efficiency of our proposed algorithm. As will be demonstrated by the numerical experiments in Section 4, the proposed algorithm reduces the modeling error by 1.31-2.74× over other conventional methods for several commercial RF circuit examples.

The remainder of this paper is organized as follows. In Section 2 we review the important background for passive macromodeling, and then describe the proposed macromodeling algorithm in Section 3. The efficacy of our proposed approach is demonstrated by several commercial circuit examples in Section 4. Finally, we conclude in Section 5.

2. BACKGROUND

We focus on an *N*-port linear macromodel represented in its state-space form:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t)\end{aligned}, \qquad (1)$$

where **A**, **B**, **C** and **D** are the state-space matrices. The corresponding transfer function can be expressed as:

$$\mathbf{H}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}, \tag{2}$$

where s is the Laplace variable and $\mathbf{H}(s)$ is the N-by-N transfer function matrix of the N-port macromodel.

Assume that we are given the tabulated S-parameter data $\mathbf{H}_0(s_k)$ from the measurement where $k \in \{1, 2, \cdots, K\}$ and K is the total number of discrete frequency samples. Our goal is to find a passive macromodel that minimizes the total squared error between the tabulated data and the response of the macromodel:

$$\begin{aligned}\varepsilon_T &= \sum_{k=1}^{K} \left\| \mathbf{H}(s_k) - \mathbf{H}_0(s_k) \right\|_F^2 \\ &= \sum_{k=1}^{K} \left\| \mathbf{C}(s_k\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} - \mathbf{H}_0(s_k) \right\|_F^2\end{aligned}, \tag{3}$$

where $\|\bullet\|_F$ denotes the Frobenius norm of a matrix.

According to the BRL [16], the macromodel in Eq. (1) is passive if there exists a matrix $\mathbf{P}$ satisfying the LMI constraints:

$$\begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B} & \mathbf{C}^T \\ \mathbf{B}^T\mathbf{P} & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0 \tag{4}$$

$$\mathbf{P} = \mathbf{P}^T > 0, \tag{5}$$

where $\mathbf{P}$ is often referred to as the Lyapunov function matrix in the literature, and the symbols "≤" and ">" denote that the matrices on the left-hand side are negative semi-definite and positive definite respectively. If the LMI constraints in Eq. (4)-(5) do not hold for any matrix $\mathbf{P}$, the macromodel is not passive.

The constraint set defined by Eq. (4)-(5) is convex in terms of the matrix $\mathbf{C}$ [16]. Hence, we can directly enforce passivity of the macromodel by solving a convex optimization to perturb the matrix $\mathbf{C}$. In other words, once we have an initial non-passive macromodel extracted by a conventional approach (e.g., vector fitting [1]), we can update the matrix $\mathbf{C}$ by solving a convex optimization to make the macromodel passive. Note that since the matrices $\mathbf{B}$ and $\mathbf{C}$ together define the zeros, we can choose either $\mathbf{B}$ or $\mathbf{C}$ to optimize the zeros, where the poles determined by the matrix $\mathbf{A}$ are fixed. Without loss of generality, we consider $\mathbf{C}$ as the optimization variable in this paper.

To sum up, the conventional SDP approaches first identify an initial macromodel with the system matrices: $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$. If the resulting macromodel is not passive, we further perturb the matrix $\mathbf{C}$ based on a convex SDP problem that minimizes the perturbation [2]:

$$\begin{aligned} &\min_{\mathbf{C}_\Delta, \mathbf{P}} \quad \left\| \mathbf{C}_\Delta \right\|_F \\ &\text{S.T.} \quad \begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B} & (\mathbf{C} + \mathbf{C}_\Delta)^T \\ \mathbf{B}^T\mathbf{P} & -\mathbf{I} & \mathbf{D}^T \\ (\mathbf{C} + \mathbf{C}_\Delta) & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0, \\ &\qquad \mathbf{P} = \mathbf{P}^T > 0 \end{aligned} \tag{6}$$

or the modeling error [3]-[4]:

$$\begin{aligned} &\min_{\mathbf{C}, \mathbf{P}} \quad \sum_{k=1}^{K} \left\| \mathbf{C}(s_k\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} - \mathbf{H}_0(s_k) \right\|_F^2 \\ &\text{S.T.} \quad \begin{bmatrix} \mathbf{A}^T\mathbf{P} + \mathbf{P}\mathbf{A} & \mathbf{P}\mathbf{B} & \mathbf{C}^T \\ \mathbf{B}^T\mathbf{P} & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0 \\ &\qquad \mathbf{P} = \mathbf{P}^T > 0 \end{aligned}. \tag{7}$$

The conventional SDP approaches directly minimize the perturbation or the modeling error. However, the resulting macromodel is optimal for the given poles only (i.e. determined by the matrix $\mathbf{A}$). These poles are pre-extracted for the initial non-passive macromodel, but not necessarily optimized for the final passive macromodel of interest. In practice, the poles extracted for the initial non-passive macromodel can be highly biased due to a limited number of frequency samples, reduced model order and/or inevitable measurement noise. For instance, it has been shown that the vector fitting algorithm [1] may converge to a wrong macromodel if the measurement data is noisy [17]. It, in turn, motivates us to propose a new macromodeling algorithm to update both zeros and poles, while simultaneously minimizing the modeling error with guaranteed passivity. In what follows, we will describe our proposed algorithm in detail.

3. PROPOSED APPROACH

Our proposed macromodeling approach iteratively updates both poles and zeros with consideration of the passivity constraint. In this section, we describe its mathematical formulation and highlight the novelty.

3.1 Mathematical Formulation

In order to optimize both poles and zeros, we must further optimize the matrix $\mathbf{A}$, since $\mathbf{A}$ defines the poles. In other words, we must simultaneously consider the matrices $\mathbf{A}$, $\mathbf{C}$ and $\mathbf{P}$ as the optimization variables. Solving such an optimization, however, is not trivial, since both the error function in Eq. (3) and the BRL constraints in Eq. (4)-(5) become non-convex.

To overcome this difficulty, we propose an iterative approach to update poles and zeros in this paper. Our proposed algorithm starts from a given non-passive macromodel initially extracted by a conventional method (e.g., vector fitting [1]), and it iteratively perturbs the poles and zeros to minimize the modeling error. At each iteration step, we perform a first-order approximation to both the error function in Eq. (3) and the BRL constraints in Eq. (4)-(5), resulting in a convex SDP problem to locally update poles and zeros. The aforementioned local update is repeated until the modeling error cannot be further reduced.

To derive the detailed mathematical formulation of the proposed algorithm, we consider the i-th iteration step where the matrices $\mathbf{A}_i$, $\mathbf{C}_i$ and $\mathbf{P}_i$ should be perturbed by a small amount $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$:

$$\begin{aligned} \mathbf{A}_{i+1} &= \mathbf{A}_i + \mathbf{A}_{\Delta i} \\ \mathbf{C}_{i+1} &= \mathbf{C}_i + \mathbf{C}_{\Delta i} \\ \mathbf{P}_{i+1} &= \mathbf{P}_i + \mathbf{P}_{\Delta i} \end{aligned}. \tag{8}$$

In Eq. (8), $\mathbf{A}_{i+1}$, $\mathbf{C}_{i+1}$, and $\mathbf{P}_{i+1}$ are the new matrices of the perturbed macromodel. Based on Eq. (4)-(5) and (8), the following BRL constraints can be derived for the perturbed macromodel:

$$\begin{bmatrix} \begin{pmatrix} (\mathbf{A}_i + \mathbf{A}_{\Delta i})^T (\mathbf{P}_i + \mathbf{P}_{\Delta i}) + \\ (\mathbf{P}_i + \mathbf{P}_{\Delta i})(\mathbf{A}_i + \mathbf{A}_{\Delta i}) \end{pmatrix} & (\mathbf{P}_i + \mathbf{P}_{\Delta i})\mathbf{B} & (\mathbf{C}_i + \mathbf{C}_{\Delta i})^T \\ \mathbf{B}^T (\mathbf{P}_i + \mathbf{P}_{\Delta i}) & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C}_i + \mathbf{C}_{\Delta i} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0 \tag{9}$$

$$\mathbf{P}_i + \mathbf{P}_{\Delta i} = (\mathbf{P}_i + \mathbf{P}_{\Delta i})^T > 0. \tag{10}$$

Note that Eq. (9) does not represent an LMI constraint in terms of the matrices $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$, because it contains the cross-product terms:

$$\begin{aligned} (\mathbf{A}_i + \mathbf{A}_{\Delta i})^T (\mathbf{P}_i + \mathbf{P}_{\Delta i}) + (\mathbf{P}_i + \mathbf{P}_{\Delta i})(\mathbf{A}_i + \mathbf{A}_{\Delta i}) = \mathbf{A}_i^T \mathbf{P}_i + \mathbf{P}_i \mathbf{A}_i \\ + \mathbf{A}_i^T \mathbf{P}_{\Delta i} + \mathbf{P}_{\Delta i} \mathbf{A}_i + \mathbf{A}_{\Delta i}^T \mathbf{P}_i + \mathbf{P}_i \mathbf{A}_{\Delta i} + \mathbf{A}_{\Delta i}^T \mathbf{P}_{\Delta i} + \mathbf{P}_{\Delta i} \mathbf{A}_{\Delta i} \end{aligned}. \tag{11}$$

Therefore, the constraint set specified by Eq. (9) is not convex.

In order to set the BRL constraint in Eq. (9) as an LMI (hence, convex) constraint for the perturbed macromodel, we apply a first-order approximation to Eq. (11). In other words, we discard the cross-product terms $\mathbf{A}_{\Delta i}^T \mathbf{P}_{\Delta i}$ and $\mathbf{P}_{\Delta i} \mathbf{A}_{\Delta i}$ by assuming that both $\mathbf{A}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$ are sufficiently small. In practice, this assumption can be made valid by further constraining the norm of $\mathbf{A}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$ at each iteration step, as will be discussed later in this sub-section. After the cross-product terms are removed, the BRL constraint in Eq. (9) can be re-written as:

$$\begin{bmatrix} \begin{pmatrix} \mathbf{A}_i^T\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_{\Delta i} \\ +\mathbf{P}_{\Delta i}\mathbf{A}_i + \mathbf{A}_{\Delta i}^T\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_{\Delta i} \end{pmatrix} & (\mathbf{P}_i + \mathbf{P}_{\Delta i})\mathbf{B} & (\mathbf{C}_i + \mathbf{C}_{\Delta i})^T \\ \mathbf{B}^T(\mathbf{P}_i + \mathbf{P}_{\Delta i}) & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C}_i + \mathbf{C}_{\Delta i} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0 . \quad (12)$$

Eq. (12) represents an LMI constraint and, hence, the constraint set is convex in terms of the matrices $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$.

In addition to the passivity constraints based on the BRL, we need to further formulate the cost function to minimize the modeling error. To this end, we re-write the transfer function in Eq. (2) and the total squared error in Eq. (3) in terms of the matrices $\mathbf{A}_{\Delta i}$ and $\mathbf{C}_{\Delta i}$:

$$\mathbf{H}_{i+1}(s) = (\mathbf{C}_i + \mathbf{C}_{\Delta i})(s\mathbf{I} - (\mathbf{A}_i + \mathbf{A}_{\Delta i}))^{-1}\mathbf{B} + \mathbf{D} \quad (13)$$

$$\varepsilon_T = \sum_{k=1}^{K} \left\| (\mathbf{C}_i + \mathbf{C}_{\Delta i})(s_k\mathbf{I} - (\mathbf{A}_i + \mathbf{A}_{\Delta i}))^{-1}\mathbf{B} + \mathbf{D} - \mathbf{H}_0(s_k) \right\|_F^2 . \quad (14)$$

It is straightforward to observe that the transfer function $\mathbf{H}_{i+1}(s)$ at the i-th iteration step involves the inverse of a matrix containing $\mathbf{A}_{\Delta i}$. Moreover, the matrix inverse must be multiplied by $\mathbf{C}_{\Delta i}$ to obtain the transfer function value. For these reasons, the total squared error in Eq. (14) is not a convex function of $\mathbf{A}_{\Delta i}$ and $\mathbf{C}_{\Delta i}$.

To address this issue, we first simplify the matrix inverse in Eq. (13):

$$\begin{aligned} (s\mathbf{I} - (\mathbf{A}_i + \mathbf{A}_{\Delta i}))^{-1} &= ((s\mathbf{I} - \mathbf{A}_i) - \mathbf{A}_{\Delta i})^{-1} \\ &= \left((s\mathbf{I} - \mathbf{A}_i)\left(\mathbf{I} - (s\mathbf{I} - \mathbf{A}_i)^{-1} \cdot \mathbf{A}_{\Delta i}\right)\right)^{-1} \\ &= \left(\mathbf{I} - (s\mathbf{I} - \mathbf{A}_i)^{-1} \cdot \mathbf{A}_{\Delta i}\right)^{-1}(s\mathbf{I} - \mathbf{A}_i)^{-1} \end{aligned} . \quad (15)$$

Based on the matrix theory [18], the following first-order approximation holds for any square matrix $\mathbf{X}$ if the matrix $\mathbf{I} - \mathbf{X}$ is invertible:

$$(\mathbf{I} - \mathbf{X})^{-1} = \mathbf{I} + \mathbf{X} + \mathbf{X}^2 + \cdots \approx \mathbf{I} + \mathbf{X} . \quad (16)$$

Applying the aforementioned first-order approximation to Eq. (15), we have:

$$\begin{aligned} (s\mathbf{I} - (\mathbf{A}_i + \mathbf{A}_{\Delta i}))^{-1} &\approx \left(\mathbf{I} + (s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}\right)(s\mathbf{I} - \mathbf{A}_i)^{-1} \\ &= (s\mathbf{I} - \mathbf{A}_i)^{-1} + (s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1} \end{aligned} . \quad (17)$$

Note that the matrix inverse in Eq. (17) is now approximated as a linear function of $\mathbf{A}_{\Delta i}$.

We further substitute Eq. (17) into Eq. (13), yielding:

$$\begin{aligned} \mathbf{H}_{i+1}(s) \approx\ & \mathbf{D} + \mathbf{C}_i(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \mathbf{C}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \\ & \mathbf{C}_i(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \\ & \mathbf{C}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1} \end{aligned} . \quad (18)$$

Assume that both $\mathbf{A}_{\Delta i}$ and $\mathbf{C}_{\Delta i}$ are sufficiently small and the last term in Eq. (18) can be ignored, resulting in the following first-order approximation:

$$\begin{aligned} \mathbf{H}_{i+1}(s) \approx\ & \mathbf{D} + \mathbf{C}_i(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \mathbf{C}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \\ & \mathbf{C}_i(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} \end{aligned} . \quad (19)$$

The transfer function $\mathbf{H}_{i+1}(s)$ is now approximated as a linear function of $\mathbf{A}_{\Delta i}$ and $\mathbf{C}_{\Delta i}$ in Eq. (19).

Based on the first-order approximation in Eq. (19), the modeling error in Eq. (14) can be re-written as:

$$\varepsilon = \sum_{k=1}^{K} \left\| \begin{aligned} & \mathbf{D} + \mathbf{C}_i(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \mathbf{C}_{\Delta i}(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \\ & \mathbf{C}_i(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} - \mathbf{H}_0(s_k) \end{aligned} \right\|_F^2 . \quad (20)$$

The error function in Eq. (20) is quadratic in terms of $\mathbf{A}_{\Delta i}$ and $\mathbf{C}_{\Delta i}$. In addition, the model error is defined by the Frobenius norm and must be non-negative. Hence, the quadratic function in Eq. (20) is convex [16].

It is important to emphasize that our proposed first-order approximations in Eq. (12) and (20) are valid if and only if the perturbations $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$ at the i-th iteration step are sufficiently small. In order to guarantee this assumption, we must bound the maximum perturbations that are allowed. Such a goal can be accomplished by defining the upper bounds for the Frobenius norm of $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$:

$$\|\mathbf{A}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{A}_i\|_F \quad (21)$$

$$\|\mathbf{C}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{C}_i\|_F \quad (22)$$

$$\|\mathbf{P}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{P}_i\|_F , \quad (23)$$

where α is a user-defined parameter. Eq. (21)-(23) essentially implies that the matrices $\mathbf{A}_i$, $\mathbf{C}_i$, and $\mathbf{P}_i$ can be perturbed by a relatively equal amount at the i-th iteration step. From this point of view, the parameter α controls the "step size" of our proposed algorithm to update poles and zeros.

Finally, we combine the BRL constrains in Eq. (10) and (12), the error function in Eq. (20) and the perturbation constraints in Eq. (21)-(23), resulting in the following optimization problem:

$$\begin{aligned} \min_{\mathbf{C}_{\Delta i}, \mathbf{A}_{\Delta i}, \mathbf{P}_{\Delta i}} \quad & \sum_{k=1}^{K} \left\| \begin{aligned} & \mathbf{D} + \mathbf{C}_i(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \mathbf{C}_{\Delta i}(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} + \\ & \mathbf{C}_i(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{A}_{\Delta i}(s_k\mathbf{I} - \mathbf{A}_i)^{-1}\mathbf{B} - \mathbf{H}_0(s_k) \end{aligned} \right\|_F^2 \\ \text{S.T.} \quad & \begin{bmatrix} \begin{pmatrix} \mathbf{A}_i^T\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_{\Delta i} \\ +\mathbf{P}_{\Delta i}\mathbf{A}_i + \mathbf{A}_{\Delta i}^T\mathbf{P}_i + \mathbf{P}_i\mathbf{A}_{\Delta i} \end{pmatrix} & (\mathbf{P}_i + \mathbf{P}_{\Delta i})\mathbf{B} & (\mathbf{C}_i + \mathbf{C}_{\Delta i})^T \\ \mathbf{B}^T(\mathbf{P}_i + \mathbf{P}_{\Delta i}) & -\mathbf{I} & \mathbf{D}^T \\ \mathbf{C}_i + \mathbf{C}_{\Delta i} & \mathbf{D} & -\mathbf{I} \end{bmatrix} \le 0 \\ & \mathbf{P}_i + \mathbf{P}_{\Delta i} = (\mathbf{P}_i + \mathbf{P}_{\Delta i})^T > 0 \\ & \|\mathbf{A}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{A}_i\|_F \\ & \|\mathbf{C}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{C}_i\|_F \\ & \|\mathbf{P}_{\Delta i}\|_F \le \alpha \cdot \|\mathbf{P}_i\|_F \end{aligned} . \quad (24)$$

Eq. (24) represents a convex SDP that can be solved both efficiently (i.e., with low computational cost) and robustly (i.e., with guaranteed global optimum) [16]. By solving Eq. (24), we are able to incrementally update both poles and zeros at the i-th iteration step to explicitly minimize the modeling error. Such an update is repeated until the modeling error cannot be reduced any more.

3.2 Implementation Issues

To efficiently implement our proposed algorithm for pole/zero update, three practical issues must be carefully addressed: (i) model legalization, (ii) step size control, and (iii) frequency/port weighting. In this sub-section, we discuss these implementation issues in detail.

First, since the macromodel solved by Eq. (24) is based on first-order approximation, it may not exactly satisfy the BRL constraints. Hence, an additional "legalization" operation must be applied to further update the zeros so that the resulting macromodel is guaranteed to be passive. The aforementioned model legalization can be pursued by solving a convex SDP problem that is similar to Eq. (7).

Second, as shown in Eq. (24), the value of the step size α must be pre-determined, before we solve the SDP problem to update the macromodel. In practice, the best value of α is unknown in advance. Hence, we must adaptively determine its optimal value over iterations. In this paper, we adopt a simple heuristic approach where a set of possible α values are pre-defined over a wide range. At each iteration step, we solve the SDP problem in Eq. (24) with different values of α and, consequently, get a set of possible solutions for $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$. Among all these solutions, we choose the optimal one that leads to the minimum modeling error.

The corresponding value of α is then considered as the optimal choice for the current iteration step. Note that the proposed heuristic method can possibly choose different optimal values of α at different iteration steps.

It is also possible to apply different values of α to bound the norms of $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$ in Eq. (24) respectively. Such an approach, however, results in three independent parameters that we have to adaptively control. It, in turn, substantially increases the computational cost. For this reason, we simply use the same step size α for $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$ in order to make the computational cost tractable.

Finally, frequency/port weighting is another important implementation issue that we should study. In practice, there are many cases where the modeling error should be minimized in a weighted manner. For example, when modeling a passive bandpass filter, we can explore the trade-off between the modeling accuracy of the stop-band response and that of the pass-band response by adjusting the weight for the measurement data in different frequency bands. Consider an SPNT (single-pole-N-throw) multiport switch as another example. For this example, we are most interested in the frequency response of the ports in the ON state, instead of those in the OFF state. Hence, the ports in the ON state should be assigned to a larger weight than those in the OFF state.

In general, the total squared error in Eq. (3) can be replaced by the following weighted error for an N-port macromodel:

$$\varepsilon_W = \sum_{k=1}^{K}\sum_{m=1}^{N}\sum_{n=1}^{N} w_{k,m,n} \cdot \left|\mathbf{H}_{m,n}\left(s_k\right) - \mathbf{H}_{0,m,n}\left(s_k\right)\right|^2 \tag{25}$$

where $\mathbf{H}_{m,n}(s_k)$ and $\mathbf{H}_{0,m,n}(s_k)$ denote the (m, n)-th entry of $\mathbf{H}(s_k)$ and $\mathbf{H}_0(s_k)$ respectively, and $w_{k,m,n}$ is the corresponding weight defined by the user. By appropriately defining these weight values, the "importance" of the modeling error can be distinguished for different frequency bands and/or ports.

3.3 Summary

Algorithm 1: Passivity-enforced Macromodeling via Iterative Zero/Pole Update

1. Obtain an initial non-passive macromodel by applying a conventional approach (e.g., vector fitting [1]).
2. Enforce passivity by solving the SDP problem in Eq. (7). Generate a new passive macromodel and calculate the modeling error.
3. Set the iteration index $i = 1$ and initialize a set of possible step sizes $\{\alpha_1, \alpha_2, \cdots, \alpha_M\}$.
4. For each $\alpha \in \{\alpha_1, \alpha_2, \cdots, \alpha_M\}$
5. Solve the SDP problem in Eq. (24) to obtain $\mathbf{A}_{\Delta i}$, $\mathbf{C}_{\Delta i}$ and $\mathbf{P}_{\Delta i}$. If frequency/port weighing is applied, the cost function in Eq. (24) should be replaced by the weighted error function in Eq. (25).
6. Update $\mathbf{A}_{i+1}$, $\mathbf{C}_{i+1}$, and $\mathbf{P}_{i+1}$ by using Eq. (8).
7. Given the fixed $\mathbf{A}_{i+1}$, solve the SDP problem in Eq. (7) to obtain $\mathbf{C}_{i+1}$ and $\mathbf{P}_{i+1}$. If frequency/port weighing is applied, the cost function in Eq. (7) should be replaced by the weighted error function in Eq. (25).
8. Calculate the modeling error.
9. End For
10. Select the optimal macromodel with the least modeling error.
11. Set $i = i + 1$, and repeat Step 4-10 until the modeling error cannot be further reduced.

Algorithm 1 outlines the major steps of the proposed macromodeling method. It starts from an initial macromodel generated by the conventional method and iteratively updates the poles/zeros by solving a sequence of SDP problems. To find the optimal value of α, an inner loop is used to calculate the modeling error for each possible value of α. For a given α, we first update the poles and zeros by applying a first-order approximation to both the BRL constraints and the error function, as shown in Step 5. Next, an additional "legalization" operation is applied in Step 7 to further update the zeros so that the resulting macromodel is guaranteed to be passive. The aforementioned update on poles/zeros stops if the modeling error cannot be further reduced.

The proposed macromodeling algorithm relies on first-order approximation and, hence, is based on local optimization. It may not converge to the globally optimal model that minimizes the modeling error and simultaneously guarantees passivity. However, since our proposed approach takes the initial macromodel generated by a conventional approach, it always improves the modeling accuracy by further updating the poles and zeros. For this reason, the proposed method is able to offer superior modeling accuracy over the conventional approach, as will be demonstrated by the experimental results in Section 4.

Finally, it is important to mention that our proposed macromodeling algorithm is only described for S-parameters in this paper. It can be directly extended to model other network parameters (e.g., Y-parameters, Z-parameters, etc.), if we appropriately incorporate the PRL constraints into the optimization formulation. Due to limited space, the details of these extensions are not included in the paper.

4. NUMERICAL EXPERIMENTS

In this section we demonstrate the efficacy of the proposed passive macromodeling algorithm by two commercial RF switch examples. For testing and comparison purposes, three different macromodeling algorithms are implemented: (i) the conventional vector fitting method [1] (VF), (ii) the conventional passive macromodeling method [3]-[4] where the poles are first extracted by vector fitting and then the zeros are updated by SDP to meet the passivity constraints (SDP), and (iii) the proposed passive macromodeling method where both poles and zeros are simultaneously updated to minimize the modeling error (Proposed). In our implementation, the SDP problem is solved by the commercial solver MOSEK (www.mosek.com). All experiments are run on an i7-2600 3.4GHz computer with 8 GB memory.

4.1 Hittite RF Switch (HMC232LP4)

We first apply the proposed passive macromodeling algorithm to HMC232LP4, a broadband high isolation non-reflective GaAs SPDT switch from Hittite Microwave [19]. The 3-port scattering parameters are measured over the frequency range from 3.03GHz to 14.95GHz with the step size of 167.9MHz (i.e., 76 sampled frequencies in total). The measurement data is first processed by VF to generate a 10-pole macromodel as the initial starting point for both the conventional SDP approach and our proposed macromodeling approach.

Figure 1 and Figure 2 compare the modeling results for three different methods. Due to limited space, we only show the responses of S_{11} in this example. Figure 1 shows the in-band frequency responses, while Figure 2 plots the frequency responses over a substantially wider range.

Studying Figure 1 and Figure 2 reveals two important observations. First, the macromodel extracted by VF accurately matches the measurement data. It results in the least modeling error within the frequency range where the measurement data is available. In other words, VF can capture the in-band frequency response with high accuracy. However, the VF model shows an abnormally large gain over the frequency range where the measurement data is unavailable. As shown by the out-of-band frequency response in Figure 2, the maximum gain of the VF model reaches 20dB, implying that the macromodel is non-passive. Here, VF fails to guarantee passivity, because it does not force any passivity constraint during the modeling process. The aforementioned violation of passivity is a major limitation of the VF method, since a non-passive model may result in unstable transient response, when it is used to simulate the entire RF system in time domain. On the other hand, both SDP and our proposed method are able to generate a passive macromodel. As shown in Figure 2, the out-of-band frequency responses are always less than or equal to 0dB for both SDP and the proposed method, demonstrating that the corresponding macromodels are passive.

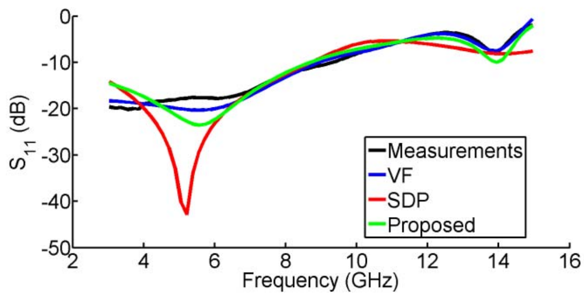

Figure 1. In-band frequency responses of S_{11} are shown for HMC232LP4 and its macromodels generated by three different methods.

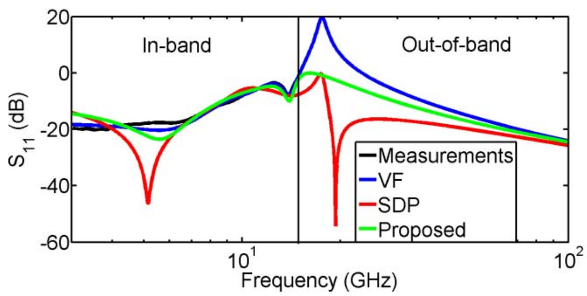

Figure 2. Out-of-band frequency responses of S_{11} are shown for HMC232LP4 and its macromodels generated by three different methods.

Table 1. Macromodeling error for HMC232LP4

	VF	SDP	Proposed
Modeling Error	0.87	5.10	1.86

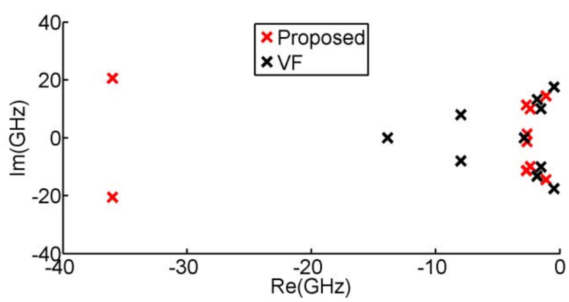

Figure 3. The poles are plotted on the complex plane for the HMC232LP4 macromodels generated by two different methods.

Second, Figure 1 reveals another important observation that the SDP model is not highly accurate, since its in-band frequency response substantially deviates from the measurement data. Namely, the conventional SDP approach sacrifices its modeling accuracy in order to enforce passivity. Our proposed method, however, closely matches the measurement data, while simultaneously preserving passivity. Table 1 further shows the modeling error defined by Eq. (3) for three different macromodeling methods. Note that the proposed approach reduces the modeling error by 2.74× over SDP in this example.

Finally, Figure 3 shows the poles of the macromodels generated by two different methods: the conventional VF method and our proposed approach. Based on Figure 3, we observe that most poles are perturbed locally to preserve passivity. However, there are two poles that are substantially changed. It, in turn, implies that VF cannot accurately capture all poles due to limited measurement data and, therefore, it is important to move the poles of the macromodel in order to enforce passivity in this example.

4.2 Mini-Circuits RF Switch (GSWA-4-30DR+)

In this example we consider an SP4T switch GSWA-4-30DR+ from Mini-Circuits [20]. The 5-port scattering parameters are measured over the frequency range from 300kHz to 4GHz. Similar to the previous example, we first generate a 10-pole macromodel by using VF and then update this initial macromodel to enforce passivity by using the conventional SDP method and our proposed method respectively.

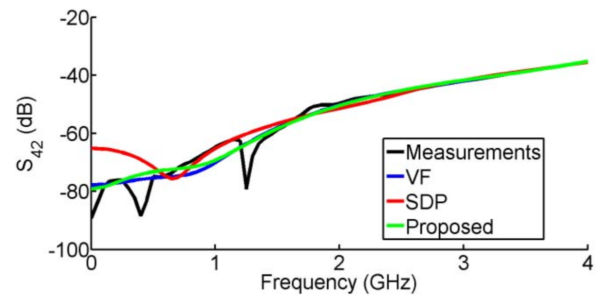

Figure 4. In-band frequency responses of S_{42} are shown for GSWA-4-30DR+ and its macromodels generated by three different methods.

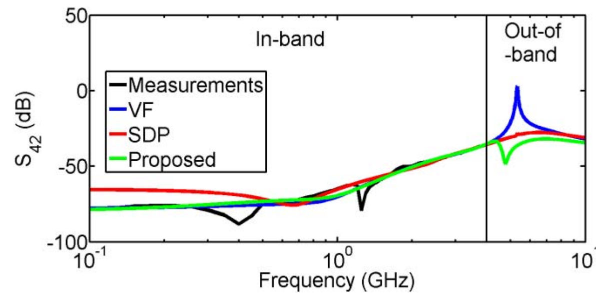

Figure 5. Our-of-band frequency responses of S_{42} are shown for GSWA-4-30DR+ and its macromodels generated by three different methods.

Table 2. Macromodeling error for GSWA-4-30DR+

	VF	SDP	Proposed
Modeling Error	0.12	0.17	0.13

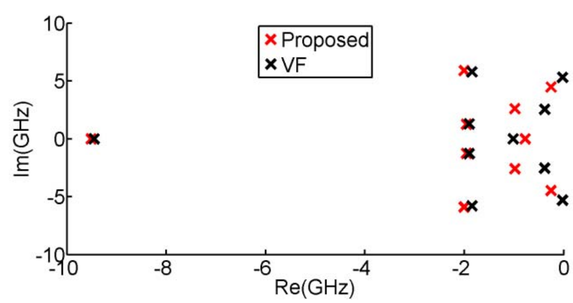

Figure 6. The poles are plotted on the complex plane for the GSWA-4-30DR+ macromodels generated by two different methods.

Figure 4 and Figure 5 summarize the frequency responses of S_{42} for three different macromodels. Similar to the previous example, VF generates a macromodel with highly accurate in-band response. However, its gain at high frequency can be as large as 2.5dB, thereby violating the passivity constraints. SDP enforces model passivity without updating the poles, yielding an inaccurate macromodel. Finally, our proposed method results in a macromodel that is almost as accurate as the VF macromodel, while simultaneously maintaining the passivity property.

Table 2 shows the modeling error for three different macromodeling methods. Since VF does not consider the passivity constraint, it is able to generate a macromodel with the most accurate in-band frequency response. Using such a non-passive macromodel for system-level simulation, however, may suffer from stability issues. On the other hand, the macromodel generated by our proposed method is also highly accurate. Compared to the conventional SDP method, our method reduces the modeling error by 1.34× in this example.

Figure 6 further plots the poles for the conventional VF method and our proposed approach. Note that most poles are not significantly changed in order to enforce passivity in this example. However, even with a small perturbation for the poles, the modeling error can be substantially reduced over the SDP method, as shown in Table 2.

5. CONCLUSIONS

In this paper, we propose a passive macromodeling algorithm that iteratively finds the optimal poles and zeros to minimize the modeling error based on measurement data, while simultaneously guaranteeing passivity. At each iteration step, a convex SDP problem is formulated and it can be solved both efficiently and robustly. The proposed macromodeling method has been validated for two commercial RF circuit examples where it achieves 1.31-2.74× error reduction over the conventional approaches. In our future research, we will further demonstrate our proposed macromodeling algorithm for a broad range of RF circuit blocks and incorporate the extracted macromodels into a system-level simulation tool to efficiently evaluate the system-level performance (e.g., bit error rate) of large-scale RF systems.

6. ACKNOWLEDGEMENTS

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