CURRENT DISTRIBUTION ALONG A PROBE IN WAVEGUIDE

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ABSTRACT: There have been three different formulae for the current distritution along a probe in rectangular waveguide[1], but we did not know exactly which one is more accurate to describe the current distribution. In this paper, the question is answered using the moment method. The numerical solution of the current along the probe is obtained solving the surface integral equation, which is set up utilizing the boundary condition. The pulse bases and point-matching moment procedure is applied. A new approximate expression of the current is given using the curve-fitting techenique. The results show that the new expression is more accurate than any other available expressions,and it provides a good bases in solving other problems of posts in waveguide.

INTRODUCTION

It is well known that there are many kinds of discontinuties in a waveguide. One of the most common used is a probe in rectangular waveguide, shown in Fig.1. The conducting probe is with samll radius, variable-height and located arbitrarily on the broad wall of the waveguide. The usual sitiation of interest is that TE_{10} mode is the only propagation mode. The characteristics of the probe is well known qualitatively, that is to say the variation resulting, for example, from a change in height. There have been, however, few attempts at the analysis of the problem.

One of the earliest theoretical treatments of the problem was presented by Lewin[2] who used a variational procedure, the current distribution of the form $I_1(y)=\sin k(h-y)$ with a zero at the open end of the probe, $k=2\pi/\lambda$. Later, many other scholars used the same assumption to analyze such a problem. However, experimental measurments by Al-Hakkak[3] agree with the variational results using the expression I_1 above provided only the probe length does not exceed 0.6 of the guide height. Detinko and Levdikova[4] have considered the case where the current is constant along the probe, in attempt to reconcile the theory with experimental measurments on long probes. More recently, the problem has been investigated by Chang and Khan[1], their formulations actually relate to the problem of an infinitly thin strip rather than a probe, although it is possible to relate empirically the strip width and probe radius. Their analysis is similar to Lewin's, except that as well as investigated using I_1 as the assumed current distribution, they also investigated the use of $I_2(y)=\sin k_1(h-y)$ and $I_3(y)=\cos kz-\cos kh$, k_1 being determined by a variational approach. It was known

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that k_1 approximates $\pi/2h$. The theoretical results using I_2 or I_3 is closer to the measured results than those using I_1 below the cut-off frequency, or at higher, or large probe depth such that $h > \lambda/4$.

The key to achieving an accurate solution for this problem, is the accurate determination of the current distribution along the probe. Hence there is necessity to pay our attention to discuss the current distribution.

THEORY

The structure to be analyzed and the coordinates to be used are shown in Fig.1. The incidant electric field is assumed as follows:

$$\vec{E}^i = E_y^i \hat{Y} = \sin(\pi x/a)\exp(j\beta z)\hat{Y} \qquad (1)$$

where $\beta = \sqrt{(\pi/a)^2 + k^2}$, $k = 2\pi/\lambda$

When the probe radius is very small, as compared to waveguide dimensions, there is no appreciable phase change of incident wave across the probe, and hence the effect of the current will be in phase all around the probe. Now the current along the probe may be expressed as $\vec{I} = I(y)\hat{Y}$.

Since the problem is equivalent to that of an antenna radiating in a closed space, the radiation field of the probe can be expaned due to the eigenfunctions which form the solution space of the region under consideration. In the present case, the fields will be expanded in terms of the rectangular waveguide modes, that is

$$E_y^s = -jw\mu \int_0^h Gyy(r,r')I(y)dy' \qquad (2)$$

where the Green's function used here is given by Tai[5] in the form

$$Gyy = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{(2-\delta n)(k^2-k_y^2)}{abk^2\Gamma mn} \sin k_x x \sin k_x x' \cos k_y y \cos k_y y' \exp(-\Gamma nm|z|) \qquad (3)$$

with $kx = m\pi/a$, $ky = n\pi/b$, $\Gamma mn = \sqrt{k_x^2 + k_y^2 - k^2}$, $\delta n = 1$ (when $n=0$), or 0(when $n \neq 0$)

Since the total tangential electric field should be zero on the perfectly conducting probe surface S, then

$$\sin(\pi x/a)\exp(-j\beta z) + jw\mu \int_0^h Gyy(r,r')I(y')dy' = 0 \qquad \text{on S} \quad (4)$$

An exact solution of (4) can rarely be obtained. An approximate solution can be got by using the moment method. As shown in Fig.2 the probe is divided equally into M segments. each segment of which carries a constant current whose value is to be determined,

$$I(y) = \sum_{j=1}^{M} I_j P_j(y) \qquad (5)$$

where I_j is the unknow constant current on C_j. Pulse function

$$P_j(y) = \begin{cases} 1, & y \in C_j \\ 0, & y \notin C_j \end{cases} \qquad (6)$$

Just as the moment procedure used in analysis of a wire antenna in free space[6], the point-matching method is chosen and the integral equation (4) may be transferred into a matrix form

$$[Z_{ij}][I_j] = [V_i] \qquad (7)$$

where $[V_i]$ is the general voltage matrix, whose element is

$$V_i = \sin(\pi(x_0+R)/a) \qquad (8)$$

$[I_j]$ is the general current matrix, whose element is the unkown constant in (5), $j=1,2,\ldots,M$, $i=1,2,\ldots,M$.

[Zij] is the general impedance matrix, whose element is

$$Z_{ij}=Z_o+Z_n$$

$$Z_o=j\omega\mu\Delta l/(ab)\sum_{m=1}^{\infty}\sin(k_x(x_o+R))\sin(k_x x_o)/\Gamma_{mo} \qquad (9)$$

with $\Delta l=h/(M+1)$

$$Z_n=j\omega\mu/(abk^2)\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}2(k^2-k_y^2)\sin(k_x(x_o+R))\sin(k_x x_o)\cos(k_y y_i)/\Gamma_{mn}\int_{y_j}^{y_{j+1}}\cos(k_y y')dy' \qquad (10)$$

Zij is the sum of general voltages at matching point $(x +R,y\ ,0)$ of all Hmn modes simulated by unit current on Cj.

Since the convergent speeds of the series in (9) and (10) are very slow, especially that in(10). We must take some mathematical process to fast the speeds, so as to calculate the series approximately by a small number of terms, and the errors may be cotrolled.

Taking similar steps as that in [7], we obtain

$$Z_o=j\omega u\Delta l\ /(ab)\{-\sin\frac{\pi}{a}(x_o+R)\sin k_x x_o(j/\beta+a/\pi)+\sum_{m=2}^{\infty}\sin k_x(x_o+R)\sin k_x x_o(1/\Gamma_{mo}-a/m\pi)+ .5a/\pi\ln|\sin(\pi(2x_o+R)/2a)/\sin(\pi R/2a)|\} \qquad (11)$$

$$Z_n=j\omega u/(\pi R)^2\sum_{n=1}^{\infty}(k^2-k_y^2)/n(\sin k_y y'_{j+1}-\sin k_y y'_j)(K_o(k_n R)-K_o(k_n(2x_o+R)-K_o(k_n(2a-2x_o-R))) \qquad (12)$$

where $k_n=\sqrt{k_y^2-k^2}$, Ko is the modified Bessel function of the second kind,which decays rapidly to zero.

It shoud be pointed out that all the three terms in approximate expression (12) are important. But the authors of paper [1] and [7] have chosen only the first two terms. Usually, $2x_o+R$ and $2a-2x_o-R$ are at the same lever, when $x_o\rightarrow a$, the later is more important than the former.

Now, we can easily compute the current distribution by solving matrix equation (7).

RESULTS

With the help of computer, we have calculated the problem with many cases, Fig.3 shows two of them. The computing results show that the current distribution along the probe is

a). dependent on frequency and height, but almost independent on the ratio h/λ.

b). almost independent on position and radius.

Those characteristics may be explaimed by EM theory.Obviously, none of I_1, I_2, or I_3 can completly reflect those characteristics. We try to figuer out a more accurate pexpression for the current distribution. The curve-fitting techniqu is applied to the moment solutions, and a new formula for the distribution is yielded by

$$I_4=[1+\lambda/(8h)(y/h)^3]\cos(\pi y/2h) \qquad (13)$$

Compare I_4 and I_2, it is not difficult to find that I_4 is consist of I_2 and a modiyied term and can satisfy all the characteristics above. Fig.3 also shows the distrbutions of I_1, I_2, I_3 and I_4. It is observed that all the expressions are with zero at the end of the probe, I_4 is the closest with the moment solution, and I_1 the farest, especially with large h.

Furthermore, it is evident that the computing results of other characteristics of the probe, sush as equivelant impedence and resonance height, are more closer to the measured by using I_4 than using any others. The resonace frequency vs. probe height is shown in Fig.4. More results will be presented in the syposium.

A direct application of the procedure reported here is to analyze other characteristics of the probe. There is no need to assume the current distribution, unlike other papers, so more accurate results may be obtained. In addition, I_4 makes it possible to get a good result in analysis of a post or strip in waveguide.

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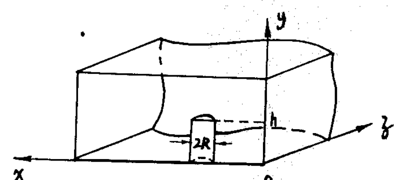

Fig.1

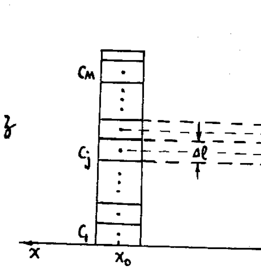

Fig.2

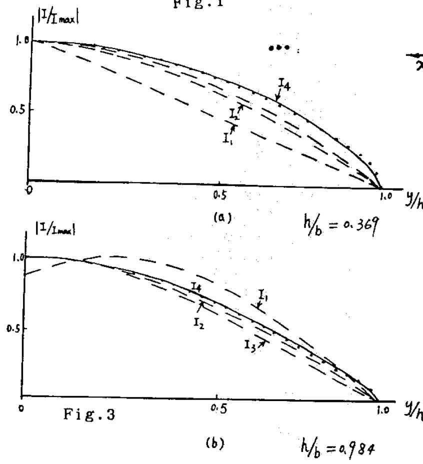

Fig.3

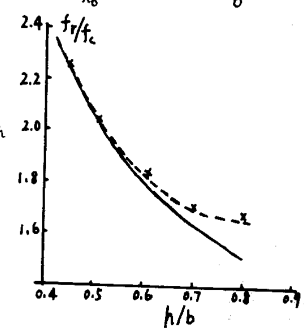

Fig.4