A Theoretical Framework for On-chip Stochastic Communication Analysis

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Abstract — **One of the greatest challenges of the emerging VLSI technology is the shift from design determinism to design uncertainty. Indeed, at nanoscale, the chip manufacturability entails failure increase and unpredictable behavior and thus, in order to ensure system-level fault-tolerance, we need to consider alternative paradigms for on-chip communication. This paper investigates the theoretical foundations and the design implications of a biologically-inspired communication approach which can be used for on-chip multiprocessor communication.**

I. INTRODUCTION

Due to the recent advances of CMOS technology in the deep sub-micron domain, there is an increasing need for new design and verification methodologies. From this perspective, moving towards packet-based communication via the Network-on-Chip (NoC) approach seems to be a natural choice [13]. However, the traditional acknowledgement/request protocols are not suitable for NoCs due to the communication overhead. At the same time, as the transistor size scales down, new quantum effects, such as neutron and alpha radiation, can affect the reliability and performance of multicore designs implemented on the same chip. As predicted by ITRS [11], various effects generated by cross-talk noise, soft errors, multiple clock domains, *etc.* can make the designs more unpredictable and more constrained in terms of power, performance and fault-tolerance. As a result, a new communication paradigm is needed to deal with the occurrence of unpredictable faults in future technologies [7].

Although not immediately obvious, the use of epidemic models to NoC communication looks like a win-win situation. Indeed, the epidemic-style communication can provide robust and scalable ways of disseminating information among multiple cores residing on the same chip. The process of information diffusion within a network resembles the propagation of a virus (or a rumor) in a group of individuals or population [1][3][12]; in our case, the "individuals" are just the NoC nodes consisting of routers and processing elements. The epidemic-style dissemination scheme exhibits a stable and reliable behavior in the presence of faults (*e.g.* damaged links, corrupted packets due to cosmic rays' radiation, *etc.*).

Several gossip protocols and simulation results were reported in [5][6], but a theoretical model for their analysis has not been proposed yet. To remedy this situation, we propose an analytical model for NoC stochastic communication and investigate its applicability to deriving performance metrics that can be used to asses various practical scenarios. At the same time, due to the architectural constraints specific to NoCs, the analytical model developed for on-chip stochastic communication is more general than the models proposed in epidemics [2] or gossiping [5]. As such, this new model can potentially advance the state-of-the-art in biological research too.

II. PREVIOUS WORK

The basic model for the rumor spreading process was introduced by Daley and Kendall [1]. Their model consists of three types of subpopulations, namely spreaders (S), ignorants (I) and stiflers (R). A *spreader* is an individual (or a node) who decides to send a distinct message (packet) to its neighbors according to a certain probabilistic rule. Further, we denote by $S(t)$ ($t > 0$) the total number of spreaders. An *ignorant* denotes a node which didn't receive any packet yet; that is, a node not yet included in the communication area. We denote by $I(t)$ ($t > 0$) the total number of ignorants. A *stifler* refers to an individual who received the packet and chooses to cease its transmission. Further on, we denote the number of stiflers by $R(t)$ ($t > 0$).

Similarly to epidemics, the interaction between a spreader and an ignorant results in converting the ignorant into a spreader at a rate proportional with SI, where S and I are the number of spreaders and ignorants. The interaction between two spreaders or between a spreader and a stifler may result in two stiflers or one stifler and one spreader. The rates at which these transitions can take place are considered to be proportional to $0.5S(S-1)$ for spreader-spreader interaction, and to SR for spreader-stifler interaction. Another approach to rumor spreading is the Maki-Thompson model [2]. The fundamental difference between these two approaches is that the Maki-Thompson model considers that the interaction between two spreaders results into a new stifler at a rate twice faster compared to the Daley-Kendall model, *i.e.,*$S(S-1)$. A unified treatment of these two models is given by Pearce in [4].

The above mentioned approaches inspired the so-called *gossip protocols* for information dissemination in computer systems. Gossip protocols provide robustness, simplicity, and scalability to the communication paradigm. Examples include the USENET News protocol [8] and the randomized gossip algorithms for lazy update of objects in a wide area data-base system [9]. One step further was established in [5] which addresses the reliability of multicast protocols and defines some strong reliability properties, such as atomicity, ordering, multicast stability, detection of lost packets and scalability.

A probabilistic broadcast algorithm for on-chip fault-tolerant communication is proposed in [6]. Similar to the randomized gossip protocols in [9], the algorithm in [6] diffuses the packets from one node to its immediate neighbors in a randomized fashion. The authors illustrate some important properties, *e.g.* fault-tolerance, low latency, design flexibility, *etc.* but no theory for their approach is proposed. This is precisely the issue which motivates the present work. Specifically, we make two contributions: First, we propose a Markovian model for NoC stochastic communication. Second, we derive an analytical solution for the master equation which enables model validation via a coverage metric. In other words, the present work explains *why* the on-chip stochastic communication is possible and *how* its coverage (*i.e.,* the number of reached nodes) can be analytically estimated.

A fundamental difference between our work and other studies in epidemics is that the theoretical analysis for the NoC stochastic communication considers explicitly the network topology. Indeed, as opposed to previous work in epidemics (or rumor spreading) where the interactions between spreaders and ignorants take place randomly, in our analysis the spreaders always interact *only* with their immediate neighbors; this limited interaction is dictated by the network topology which also influences the dynamics at which the interactions occur. Considering the network topology explicitly adds a significant complexity to modeling and analysis of such systems and makes our contribution quite unique.

To summarize, the fixed topology among different cores residing on the same chip introduces new constraints and challenges to the random communication problem. This issue is addressed next.

III. PROBLEM FORMULATION

A. Modeling the Information Dissemination Process

Starting from Daley and Kendall's framework [1], we consider a grid network of N^2 nodes able to communicate as in a NoC architecture. Moreover, we classify the entire population of nodes into spreaders, ignorants and stiflers. Initially, $S(0)=N_1$, $I(0)=N_2$ and $R(0)=N^2-N_1-N_2$, while for time $t>0$, the following conservation law is satisfied:

$$S(t)+I(t)+R(t) = N^2 \qquad \textbf{(1)}$$

The process $\{(S, I, R)(t), t \geq 0\}$ can be described by a continuous time Markov process for which the initial state and the infinitesimal transition probabilities are specified. Since at $t=0$, $(S, I, R)(0) = (N_1, N_2, N^2-N_1-N_2)$, it is sufficient to deal with the stochastic processes $S(t)$ and $I(t)$ in order to ensure the system evolution. We consider next the following interactions:

a) **Spreader-Ignorant:** For a mesh topology, using the law of mass action[1] [2], we can distinguish four cases of spreader-ignorant interaction as follows:

$$P\{S(t+h)=s, I(t+h)=i|S(t)=s-k, I(t)=i+k\}= \\ =\alpha_k(s-k)(i+k)h+O(h), \quad k = 1, 2, 3, 4 \qquad \textbf{(2)}$$

where $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are the rates that characterize the interaction between any spreader and one, two, three, and four ignorant neighbors, respectively, and $O(h) \to 0$. For instance if k=1, starting with s-1 spreaders and i+1 ignorants (*i.e.*, initial state *(s-*1*,i+*1*)*), we end up with s spreaders and i ignorants (*i.e.*, final state *(s,i)*) over an h interval of time. From the law of mass action, this happens at rate proportional with $\alpha_1(s-1)(i+1)$. It is important to note that the interaction in Equation 2 is ruled by the network topology. For the mesh topology, each node can potentially have four such interactions (each for k=1÷4). Simply speaking, Equation 2 gives the probability of having s spreaders and i ignorants at time t+h, given the system previous state (at time t) with s-k spreaders and i+k ignorants.

b) **Spreader-Spreader:** The interaction between two spreaders has two possible outcomes: Either both spreader nodes decide to cease the dissemination process with a rate β_2 (case in which the number of s senders decreases to *(s-*2*)*) or, one node stops sending packets, while the other continues forwarding them into the network with a rate β_1(case in which the s senders decrease to *(s-*1*)*). These two cases can model, for instance, the behavior of a link between two congested nodes. The transition probability for these first two cases is given by:

$$P\{S(t+h)=s, I(t+h)=i|S(t)=s+m, I(t)=i\}= \\ =\beta_m(s+m)h+O(h), \quad m = 1, 2 \qquad \textbf{(3)}$$

c) **Spreader-Stifler:** This interaction transforms the spreader node into a stifler one with a rate α_5. This is equivalent to the case where the corrupted packets from a faulty node affect an otherwise healthy neighbor. The transition probability is given by:

$$P\{S(t+h)=s, I(t+h)=i|S(t)=s+1, I(t)=i\}= \\ =\alpha_5(s+1)(N^2-s-i-1)h+O(h) \qquad \textbf{(4)}$$

d) **State Preservation:** The last non-zero transition probability can occur for $s+i \leq N_1+N_2$ and $i \leq N_2$, and it is given by:

$$P\{S(t+h)=s, I(t+h)=i|S(t)=s, I(t)=i\}= \\ =1-s\left\{\sum_{k=1}^{4}\alpha_k i+\sum_{m=1}^{2}\beta_m+\alpha_5(N^2-s-i)\right\}h-O(h) \qquad \textbf{(5)}$$

To give some intuition, we consider stochastic packet dissemination in a 5×5 mesh network. In Figure 1, we set the nodes 9 and 21 to be the sources for packet dissemination. Source 21 can follow the spreader-ignorant interaction and send packets to its neighbors (*i.e.* nodes 16 and 22) with rate α_2. Further, nodes 16 and 22 can forward their packets with rates α_2 and α_1, respectively. Node 22 sends the packet with α_1 rate (instead of α_2) because the dotted link between nodes 22 and 17 is assumed to be damaged. We observe that node 17 does not send any packet and becomes a stifler with rate α_5. Subsequently, nodes 11, 12 and 13 behave similarly as they describe new spreader-ignorant interactions.

By the same token, the links between the neighboring nodes 6, 11 and 13, 18 are considered damaged. Source 9 can send a packet to its neighbors with probability α_4. Later, nodes 8, 13, and 14 try to disseminate their packets. If the network is overloaded, we may notice new interactions. For instance, it can happen that only one of the nodes 8 or 13 disseminates its packets; this transition probability (β_1) is shown on the link between them (spreader-spreader interaction). Moreover, if nodes 13 and 14 are blocked due to congestion, then we end up with two nodes blocking the dissemination process (β_2). The next analytical derivations provide a closed-form solution for the packet diffusion process.

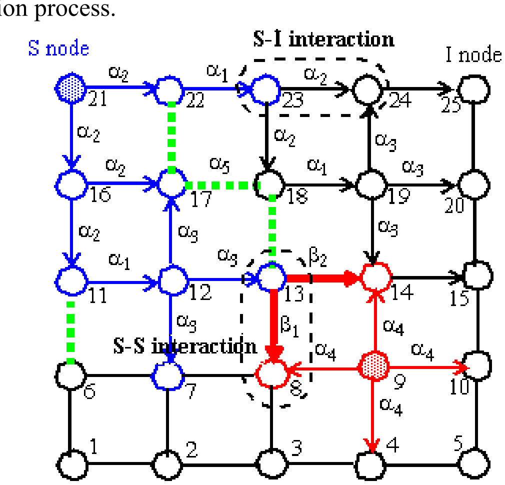

Figure 1. Stochastic dissemination in a 5x5 mesh network. The dotted lines show damaged links. The lines with arrows represent packet communication

B. Master Equation for Stochastic Packet Dissemination

We denote by *P(s,i,t)* the probability that, at time t, we have s spreaders and i ignorants which did not hear the rumor in a closed population of N^2 members. The number of stifler nodes is simply $r = N^2-s-i$. We can now write the following forward Kolmogorov equation for the considered Markov chain:

$$\frac{dP(s,i,t)}{dt} = -s\left\{\sum_{k=1}^{4}\alpha_k i+\sum_{k=1}^{2}\beta_k+\alpha_5(N^2-s-i)\right\}P(s,i,t)+ \\ +\sum_{k=1}^{4}\alpha_k(s-k)(i+k)P(s-k,i+k,t)+ \qquad \textbf{(6)} \\ \sum_{k=1}^{2}\beta_k(s+k)P(s+k,i,t)+\alpha_5(s+1)(N^2-s-i-1)P(s+1,i,t)$$

Since the model considered above describes a finite-state Markov chain, it is possible to determine the exact value of probability $P\{S(t)=s, I(t)=i|S(0)=N_1, I(0)=N_2\}$ by enu-

1.The law of mass action states that the transition probabilities scale with the size of the system.

merating the finite set of paths that connect the two states $\{(S, I)(t) = (s, i)\}$ and $\{(S, I)(0) = (N_1, N_2)\}$.

Multiplying the forward Kolmogorov Equation 6 by x^s, doing summation over $s > 0$ and substituting the definition of the *probability generating function* $f_i(x,t)$, we obtain the following relation:

$$\frac{\partial f_i(x, t)}{\partial t} = \sum_{k=1}^{4} \alpha_k(i+k)\frac{\partial f_{i+k}(x, t)}{\partial x} + \alpha_5 x(x-1)\frac{\partial^2 f_i(x, t)}{\partial x^2} + \left\{\sum_{k=1}^{2}\frac{\beta_k}{x^{k-1}} + \alpha_5(N^2 - i - 1) - x\left(\sum_{k=1}^{4} i\alpha_k + \sum_{k=1}^{2}\beta_k + \alpha_5(N^2 - i)\right)\right\}\frac{\partial f_i(x, t)}{\partial x} \quad \textbf{(7)}$$

We can encapsulate the generating functions into a column vector as: $F(x, t) = \left[f_{N_2}(x, t), f_{N_2-1}(x, t), \ldots, f_0(x, t)\right]^T$ and transform the linear system in Equation 7 of partial differential equations into:

$$\frac{\partial F(x, t)}{\partial t} = B(x)\frac{\partial^2 F(x, t)}{\partial x^2} + A(x)\frac{\partial F(x, t)}{\partial x} \quad \textbf{(8)}$$

where the terms A and B have the following expressions:

$$A(x) = \begin{bmatrix} a(N_2) & 0 & 0 & \ldots\ldots & 0 & 0 \\ b(N_2-1) & a(N_2-1) & 0 & \ldots\ldots & 0 & 0 \\ c(N_2-2) & b(N_2-2) & a(N_2-2) & \ldots\ldots & 0 & 0 \\ d(N_2-3) & c(N_2-2) & b(N_2-3) & \ldots\ldots & 0 & 0 \\ e(N_2-4) & d(N_2-4) & c(N_2-4) & \ldots\ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots\ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots\ldots & b(1) & a(0) \end{bmatrix} \quad \textbf{(9)}$$

$$b(i) = \alpha_1(i+1)x^2 \quad c(i) = \alpha_2(i+2)x^3 \quad d(i) = \alpha_3(i+3)x^4$$

$$a(i) = \sum_{k=1}^{2}\frac{\beta_k}{x^{k-1}} + \alpha_5(N^2 - i - 1) - x\left(\sum_{k=1}^{4} i\alpha_k + \sum_{k=1}^{2}\beta_k + \alpha_5(N^2 - i)\right)$$

$$e(i) = \alpha_4(i+4)x^5 \qquad B(x) = \alpha_5 x(x-1)$$

C. Frequency domain analysis

To find out the solution of the forward Kolmogorov Equation 7, we use the method proposed in [2] and [4]. Using the Laplace transform for the probability generating function associated with the forward Kolmogorov equation, we have:

$$\theta f_i(x, \theta) - \delta_{N_2} x^{N_2} = \left\{\sum_{k=1}^{2}\frac{\beta_k}{x^{k-1}} + \alpha_5(N^2 - i - 1)\right\}\frac{\partial f_i(x, \theta)}{\partial x} + \sum_{k=1}^{4}\alpha_k(i+k)x^{k+1}\frac{\partial f_{i+k}(x, \theta)}{\partial x} + \alpha_5 x(x-1)\frac{\partial^2 f_i(x, \theta)}{\partial x^2} - x\left[\sum_{k=1}^{4} i\alpha_k + \sum_{k=1}^{2}\beta_k\alpha_5(N^2 - i)\right]\frac{\partial f_i(x, \theta)}{\partial x} \quad \textbf{(10)}$$

In order to solve the differential equation, we form a column vector: $F(x, \theta) = \left[f_{N_2}(x, \theta), f_{N_2-1}(x, \theta), \ldots, f_0(x, \theta)\right]^T$; this leads to:

$$B(x)\frac{\partial^2 F(x, \theta)}{\partial x^2} + A(x)\frac{\partial F(x, \theta)}{\partial x} - \theta F(x, \theta) = -x^{N_2}E_{N_2+1} \quad \textbf{(11)}$$

where A(x) is given by Equation 9, and $B(x) = \alpha_5 x(x-1)I_{N_2+1}$. If we differentiate the above equation with respect to variable x, and use the notation:

$$F_d = F_d(x, \theta) = \frac{\partial^d F(x, \theta)}{\partial x^d}$$

then the Equation 11 takes the form:

$$B(x)F_{d+2} + [z(d)B^{(1)}(x) + A(x)]F_{d+1} + [w(d)B^{(2)}(x) + z(d)A^{(1)}(x) - \theta I_{N_2+1}]F_d + w(d)A^{(2)}(x)F_d = -\frac{N_2!x^{N_2-d}}{(N_2-d)!}E_{N_2+1} \quad \textbf{(12)}$$

with $z(d) = d$ and $w(d) = d(d-1)/2$, for $d \geq 0$. Considering that the probability generating functions $f_i(x, \theta)$ are polynomial functions of variable x of degree $N_1 + N_2 - i$, we have a stopping criterion as follows:

$$\begin{bmatrix} 0 \\ 0 \\ F_{N_1+N_2} \end{bmatrix} = \left(\prod_{d=0}^{N_1+N_2} A_d\right)\begin{bmatrix} F_1 \\ F_0 \\ 0 \end{bmatrix} + \sum_{d=0}^{N_1+N_2}\prod_{k=d+1}^{N_1+N_2} A_k B_d \quad \textbf{(13)}$$

where

$$B_d = \begin{bmatrix} -\frac{N_2!x^{N_2-d}}{(N_2-d)!}E_{N_2+1} \\ 0 \\ 0 \end{bmatrix} \quad A_d = \begin{bmatrix} A_d^1 & A_d^2 & A_d^3 \\ I_{N_2+1} & O_{N_2+1} & O_{N_2+1} \\ O_{N_2+1} & I_{N_2+1} & O_{N_2+1} \end{bmatrix} \quad \textbf{(14)}$$

with

$$A_d^1 = B^{-1}(x)[z(d)B^{(1)}(x) + A(x)] \quad A_d^3 = w(d)B^{-1}(x)A^{(2)}(x)$$
$$A_d^2 = B^{-1}(x)[w(d)B^{(2)}(x) + z(d)A^{(1)}(x) - \theta I_{N_2+1}] \quad \textbf{(15)}$$

and symbols I_{N_2+1} and O_{N_2+1} stand for the identity and zero matrices, respectively, of dimension N_2+1. Finally, the general solution has the form:

$$F(x, \theta) = F_0 + \sum_{d=1}^{N_1+N_2+2}\frac{(x-x_0)^d}{d!}\cdot\frac{\partial^d F(x, \theta)}{\partial x^d} \quad \textbf{(16)}$$

where F_0 represents the initial condition, and

$$\frac{\partial^d F(x, \theta)}{\partial x^d} = \left[\prod_{d=0}^{N_1+N_2} A_d\begin{bmatrix} F_1 \\ F_0 \\ 0 \end{bmatrix}\right]_{N_2+1} + \sum_{d=0}^{N_1+N_2}\prod_{k=d+1}^{N_1+N_2} A_k B_d \quad \textbf{(17)}$$

In summary, Equation 16 characterizes the evolution of packet dissemination on a grid. This equation lies at the heart of on-chip stochastic communication; more precisely, it captures the transition probabilities of the Markov process associated to the *SIR* interactions, as well as the effects induced by the network topology during the stochastic dissemination process.

IV. PERFORMANCE ANALYSIS

A. Number of reached nodes vs. number of rounds

So far, the packet spreading process was described in terms of static transition probabilities. We need to consider now its evolution in time, where we use the term *epoch* or *round* to distinguish different time evolution steps [3]. For convenience, we consider the discrete version of the Markov chain $\{(S, I, R)(k), k = 0, 1, 2, \ldots\}$ and return to Equation 1 where, for simplicity reasons, assume that the initial state is given by $(S, I, R)(0) = (1, N^2 - 1, 0)$. The state of the Markov process after one time step is given by:[1]

$$(S, I, R)(1) = \begin{cases} (2, N^2-2, 0), & \text{w.r. } \alpha_1 \\ (3, N^2-3, 0), & \text{w.r. } \alpha_2 \\ (4, N^2-4, 0), & \text{w.r. } \alpha_3 \\ (5, N^2-5, 0), & \text{w.r. } \alpha_4 \\ (0, N^2-1, 1), & \text{w.r. } \alpha_5 \end{cases} \quad \textbf{(18)}$$

For instance, the first branch given by the relation $(S, I, R)(1) = (2, N^2-2, 0)$ in Equation 18 means that starting with 1 spreader and N^2*-1* ignorants, leads us to a new transition in the Markov chain with rate α_1 which transforms one ignorant into a new spreader and thus S=2 and $I = N^2$-2. Note that, in the continuous case discussed in Section III, this transition would correspond to the case k=1 in the Spreader-Ignorant interaction in Equation 2. Again, the network topology governs which of the above transitions is actually taken. For the next time step

1.In these expressions, "*w.r.*"- stands for "*with rate*".

(k=2), the number of spreaders $S(2)$, ignorants $I(2)$, and stiflers $R(2)$ is more difficult to approximate.

Let T be the entire duration of the spreading process. For any time $k \le T$, we have:

$$(S, I, R)(k) = \begin{cases} (S(k-1)+1,\ I(k-1)-1,\ R(k-1)), \text{w.r.}\ \alpha_1 \\ (S(k-1)+2,\ I(k-1)-2,\ R(k-1)), \text{w.r.}\ \alpha_2 \\ (S(k-1)+3,\ I(k-1)-3,\ R(k-1)), \text{w.r.}\ \alpha_3 \\ (S(k-1)+4,\ I(k-1)-4,\ R(k-1)), \text{w.r.}\ \alpha_4 \\ (S(k-1)-1,\ I(k-1),\ R(k-1)+1), \text{w.r.}\ \beta_1 \\ (S(k-1)-2,\ I(k-1),\ R(k-1)+2), \text{w.r.}\ \beta_2 \\ (S(k-1)-1,\ I(k-1),\ R(k-1)+1), \text{w.r.}\ \alpha_5 \end{cases} \quad \textbf{(19)}$$

Consequently, if $|S(k-1)I(k-1) \cdot h| \cong 1$, then the following equations approximate the packet dissemination in early stages ($k \le T$) of stochastic communication:

$$\begin{aligned} S(k) &= C[(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\beta_1+\beta_2+\alpha_5)S(k-1) \\ &\quad +\alpha_1+2\alpha_2+3\alpha_3+4\alpha_4-\beta_1-2\beta_2-\alpha_5] \qquad \textbf{(20)} \\ I(k) &= C[(\alpha_1+\alpha_2+\alpha_3+\alpha_4)I(k-1)-\alpha_1-2\alpha_2-3\alpha_3-4\alpha_4] \end{aligned}$$

where C is a normalization constant.

In Figure 2, we compare the results predicted by this equation (shown with red line) and the results obtained from simulation (green line). As we can see, the match is quite good, the difference being less than 5%, on average.

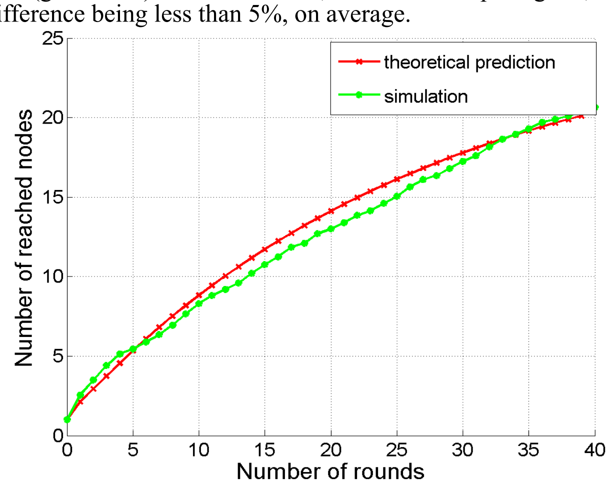

Figure 2. Coverage comparison between the analytical approximation of *S(k)* in Section IV.A and the simulation results on a 6x6 mesh network.

B. Deterministic approximation

In order to investigate the asymptotic behavior of the spreaders coverage (*i.e.*, $S(k)$ behavior when $k \to \infty$), we consider the deterministic differential equations that characterize the stochastic model. With the above notations, we have:

$$\begin{aligned} \frac{ds}{dt} &= \left[\sum_{k=1}^{5}\alpha_k\right]si + \left[\sum_{k=1}^{4}k\alpha_k - \sum_{k=1}^{2}k\beta_k - \alpha_5(N^2-2)\right]s + \\ &\alpha_5 s^2 + \left[\alpha_5 - \sum_{k=1}^{4}k\alpha_k\right]i + \sum_{k=1}^{4}k^2\alpha_k - \sum_{k=1}^{2}k^2\beta_k - \alpha_5(N^2-1) \qquad \textbf{(21)} \\ \frac{di}{dt} &= \left[-\sum_{k=1}^{4}\alpha_k\right]si - \left[\sum_{k=1}^{4}k\alpha_k\right]s + \left[\sum_{k=1}^{4}k\alpha_k\right]i + \sum_{k=1}^{4}k^2\alpha_k \end{aligned}$$

To verify this analytical approximation, we consider two cases (Figure 3): In the first case, we assume that there exist neither damaged links, nor dead nodes (*i.e.* $\beta_1 = \beta_2 = \alpha_5 = 0$) in the network. In the second case, we consider that faults are present (*i.e.* $\beta_1 = \beta_2 = \alpha_5 = 0.1$). As we can see in Figure 3, the number of nodes that become aware of the packet dissemination reaches

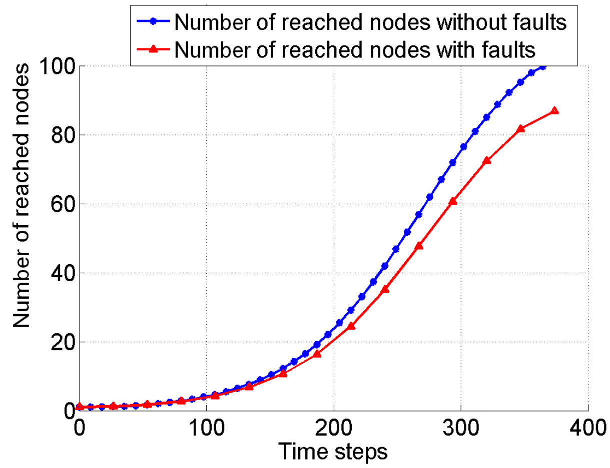

Figure 3. Coverage comparison with (red line) and without (blue line) faults in a 10x10 mesh network obtained from the deterministic approximation in Section IV.B.

a maximum point. In the presence of faults, however, the number of reached nodes is significantly smaller compared to the first case. This can be explained with the Spreader-Spreader and Spreader-Stifler interactions in Section III.

V. EXPERIMENTAL RESULTS

The main objective of our experiments is to validate the analytical expressions derived so far, primarily the coverage metric in Section IV.A. Toward this end, we use an in-house cycle accurate simulator which implements a probabilistic scheme which use a small bias towards destination when forwarding the packets among the intermediate nodes. For simplicity, we assume that each packet forwarding action needs only a single clock cycle to cross the router at each node. Subsequently, the probabilistic rule returns a vector of specified neighbors to which the packet is forwarded based on a threshold condition. The pseudo-code is given below:

Probabilistic Routing: The algorithm receives a *Source*, a *Destination* and a *Packet* ID and returns an output *direction*.

- Annotate power at current time step;
- If (Packet=corrupted OR time_to_live=0) then drop_Packet();
- Else
  - decrement_time_to_live(Packet)
  - For each neighbor in the *destination* direction
    - If (forwarding_probability > random_number in [0,1]
      - Copy the neighbor's label in the *direction* vector
- Return *direction* to the Arbiter for granting actions.

To compare the analytical approximation in Equation 21 with the results obtained from simulation, we plot in Figure 4 the number of reached nodes as a function of various forwarding probabilities in a 10x10 mesh network. We can clearly identify a region where there is an exponential growth in the number of nodes that become aware of packet dissemination. As such, this approximation provides the first theoretical explanation for the simulation behavior reported in [6] where no theory was provided to justify such a behavior.

Further, in Figure 5, we report the simulation results obtained on a 10×10 network, with (red line) and without (blue line) faulty links, using a forwarding probability of 0.5. As we can observe there are some similarities between the analytical approximation in Figure 3 and the simulation results in Figure 5. As expected, due to the presence of faults, the number of reached nodes decreases in both cases (red lines). Moreover, in the early stages of the process, both graphs show an exponential growth in the number of reached nodes.

TABLE 1 Average Latency and Energy consumption results for the XY and Probabilistic Routing algorithms

Network Size	Transferred Packets	XY routing			PR routing		
		Average Latency (cycles)	Link Energy (Joules)	Total Energy (Joules)	Average Latency (cycles)	Link Energy (Joules)	Total Energy (Joules)
5×5	4899	23.3944	2.327903e-6	3.051716e-5	26.9508	2.355346e-6	3.080093e-5
6×6	7013	26.1878	4.035253E-6	5.092503E-5	30.4609	4.060454e-6	5.118605e-5
7×7	9641	28.9207	6.473071E-6	7.954121E-5	33.9768	6.538992e-6	8.022076e-5
8×8	19035	31.4833	1.450944e-5	1.748499e-4	37.182	1.460323e-5	1.758179e-4
9×9	24180	34.3769	2.090964e-5	2.476368e-4	40.6995	2.091426e-5	2.476883e-4
10×10	300050	37.0597	2.882048e-5	3.369460e-4	44.3136	2.901997e-5	3.390023e-4

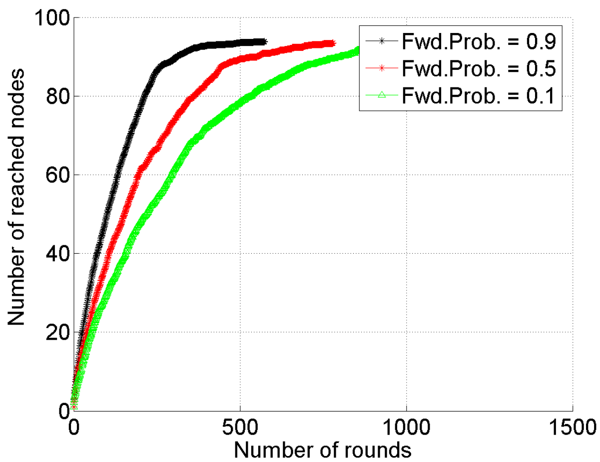

Figure 4. Number of reached nodes for different forwarding probabilities obtained via simulation in a 10x10 network.

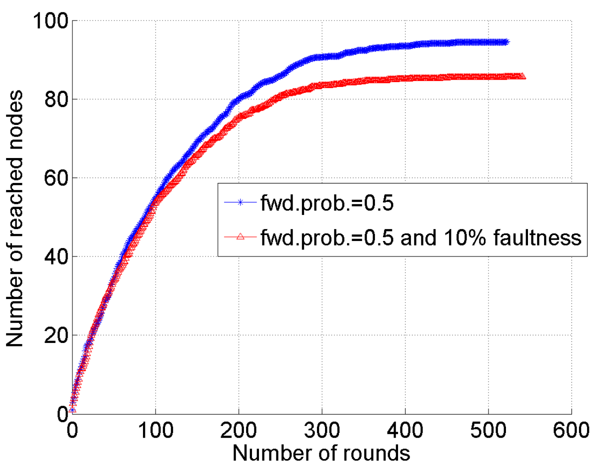

Figure 5. Coverage comparison with (red line) and without (blue line) faults in a 10x10 mesh network obtained via simulation.

We also use the simulator to investigate the performance degradation of the probabilistic scheme compared to the performance of the classical XY deterministic routing scheme used in [10]. We present in Table 1 the average latency and total energy consumption[1] values reported by the simulator under uniform traffic. For instance, for a 5×5 mesh network and 4899 transferred packets, the average latency for the XY scheme is 23.39 cycles, the link energy about 2.32μJ, and the total energy consumption 30.51μJ. Using the probabilistic routing scheme, the average latency becomes 26.95 cycles, the link energy 2.355μJ, and the total energy 30.80μJ. Overall, compared to the XY routing scheme, the proposed probabilistic algorithm experiences a 15-20% increase in the average latency, and about 10% increase in total energy consumption; this small increase can be explained by the biasing towards the destination used in the routing scheme. As expected, the higher energy consumption is due to packet duplication and read/write buffer operations. However, this seems to be well worth it, given the improvements achieved in the level of fault-tolerance.

1. The energy estimates are obtained using the model proposed in [10].

VI. CONCLUSION AND FUTURE WORK

In this paper, we have investigated the theoretical foundations of an epidemic-style communication paradigm. More precisely, we have proposed a stochastic model for the on-chip communication and validated it through coverage simulation. As opposed to classical models in epidemics and gossiping, we have considered the effects induced by the network topology on packet dissemination process.

Our future research will focus on investigating how hardware modules can be built and operated based on stochastic rules. This will help us gain insight on other performance parameters, such as average latency, energy consumption, buffer sizes, *etc*. Longer term, these new achievements rooted in the SoC design may become useful in analyzing some biological systems too.

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