18-660: Numerical Methods for Engineering Design and Optimization

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Electrical & Computer ENGINEERING

Overview

- Introduction
  - Numerical computation
  - Examples and applications

Numerical Computation

- Numerical computation: approximation techniques and algorithms to *numerically* solve mathematical problems
  - Not all mathematical problems have closed-form solutions

- Mathematical examples
  - Solve nonlinear algebraic equations
  - Find minimum of a nonlinear function
  - Determine eigenvalues of a matrix

- This course will encompass various aspects of numerical methods and their applications
  - Linear & nonlinear solver, nonlinear optimization, randomized algorithm, etc.

Numerical Computation

- Numerical computation is an important tool to solve many practical engineering problems

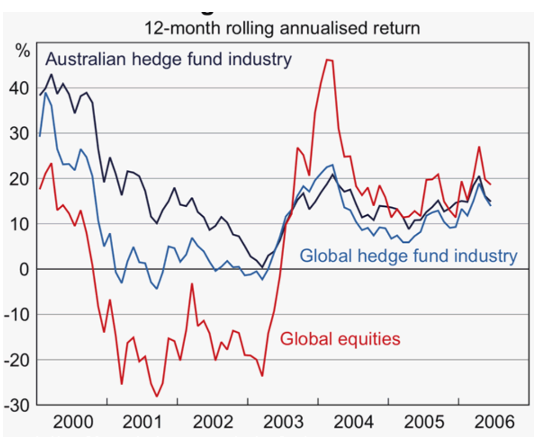
PageRank finds the eigenvector of a 10^{10}-by-10^{10} matrix – the largest matrix computation in the world

Airlines use optimization algorithms to decide ticket prices, airplane assignments and fuel needs

Numerical Computation

- Several additional real-world examples:

Car companies run computer simulations of car crashes to identify safety issues



Wall street runs statistical simulation tools to predict stock prices

Brief History of Numerical Computation

- Before 1950s

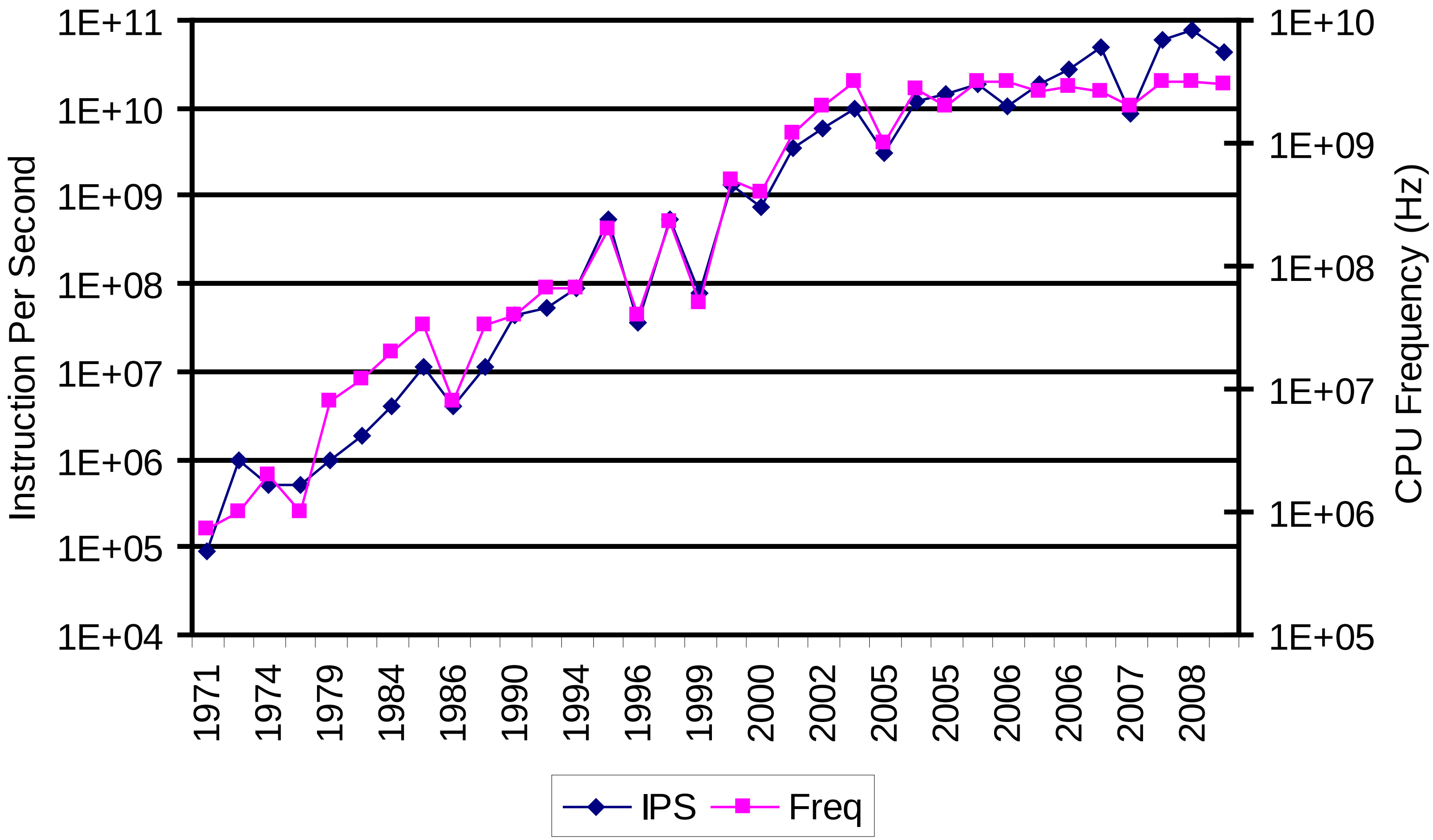
- Algorithms
  - Linear interpolation
  - Newton's method
  - Gaussian elimination

- Tools
  - Hand calculation
  - Formula handbook
  - Data table

Carl Friedrich Gauss (1777-1855)
German Mathematician

Brief History of Numerical Computation

- After 1950s
  - The invention of modern computers motivates the development of a great number of new numerical algorithms

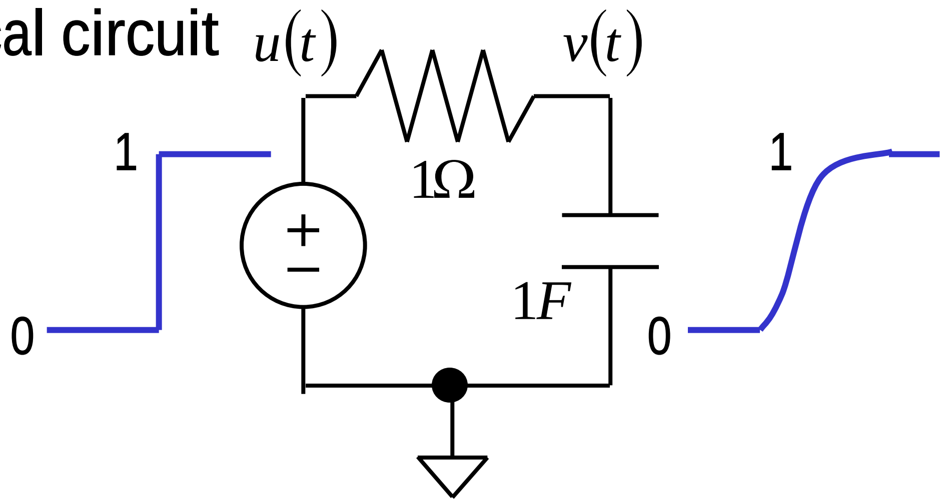


Numerical Computation Problems

- Ordinary and partial differential equations
- Regression
- Classification

Ordinary Differential Equation (ODE)

- Transient analysis for electrical circuit



$$\frac{dv(t)}{dt} = u(t) - v(t)$$

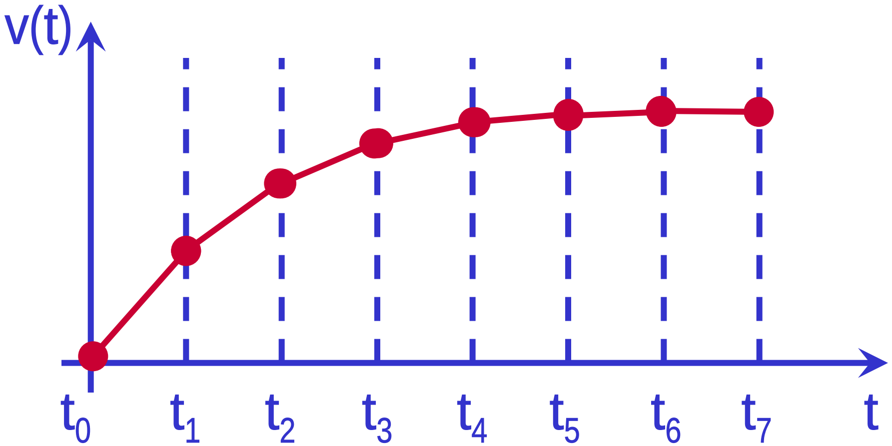
$$\frac{dv(t)}{dt} \approx \frac{v(t) - v(t-\Delta t)}{\Delta t}$$

$$\frac{v(t_1) - v(t_0)}{\Delta t} = u(t_1) - v(t_1)$$

$$\frac{v(t_2) - v(t_1)}{\Delta t} = u(t_2) - v(t_2)$$

$$\vdots$$

Algebraic equations



Partial Differential Equation (PDE)

- Thermal analysis: heat conduction is governed by partial differential equation (PDE):

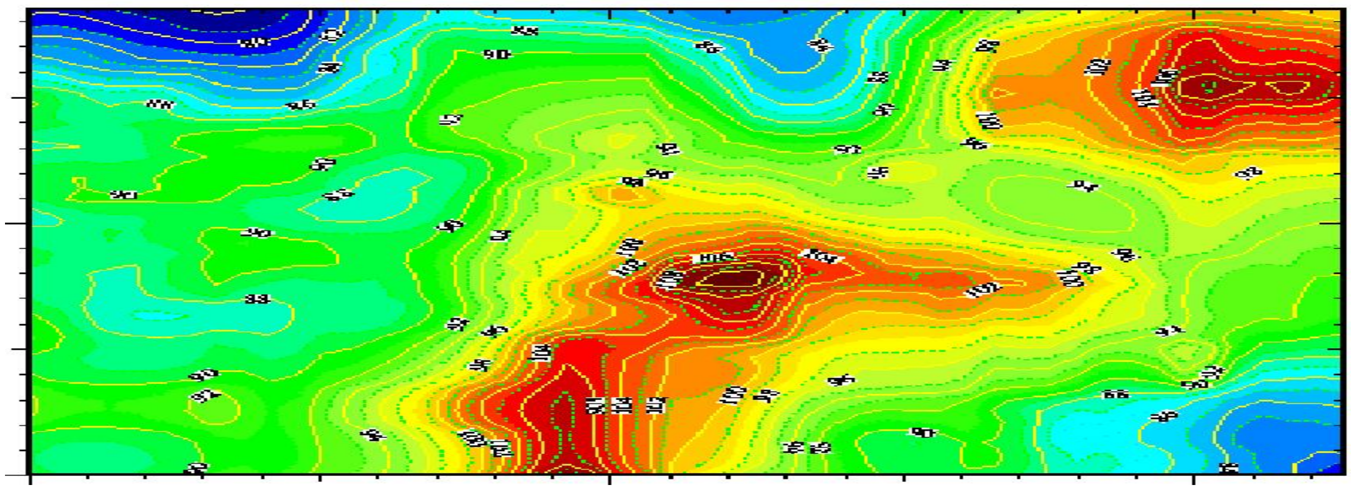
$$\rho \cdot C_p \cdot \frac{\partial T(x,y,z,t)}{\partial t} = \nabla \cdot \left[\kappa \cdot \nabla T(x,y,z,t)\right] + p(x,y,z,t)$$

Material density (kg/m^3)

Specific heat capacity (Joules/K-kg)

Thermal conductivity (Watts/K-m)

Power density of heat sources (Watts/m^3)



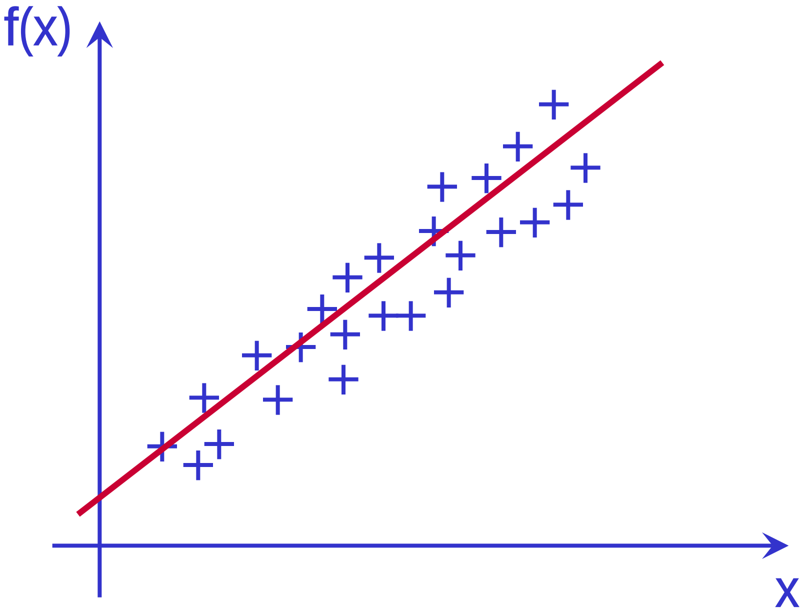
Linear Regression

- Linear regression (response surface modeling)

$$f(x) = ax + b$$

⇩

$$f(x_1) = ax_1 + b$$
$$f(x_2) = ax_2 + b$$
$$f(x_3) = ax_3 + b$$
$$\vdots$$



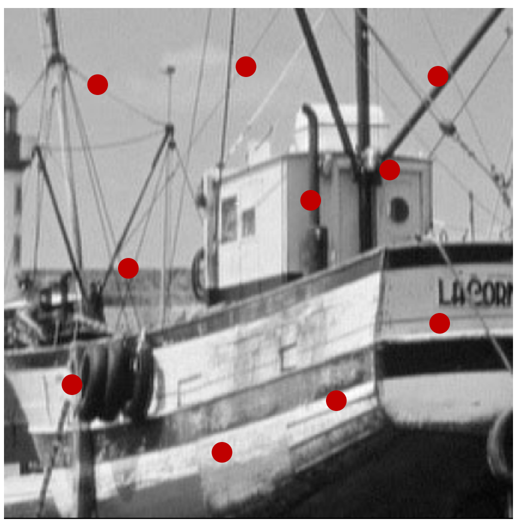
Solve over-determined linear equation to find a and b
(more equations than unknowns)

Linear Regression

- Linear regression with regularization

$$A\alpha = B \quad \Rightarrow \quad \min_{\alpha} \; \|A\alpha - B\|_2^2 + \lambda \cdot \|\alpha\|_1$$

Least-squares error — L_1-norm regularization



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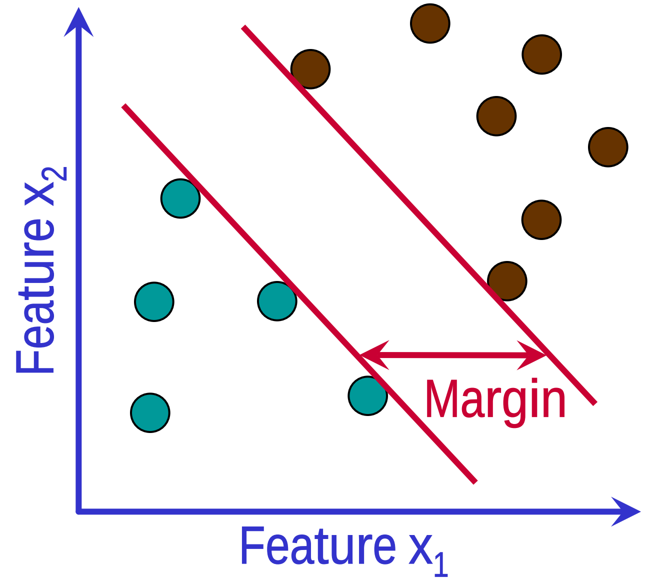
Classification

- Support vector machine (SVM)

$$f(X) = W^T X + C \begin{cases} \geq 0 & (Class\ A) \\ < 0 & (Class\ B) \end{cases}$$

SVM Coefficients Features



$$\min \quad \|W\|_2^2$$ → L_2-norm (maximize margin)

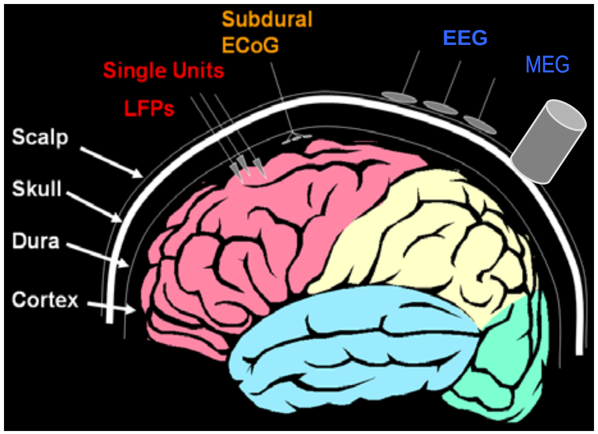
$$\text{S.T.} \quad W^T X + C \geq 0 \quad (X \in Class\ A)$$

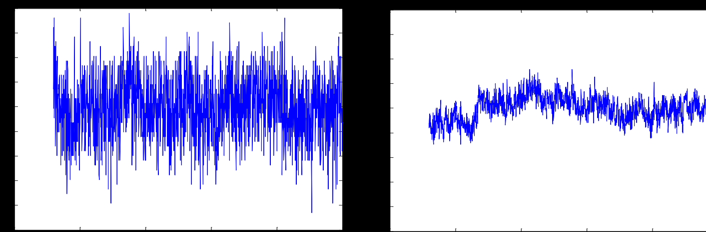
$$W^T X + C < 0 \quad (X \in Class\ B)$$

Solve convex quadratic programming to find W and C

Classification

- Brain computer interface based on classification

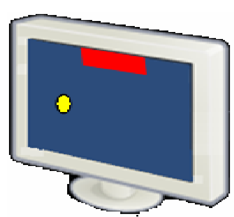




Methods: filtering; signal subspace methods …

Tool Boxes: MNE-Suite; FieldTrip; MaxFilter …

Neural signal recording → Neural signal processing → Movement decoding → Feedback



Summary

- Numerical computation
- Examples and applications