Novel mode content analysis technique for multimode waveguides

Pavel V. Nikitin, Daniel D. Stancil*, Ahmet G. Cepni*, Ariton E. Xhafa*, Ozan K. Tonguz*,
and Dagfin Brodtkorb**

University of Washington, Department of Electrical Engineering,
Seattle, WA 98195-2500, USA
*Carnegie Mellon University, Department of Electrical and Computer Engineering,
Pittsburgh, PA 15213-3890, USA
**ABB Corporate Research, Bergerveien 12, P.O.Box 90, N-1735 Billingstad, Norway

Abstract—**This paper presents a novel technique for measuring and analyzing the mode content in multimode waveguides. The technique is based on measuring the frequency response between the two probes coupled into a multimode waveguide and using that information to extract the average mode content in the frequency band of interest. This method is applicable to cases in which the mode amplitudes are approximately constant over the frequency band of interest. The direct application of this technique is channel characterization of an HVAC duct system, which behaves as a multimode waveguide network when used for indoor wireless communications at ISM band frequencies.**

I. INTRODUCTION

Using the heating, ventilation, and air conditioning (HVAC) duct system in buildings for communications is a promising way to provide a high-speed network access to offices [1]. A typical HVAC duct system is a complex network of hollow metal pipes of rectangular or circular cross-section which may extend to hundreds of meters. These pipes behave as multimode waveguides when driven at RF and microwave frequencies. The signal is coupled into and out of the ducts using coaxially-fed probe antennas mounted on duct walls.

Multimode dispersion in ducts and re-distribution of energy between different modes in such HVAC elements as T-junctions can significantly affect the capacity and the signal-to-noise ratio in the duct communication channel. Knowing the multimode content at various locations of HVAC duct system is important for characterizing channel properties and understanding the behavior of complicated HVAC elements.

When experimentally characterizing the HVAC duct system, mode content must be measured multiple times at various locations. That means that the mode content measurement technique must be simple, efficient, and non-destructive to an existing duct system. Moreover, only the modes to which a probe antenna is most sensitive to are important for channel analysis.

This paper describes a novel technique for the mode content analysis that satisfies all the aforementioned criteria. The remainder of this paper is organized as follows. Section II describes the previous work in the area of mode content measurement. The technique description is presented in Section III. Section IV contains experimental results and technique validation. Conclusions are given in Section V.

II. PREVIOUS WORK

All existing mode content determination approaches can be divided into four groups: scanning the field pattern, using mode-selective couplers, measuring open-end radiation pattern, and array processing. The first and the last approaches may overlap.

The scanning field pattern technique has been used by many researchers. Forrer and Tomiyuasu [2] used a moving probe to measure the electric field magnitude and phase at the walls of a waveguide and a Fourier analysis to compute the power flow in each mode. Fixed multiple-probe arrays were used by Price [3], Taub [4], and Levinson and Rubinstein [5]. Klinger [6] used a fixed probe and a moving short termination to measure the multimode content. Glock [7] used a fixed probe, a fixed termination, but moving (adjustable length) waveguide to perform necessary measurements.

The mode-selective coupling has been used by Lewis [8] and Beck [9], who employed a series of specially designed mode couplers to couple a mode to its own output port. Seguinot [10] used mode coupling technique for characterizing multimode microstrip lines.

Measuring the radiation pattern of an open-ended waveguide is typically used in high-power microwave engineering for extracting the mode content of a high-power source (magnetron, gyrotron, etc.) [11].

Array processing involves measuring the signal at the elements of an antenna array mounted on the waveguide and using those measurements for mode content extraction. To achieve good results, antenna locations must be carefully chosen and even optimized in the process of measurement. An excellent example of using an antenna array for mode measurement is described by Roper [12]. Other related work is presented in [13]-[14].

All of the techniques described above require a complicated experimental setup and a lengthy process of mode content measurement. Below we describe a novel technique for mode content analysis that requires only one sensing antenna and a network analyzer for determining the average mode content in the frequency band of interest.

III. TECHNIQUE DESCRIPTION

Consider a conceptual setup shown in Fig. 1. Assume that a multimode waveguide system is terminated on both ends with matched loads which prevent end reflections. The mode content is to be determined at the plane P, located at a distance L_o from the receiving antenna. The key information needed for mode content determination at this plane is the frequency response between the two coaxially-fed probe antennas coupled into a multimode waveguide system, where one antenna is transmitting, and another antenna is receiving (sensing the mode content). The frequency response measurement is performed by a network analyzer. Fixing the antenna position and sweeping the frequency allows to obtain independent measurements, somewhat similar to to fixing the frequency and moving the antenna along a slit in a waveguide wall.

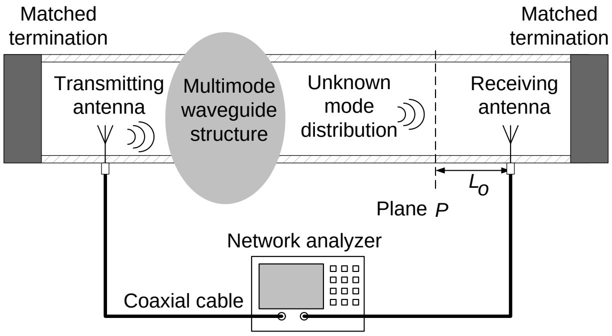

Fig. 1. Swept-frequency mode content measurement with a single antenna.

If mode amplitudes are weak functions of frequency in the band of interest, then system frequency response values measured at different frequencies can be used as independent sources of information to resolve the mode content. This allows one to find a set of approximate mode amplitudes, constant over the frequency band, which best approximate the measured frequency response, such as the one shown in Fig. 2.

Assume for simplicity that both probe antennas are identical, oriented in the same way, and the cross-section dimension of the waveguide system is the same at the transmitting and receiving antenna locations. Assume further that N modes can potentially propagate in a branch of our multimode waveguide system where the sensing antenna is located. The unknown N-dimensional vector of frequency-dependent complex mode amplitudes $\vec{X}(\omega)$ is to be measured at the cross-section plane P. The frequency response measured between the transmitting and the receiving antenna can be written in terms of $\vec{X}$ as

$$H(\omega) = \vec{C}(\omega)\vec{X}(\omega)\,, \tag{1}$$

where frequency-dependent vector $\vec{C}(\omega)$ describes a coupling between N waveguide modes and a sensing antenna.

Assume that mode amplitudes are weak functions of frequency and can be approximated as constants over the frequency band of interest:

$$\vec{X}(\omega) \approx [X_1\, X_2 \ldots X_N]\,. \tag{2}$$

Assume further that measurements are performed at M discrete frequency points. Then Equation (1) can be rewritten as

$$\vec{H} = \hat{A}\,\vec{X}\,, \tag{3}$$

where matrix $\hat{A}$ consists of elements $A_{mn} = C_n(\omega_m)$ and the components of M-dimensional vector $\vec{H}$ are $H_m = H(\omega_m)$. Linear system given by Equation (3) contains N unknowns and can be solved for $\vec{X}$ if the number of independent frequency measurement points is greater or equal to the number of modes to be determined.

Element A_{nm} represents the influence of mode n on the signal measured at frequency ω_m and can be calculated for generic antenna and waveguide using microwave theory:

$$A_{nm} = \frac{2Z_o}{(Z_o + Z_a)^2}\frac{Z_n p_n}{\mathcal{I}_n}e^{-\gamma_n L_o}\,, \tag{4}$$

where Z_o is a coaxial cable impedance, Z_a is the antenna impedance in a waveguide, p_n is the normalized power flow density in mode n, Z_n is the antenna impedance due to mode n, $\mathcal{I}_n$ is the integral that describes the interaction of antenna current with electric field of mode n, and γ_n is the waveguide propagation constant of mode n. Frequency-dependent quantities Z_a, Z_n, $\mathcal{I}_n$, γ_n can be explicitly calculated for any given frequency, waveguide cross-section, and antenna geometry. The analytical formulas for the above quantities in a special case of monopole probe antennas in cylindrical and rectangular waveguides can be found in [15] and are omitted in this paper for reasons of brevity.

IV. Results and validation

To validate our mode extraction technique, we performed experimental measurements on a straight cylindrical multimode waveguide where amplitudes of modes excited by monopole probe antennas can be theoretically calculated. We used metal cylindrical ducts of 12 inches (approximately 30.5 cm) in diameter as those are typical HVAC ducts used in the US. We performed measurements in the 2.4-2.5 GHz band which includes popular unlicensed wireless communication of 2.4-2.4835 GHz. This band allows propagation of 17 modes in 30.5 cm cylindrical ducts and does not contain any mode cutoff frequencies. The antennas used were 3.1 cm long (approximately quarter-wavelength at 2.45 GHz) coaxially-fed monopole probes located 14.6 m apart. In the experiment, waveguide ends were left open, which was a good approximation of matched load terminations. The respective distances from the antennas to open ends were 0.25 m and 0.38 m. The mode content was determined at the plane located L_o =14.5 m from the receiving antenna.

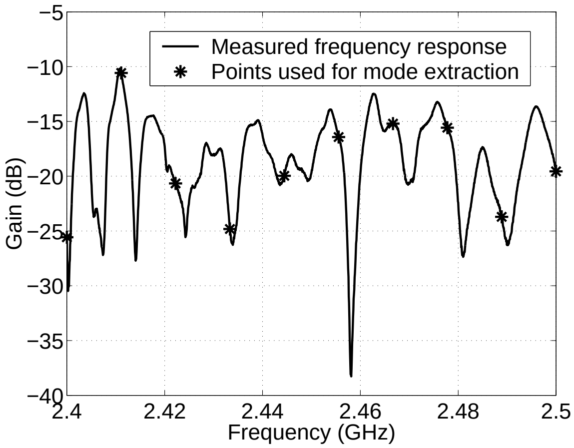

Fig. 2. Frequency response measured between two 3.1 cm monopole antennas located 14.6 m apart in a 30.5 cm straight cylindrical duct with open ends and frequency measurement points used for mode extraction.

Fig. 2 shows the frequency response measured between the antennas. The observed frequency response shape depends on the excited mode distribution and the distance between the antennas. Interference between the modes results in maxima and minima with specific widths, depths, and positions.

The assumption that mode coefficients are weak functions of frequency strongly depends on the waveguide size and operating frequency band. In the validation case considered here, analytical expressions for mode amplitudes can be theoretically found [15]. It can be shown that only the amplitude of mode TE_{61} notably changes over the 2.4-2.5 GHz band (the change is about 40%). Note that TE_{61} is a higher order mode, whose cutoff frequency is 2.35 GHz, which is very close to 2.4 GHz.

The spacing between the frequency points must be such that the frequency responses measured at different frequencies are sufficiently independent and solution to system given by Equation (3) can be found. The minimum spacing distance can be estimated from the autocorrelation function $S(\omega)$ of the frequency response defined as:

$$S(\omega) = \int_{\omega_1}^{\omega_2} H(\omega + y) H^*(y)\, dy \,, \tag{5}$$

where ω_1 and ω_2 are the lower and the upper frequencies in the band. Fig. 3 shows the normalized magnitude of the autocorrelation function computed for a frequency response shown in Fig. 2. The width of the central peak at the 50% signal correlation level (dashed line) can serve as an estimate for the coherence bandwidth, which gives a minimum frequency spacing for measurements. One can estimate from Fig. 3 that the coherence bandwidth is about 3.3 MHz, which means that 30 independent frequency measurements can fit into a 100 MHz band.

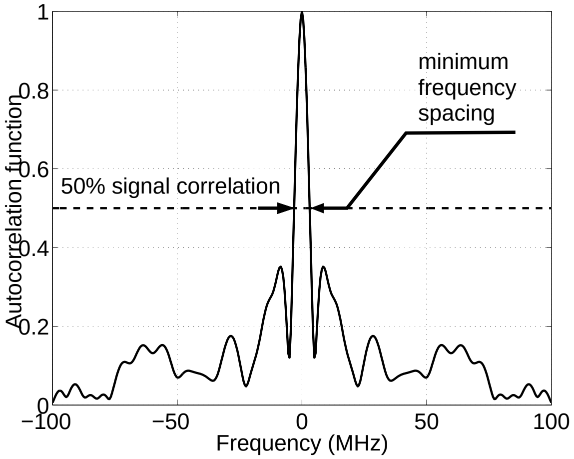

Fig. 3. Normalized magnitude of the frequency autocorrelation function for the frequency response shown in Fig. 2.

Although the number of potential frequency points that can be used for mode content analysis is larger than the maximum number of propagating modes, some modes are interacting with the sensing probe antenna very weakly, which leads to an ill-conditioned matrix $\hat{A}$. We extracted mode amplitudes for 5 modes sensed best by the 3.1 cm receiving monopole probe antenna: TE_{61}, TE_{51}, TE_{41}, TE_{31}, and TM_{01}. From reciprocity, those are also the modes most excited by the same antenna when the antenna is transmitting. Linear system given by Equation (3) was solved in *Matlab*[1] using the pseudo-

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[1] MATLAB is a registered trademark of The Mathworks, Inc.

inverse function (pinv) of matrix $\hat{A}$ for 10 equally spaced frequency points in the 2.4-2.5 GHz band.

Fig. 4 shows the normalized magnitude of theoretically calculated amplitudes of the five aforementioned modes and their values extracted from the measured frequency response using our technique. Theoretical values were averaged over the frequency band. One can see that theoretical and extracted mode amplitudes are in good agreement. The largest error is observed for the mode whose amplitude strongly varies with frequency (TE_{61}) and the modes whose interaction with the receiving antenna is very weak (TE_{31}, TM_{01}).

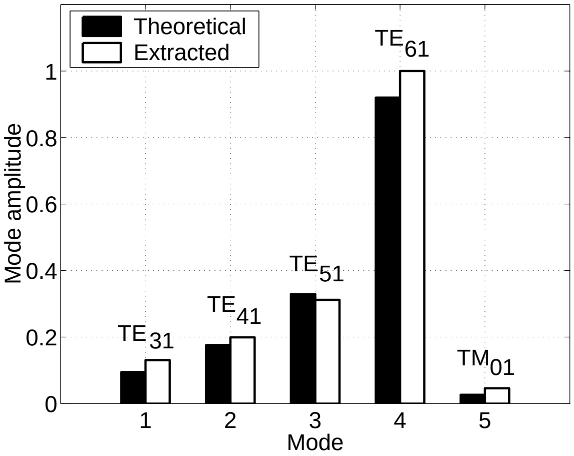

Fig. 4. Normalized magnitude of theoretically calculated mode amplitudes and their values extracted from the frequency response shown in Fig. 2 using our technique.

The accuracy of the mode extraction is determined by the condition of matrix $\hat{A}$, which depends on the characteristics of the sensing probe antenna and the location of the plane of measurement.

V. CONCLUSIONS

A novel mode content measurement technique for multimode waveguides is presented in this paper. The technique is based on using a single sensing antenna and measurements at different frequency points to obtain average mode amplitude values in the frequency band of interest.

The technique is applicable for the mode content extraction as long as no modes are present whose amplitudes are strong functions of frequency in the band of interest (this happens when mode cutoff frequencies are very near or within the operating frequency band).

The main advantage of the presented method is its simplicity, which makes it very attractive for quick estimation of mode content from the frequency response measured between any two points in a multimode waveguide system. This technique is a promising way of efficient characterization of large multimode waveguide networks, such as HVAC duct systems.

This method also allows one to analyze efficiently the mode transforming properties of multimode waveguide elements, which are difficult to model numerically or analytically. The work is under way to employ this method for obtaining the transfer matrices of such complicated HVAC elements as cylindrical T- and Y-junctions, which are known to alter the mode distribution travelling through them.

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