Signal Correlation in a Hallway Environment using Waveguide Mode Analysis

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***Abstract*—In this paper we express signal correlation in a waveguide in terms of waveguide modes. We investigate the effect of waveguide dimensions, element location and attenuation on transmitter and receiver correlation. An analytical approach for the calculation of the signal correlation is proposed, and the results are verified with simulations. Good agreement is achieved as well as huge savings in computation time. The analytical method provides intuitive explanation for the capacity behavior of a MIMO system in a hallway environment.**

Keywords—waveguide modes, signal correlation

I. INTRODUCTION

In recent years, a lot of attention has been drawn to multiple input- multiple output (MIMO) systems, because they can achieve very high spectral efficiencies [1]. This is especially important in wireless applications that are power, bandwidth and complexity limited. Several techniques that require advanced signal processing at the receiver (and possibly the transmitter) have been developed and have been demonstrated to achieve a hefty portion of the theoretically achievable capacity [2].

Assume a system with M transmitters and N receivers. Each transmitter sends an independent data stream with power Ex, so that the total transmitted power is $P_t = M E_x$. Let **T** be the channel transfer matrix, i.e. let T_{ij} be the channel gain from transmitter j to receiver i. If we assume independent zero mean additive Gaussian noise of power σ^2 on each receiver, the generalized Shannon capacity of this MIMO system is

$$C=\log_2[\det(\mathbf{I} + (E_x/\sigma^2)\,\mathbf{T}\mathbf{T}^H)] \quad (1)$$

where $\mathbf{T}^H$ is the Hermitian (complex conjugate transpose) of the matrix **T**.

The theoretical analysis conventionally treats the wireless channel between each transmitter and each receiver as a random process. A common assumption is that of a rich scattering environment, where power arrives with a uniform angle of arrival over $[0, 2\pi]$, which leads to Gaussian distributed elements of **T**. It has been theoretically and experimentally demonstrated that correlation of the elements of **T** leads to lower channel capacity [3-5].

Although general wireless environments are hard to model in a deterministic fashion, the waveguide model can be used as an approximation to describe the environment of a hallway. Propagation in this case can only occur in the waveguide modes and the channel transfer function for a given transmitter/ receiver pair is uniquely determined by the location and polarization of the transmitting/ receiving elements. It has been experimentally observed that the channel capacity is limited in a hallway [6], and the result has been theoretically interpreted with waveguide mode analysis [7].

In this paper we apply correlation calculations to the waveguide model, in order to provide an alternative explanation for the low capacity results in a hallway.

II. SYSTEM MODEL AND NOTATION

A. MIMO Capacity in terms of waveguide modes

Assume a waveguide of rectangular cross-section. The waveguide has dimensions a, b along the x- and y- axis respectively and is assumed of infinite length along the z-axis. The solution to Maxwell's equations in this structure are functions of the form $\cos(k_x(m)x)\sin(k_y(n)y)$. A signal of frequency f excites all modes (m,n) with gains a_{mn}. A mode propagates without losses if the signal frequency f is above the cut-off frequency f_c. If the frequency is less than the cut-off, then the mode is attenuated at a rate β_z (evanescent mode). For a given frequency, let L be the number of allowable propagating modes.

Following the analysis of [7], the electric field at any point (x, y, z)- z>0- is given by an equation of the form $E_{(x \text{ or } y)} = \underline{p}^T\underline{A}=\underline{A}^T\underline{p}$, where $\underline{A}$ is the vector of propagating mode coefficients (both TE and TM appropriately ordered) at z=0 and the vector $\underline{p}$ depends on the location and the polarization of the receiver.

Assume N_t sources, each one of which gives rise to a vector $\underline{A}$. Let **A** be a matrix the columns of which are those excitation vectors $\underline{A}$ (this is an LxN_t dimensional matrix). Assume also that N_r receivers are placed at different (x,y,z) locations. Let **P** be the matrix the columns of which are the position vectors $\underline{p}$ at the receiver locations. Under the narrowband assumption, the channel transfer matrix from the N_t transmitters to the N_r receivers is the matrix is $\mathbf{T}=\mathbf{P}^T\mathbf{A}$.

B. *Correlation definitions*

Let T_{ij} be the channel complex gain between transmitter j and receiver i. The complex correlation coefficient of T_{ij} and T_{kl} is defined as

$$\rho_{complex}(c,d) = \frac{E[cd^*] - E[c]E[d^*]}{\sqrt{E\left[|c|^2 - (|E[c]|)^2\right]E\left[|d|^2 - (|E[d]|)^2\right]}}, c = T_{ij}, d = T_{kl}$$

It is interesting to observe that two kinds of correlation affect the capacity of a MIMO system:

- Transmitter correlation: $\rho^{xmtr}=\rho(T_{ij}, T_{ik})$,
- Receiver correlation : $\rho^{rcvr}=\rho(T_{ij}, T_{kj})$

Using the channel transfer matrix definition introduced in section IIA, receiver correlation depends on the receiver location vector $\underline{p}$, while transmitter correlation depends on the excitation vector $\underline{A}$.

III. METHODOLOGY

A. *Receiver Correlation*

For a given excitation vector $\underline{A}$, let u, v be the values of the electric field at two locations $\underline{r}$ and $\underline{r}$+d$\underline{r}$. Without loss of generality, we assume that the receivers are on the same z-plane and that they pick up the same component of the electric field (E_x or E_y). Let $\underline{r}_{xy}$ =(Δx, Δy) be the projection of $\underline{r}$ onto the xy plane. We derive an expression for the correlation for a separation along either the x- or the y- axis (Δy=0 or Δx=0 respectively). The random variable in this case is the location $\underline{r}_{xy}$ over the allowable portion S of the waveguide cross-section C, i.e. S={$\underline{r}_{xy}$, such that $\underline{r}_{xy}$, $\underline{r}_{xy}$+d$\underline{r}_{xy}$ ∈C}. We assume uniform distribution of $\underline{r}$ on S. Then:

$u = E_{(x \text{ or } y)}(\underline{r})=\underline{A}^T\underline{p}(\underline{r})$, $v = E_{(x \text{ or } y)}(\underline{r}+d\underline{r})= \underline{A}^T\underline{p}(\underline{r}+d\underline{r})$

The purpose of this study is to express the correlation of u and v in terms of the waveguide modes. Let us define the following auxiliary vectors and matrices:

$E[u]=E[\underline{A}^T\underline{p}(\underline{r})]= \underline{A}^T E[\underline{p}(\underline{r})]= \underline{A}^T\underline{C}=\underline{C}^T\underline{A}$

$E[v]=E[\underline{A}^T\underline{p}(\underline{r}+d\underline{r})]= \underline{A}^TE[\underline{p}(\underline{r}+d\underline{r})]= \underline{A}^T\underline{D}=\underline{D}^T\underline{A}$

$|E[u]|^2=(E[u])^*(E[u])= \underline{A}^H \underline{C}^* \underline{C}^T\underline{A} = \underline{A}^H \mathbf{F} \underline{A}$

$|E[v]|^2=(E[v])^*(E[v])= \underline{A}^H \underline{D}^* \underline{D}^T\underline{A} = \underline{A}^H \mathbf{G} \underline{A}$

$E[|u|^2]=E[u^*u]= \underline{A}^H E[\underline{p}(\underline{r})^* \underline{p}(\underline{r})^T]\underline{A} = \underline{A}^H \mathbf{J} \underline{A}$

$E[|v|^2]=E[v^*v]= \underline{A}^H E[\underline{p}(\underline{r}+d\underline{r})^* \underline{p}(\underline{r}+d\underline{r})^T]\underline{A} = \underline{A}^H \mathbf{H} \underline{A}$

$E[uv^*]=E[\underline{A}^T\underline{p}(\underline{r})\underline{p}^H(\underline{r}+d\underline{r})\underline{A}^*]=\underline{A}^TE[\underline{p}(\underline{r})\underline{p}^H(\underline{r}+d\underline{r})]\underline{A}^*=\underline{A}^T\mathbf{Q}\underline{A}$

where

$$(\underline{C})_i = e^{jk_z(i)z}\int_S \cos\left(\frac{m(i)\pi}{a}x\right)\sin\left(\frac{n(i)\pi}{b}y\right)dxdy$$

$$(\underline{D})_i = e^{jk_z(i)z}\int_S \cos\left(\frac{m(i)\pi}{a}(x+\Delta x)\right)\sin\left(\frac{n(i)\pi}{b}(y+\Delta y)\right)dxdy$$

$$(\mathbf{J})_{ij} = e^{j(k_z(j)-k_z(i))z}\int_S \cos\left(\frac{m(i)\pi}{a}x\right)\cos\left(\frac{m(j)\pi}{a}x\right)\sin\left(\frac{n(i)\pi}{b}y\right)\sin\left(\frac{n(j)\pi}{b}y\right)dxdy$$

$$(\mathbf{H})_{ij} = e^{j(k_z(j)-k_z(i))z}\int_S \cos\left(\frac{m(i)\pi}{a}(x+\Delta x)\right)\cos\left(\frac{m(j)\pi}{a}(x+\Delta x)\right)\sin\left(\frac{n(i)\pi}{b}(y+\Delta y)\right)\sin\left(\frac{n(j)\pi}{b}(y+\Delta y)\right)dxdy$$

$$(\mathbf{Q})_{ij} = e^{j(k_z(i)-k_z(j))z}\int_S \cos\left(\frac{m(i)\pi}{a}x\right)\cos\left(\frac{m(j)\pi}{a}(x+\Delta x)\right)\sin\left(\frac{n(i)\pi}{b}y\right)\sin\left(\frac{n(j)\pi}{b}(y+\Delta y)\right)dxdy$$

Then the correlation can be expressed as

$$\rho^{RCVR}(\underline{dr}) = \frac{\underline{A}^T\mathbf{Q}\underline{A}^* - \underline{A}^H\underline{D}^*\underline{C}^T\underline{A}}{\sqrt{\underline{A}^H(\mathbf{H}-\mathbf{F})\underline{A}}\sqrt{\underline{A}^H(\mathbf{J}-\mathbf{G})\underline{A}}}$$

The exact expressions for the elements of the auxiliary vectors and matrices can be found in [8].

B. *Transmitter correlation*

1)Mode excitation

For the case of the transmitter correlation we need first to express the excitation vector $\underline{A}$ in terms of the transmitter location. The waveguide modes are functions of the form $\cos(k_x(m)x)\sin(k_y(n)y)$ and define a set of orthogonal eigenfunctions for the waveguide structure. The source excitation can then be written as a scaled sum of these eigenfunctions, where the scaling coefficients correspond to the projection terms. These projections are defined for all (m, n), however the modes (m, n) that do not satisfy the propagation condition are evanescent modes and are assumed to be sufficiently attenuated and can be ignored.

Let us assume that a transmitting antenna at a location (x_0, y_0) is a point source that produces a delta function field along the x- or the y- direction. The exact expressions for the mode excitation coefficients can be found in [6]. In reality, the source excitation is not a delta function and other factors such as the antenna gain pattern should be taken into consideration. However this can be performed by integration of the expressions for the delta function excitation.

2)Problem formulation

The formulation is similar to the one in Section IIIA. For a given receiver location $\underline{p}$, let $\underline{r}$ and $\underline{r}$+dr be the locations of two transmitters. We define:

$E[u]=E[\underline{p}^T\underline{A}(\underline{r})]= \underline{p}^T E[\underline{A}(\underline{r})]=\underline{p}^T\underline{C}=\underline{C}^T\underline{p}$

$E[v]=E[\underline{p}^T\underline{A}(\underline{r}+d\underline{r})]=\underline{p}^TE[\underline{A}(\underline{r}+d\underline{r})]=\underline{p}^T\underline{D}=\underline{D}^T\underline{p}$

$|E[u]|^2=(E[u])^*(E[u])= \underline{p}^H \underline{C}^* \underline{C}^T\underline{p} = \underline{p}^H \mathbf{F} \underline{p}$

$|E[v]|^2=(E[v])^*(E[v])= \underline{p}^H \underline{D}^* \underline{D}^T\underline{p} = \underline{p}^H \mathbf{G} \underline{p}$

$E[|u|^2]=E[u^*u]= \underline{p}^H E[\underline{A}(\underline{r})^* \underline{A}(\underline{r})^T]\underline{p} = \underline{p}^H \mathbf{J} \underline{p}$

$E[|v|^2]=E[v^*v]= \underline{p}^HE[\underline{A}(\underline{r}+d\underline{r})^* \underline{A}(\underline{r}+d\underline{r})^T]\underline{p} =\underline{p}^H \mathbf{H} \underline{p}$

$E[uv^*]=E[\underline{p}^T\underline{A}(\underline{r})\underline{A}^H(\underline{r}+d\underline{r})\underline{p}^*]=\underline{p}^TE[\underline{A}(\underline{r})\underline{A}^H(\underline{r}+d\underline{r})]\underline{p}^*=\underline{p}^T\mathbf{Q}\underline{p}$

Then the correlation can be expressed as

$$\rho^{XMTR}(\underline{dr}) = \frac{\underline{p}^T\mathbf{Q}\underline{p}^* - \underline{p}^H\underline{D}^*\underline{C}^T\underline{p}}{\sqrt{\underline{p}^H(\mathbf{H}-\mathbf{F})\underline{p}}\sqrt{\underline{p}^H(\mathbf{J}-\mathbf{G})\underline{p}}}$$

The exact expressions for the elements of the auxiliary vectors and matrices can be found in [8].

It is interesting to observe that the correlation coefficient depends on the exact location of the receiver. However one can take the expectation of the correlation expression over the cross-section of the hallway and derive an average value for the correlation coefficient. A similar calculation could not be performed for the receiver correlation coefficient because one would need a distribution over the space of excitation vectors.

IV. EXPERIMENTAL VALIDATION

The signal correlation over the cross-section of the hallway was also calculated by simulation in order to verify the correctness of the theoretical results. The fields u and v were calculated over a grid of $N_x x N_y$ points that uniformly covers the cross-section C of the hallway, and the correlation of u, v was then computed over this ensemble set.

The Nyquist sampling criterion states that the minimum sampling frequency is double the maximum signal frequency. In our case, the signals vary periodically in two-dimensional space, and the maximum spatial frequencies are

$$f_{x,MAX} = \frac{a}{2m_{MAX}}, m_{MAX} = \left\lfloor \frac{2a}{\lambda} \right\rfloor, f_{y,MAX} = \frac{b}{2n_{MAX}}, n_{MAX} = \left\lfloor \frac{2b}{\lambda} \right\rfloor$$

So $N_x > m_{max}, N_y > n_{max}$.

Figure 1 shows the results of the comparison for receiver correlation in a waveguide of dimensions a=0.7λ, b=1.4 λ, where all the modes are equally excited (grids:10x10 and 100x100).

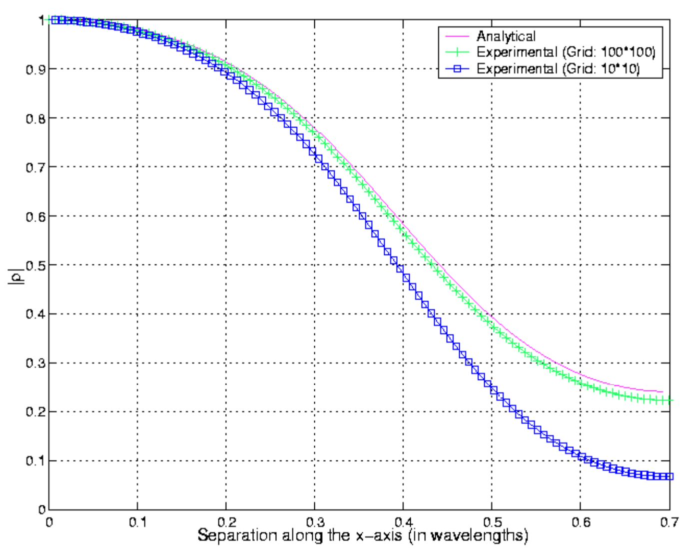

Fig. 1: Verification of theoretical results

Both methods provide similar results, and as the fineness of the grid increases, the experimental method approaches the theoretical results. The experimental method requires in the order of (N_xN_y +L N_xN_y) operations, whereas the theoretical calculation requires in the order of ($3L^2$+2L) calculations. Given that $L < m_{max}n_{max}$, and $N_x > m_{max}$, $N_y > n_{max}$, it is obvious that the theoretical calculation is preferable. This is especially true for large over-moded waveguides.

V. EFFECT OF GEOMETRY & ATTENUATION

A. Effect of waveguide dimensions

Assume that all the modes are equally excited with amplitude 1 and the same phase. We study two sizes of waveguides (for convenience all dimensions are given with respect to the wavelength):

- Small waveguide: a=0.7 λ,b=1.4 λ. Allowable modes: TE:(0,1), (0,2), (1,0, (1,1), TM: (1,1)
- Medium waveguide: a=5 λ, b = 10 λ. Allowable modes: 172 TE and 142 TM modes.

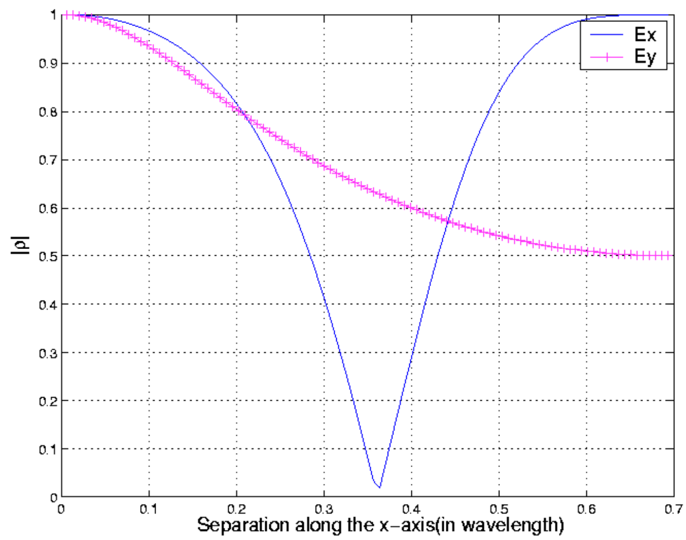

Fig. 2: Receiver correlation vs. Δx in a small waveguide

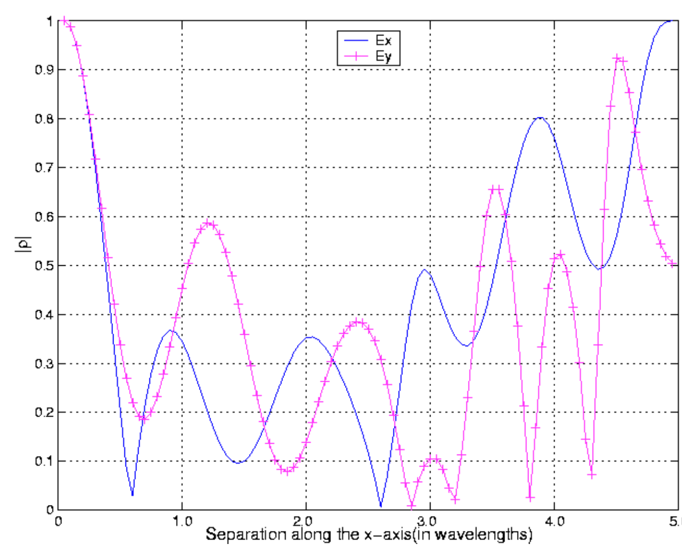

Fig. 3: Receiver correlation vs. Δx in a medium waveguide

The existence of more allowable modes in a larger waveguide gives rise to more ripples in the correlation dependence on separation. Also, the correlation coefficient is not a monotonically decreasing function of separation. This is due to the mode spatial distribution that is symmetrical or anti-symmetrical around the center of the waveguide. In a larger waveguide the range of separations over which the correlation is low (normalized to the total waveguide width) is also larger. Finally the range of separations over which the correlation is low depends on the component of the electric field that the receivers pick up.

B. *Effect of receiver location*

Figures 4 and 5 show the dependence of the transmitter correlation on the receiver location for transmitter antenna separations of $\Delta x = \lambda/2$ and $\Delta x = 4\lambda$ respectively, in a waveguide of dimensions $a=5\lambda$, $b=10\lambda$.

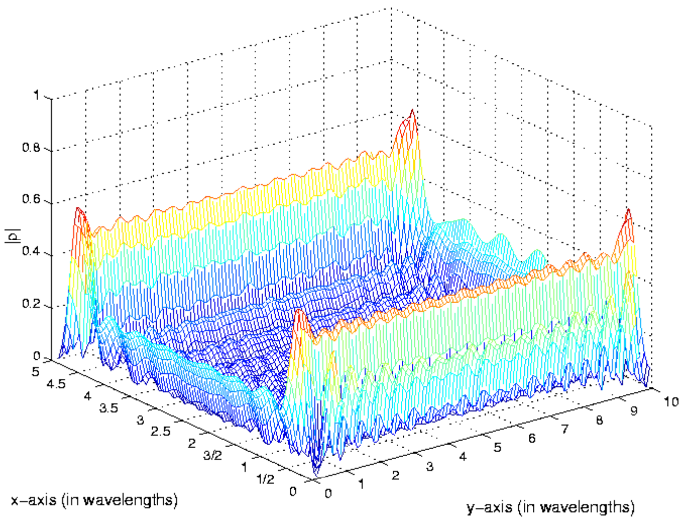

Fig. 4: Transmitter correlation for Ex field and $\Delta x= \lambda/2$

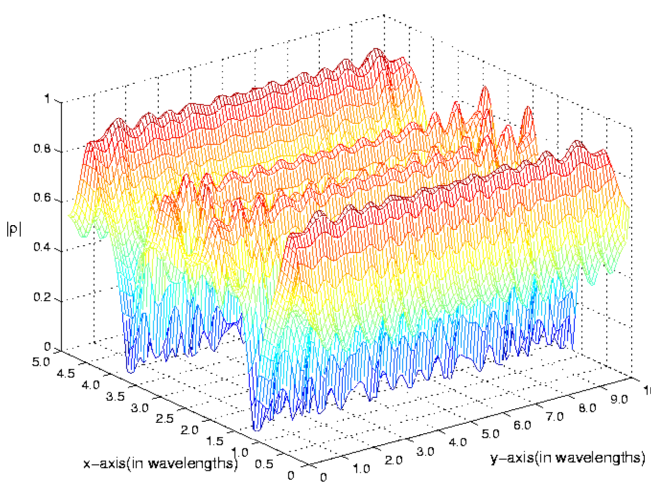

Fig. 5: Transmitter correlation for Ex field and $\Delta x=4\lambda$

The larger the transmitter separation, the larger the transmitter correlation dependence on the receiver location.

C. *Effect of attenuation*

The above analysis has been performed in the absence of attenuation. In reality, the dielectric material in the waveguide and the finite conductivity of the waveguide boundaries introduce losses. The precise expression for these losses can be found in [9].

An equivalent treatment of the problem that would better describe high losses is to assume a certain (field) reflection coefficient R<1 off the walls of the waveguide. Let the reflection coefficients for the electric fields on the boundaries parallel to the x- and y- axis respectively be R_x, R_y. Each propagating mode corresponds to a certain direction of propagation as described by the propagation vector $\underline{k} = (k_x, k_y, k_z)$, $|k|=2\pi/\lambda$, $k_x=m\pi/a$, $k_x = n\pi/b$. It has been shown in [6] that a mode (m, n) is attenuated at a rate

$$\alpha = -\left(\frac{\ln(R_x)}{a\tan\theta_x} + \frac{\ln(R_y)}{b\tan\theta_y}\right) \qquad \tan\theta_x = \frac{k_z}{k_x}, \tan\theta_y = \frac{k_z}{k_y}$$

Fig. 6 and 7 show how the receiver correlation varies with distance along the waveguide and separation Δx under the assumption of no attenuation (figure 6) and R=0.9 (figure 7). The analysis is performed for a waveguide of dimensions $a=0.7\lambda$, $b= 1.4\lambda$, assuming that all the modes are equally excited.

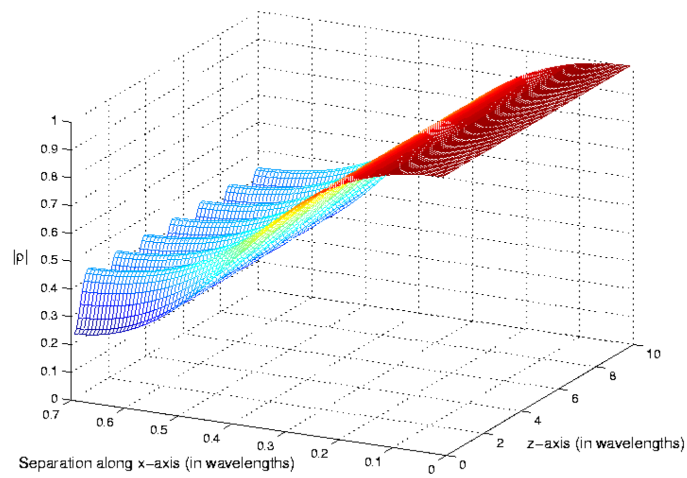

Fig. 6: Receiver correlation for Ex with a separation along the x-axis without attenuation

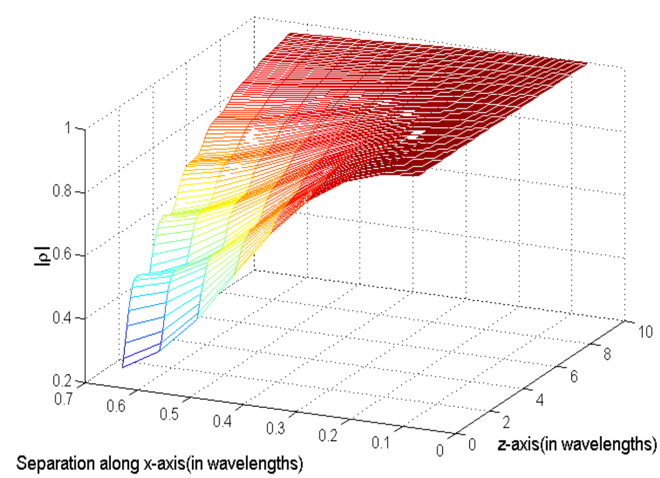

Fig. 7: Receiver correlation for Ex with a separation along the x-axis and R=0.9

Without losses, the correlation coefficient varies periodically with the distance along the hallway. This is due to the exponential dependence of the modes on z, and the same effect has been observed for the capacity dependence on distance in [7]. In the case of attenuation, the higher order modes (that correspond to more reflections per unit length) decay faster. At large distances, there are few significant modes, which do not allow enough degrees of freedom for the field to vary over the cross-section of the waveguide, and therefore the correlation coefficient approaches 1. The value of the reflection coefficient determines the rate at which the reflection coefficient increases.

D. Effect of excitation

In the examples studied above, all the modes have been assumed to be excited with the same amplitude and phase. Here, we assume that all the modes are excited with the same amplitude, and we explore the effect of phase randomization.

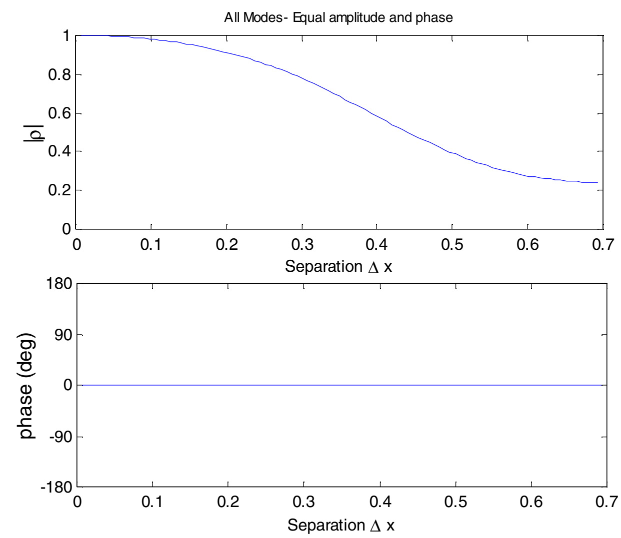

Fig. 8: ρ_{rcvr} (excitation: equal amplitudes and phases)

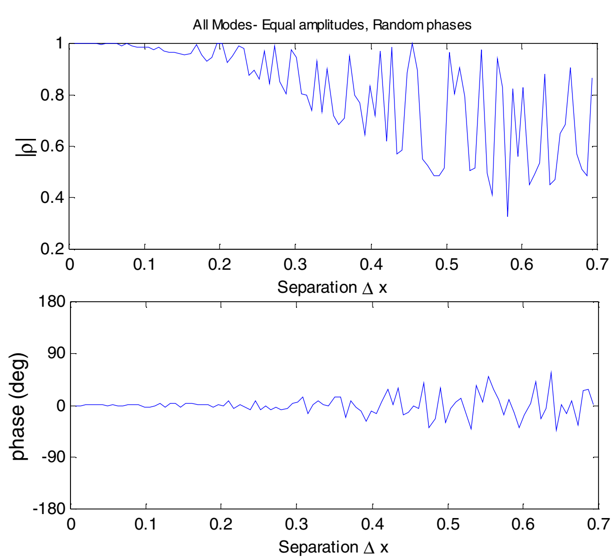

Fig. 9: ρ_{rcvr} (excitation: equal amplitudes, random phases)

Figure 8 shows the amplitude and the phase of the receiver correlation coefficient when all the modes have the same phase (0^o) and figure 9 performs a similar analysis for the case where they have the same amplitude but random phases (uniformly distributed over [0, 2π]). The calculations have been performed for the case of a small waveguide ($a=0.7\lambda$, $b=1.4\ \lambda$) and for receivers picking up the x-component of the electric field.

The introduction of phase randomization introduces a randomization in the phase and amplitude of the correlation coefficient as well. However the general trend with separation is preserved.

VI. CONCLUSIONS

In this paper, an analytical way to compute signal correlation in a waveguide structure has been proposed. The theoretical results have been verified by simulations. The size of the waveguide determines the width over which the receiver correlation coefficient is low. Moreover, the orientation of the receiver antennas affects the receiver correlation. Also, the value of the transmitter correlation for a given transmitting element separation depends on the exact receiver location. Finally the existence of losses (finite reflection coefficient off the waveguide boundaries < 1) increases the correlation coefficient. This is another way to account for the low MIMO capacity that has been experimentally observed in a hallway environment.

The results can be refined to include the reflection coefficient dependence on wave polarization and angle of incidence, and can be extended for the case of coupled waveguide modes.

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