18-799F Algebraic Signal Processing Theory
Spring 2007
Assignment 6
Due Date: Mar. 7th 2:30pm (at the beginning of class)

Chebyshev polynomials of the first kind are defined by the following recursive relation:

$$\begin{aligned} T_0(x) &= 1 \\ T_1(x) &= x \\ T_n(x) &= 2xT_{n-1} - T_{n-2}, \quad n \neq 0, 1. \end{aligned}$$

Note that this relation defines T_n for both positive and negative $n \in \mathbb{Z}$. Further, the following holds:

$$T_{n+k}(x) = 2T_k(x)T_n(x) - T_{n-k}(x).$$

The closed form for these polynomials is $T_n(x) = \cos(n \arccos x)$ (for $x \in [-1, 1]$). It follows that $T_n(x)$ has the zeros

$$\cos\frac{(k+1/2)\pi}{n}, \quad k = 0, \ldots, n-1.$$

Consider a signal model $(\mathcal{A}, \mathcal{M}, \Phi)$ given by $\mathcal{A} = \mathcal{M} = \mathbb{C}[x]/T_n(x)$ and basis $b = \{T_0(x), \ldots, T_{n-1}(x)\}$ in $\mathcal{M}$; i.e.,

$$\begin{aligned} \Phi: \quad \mathbb{C}^n &\to \mathcal{M} \\ s &\mapsto \textstyle\sum_{l=0}^{n-1} s_l T_l(x) \end{aligned}$$

(a) *(20 pts)* Determine the shift matrix $\phi(x)$ and the visualization of the signal model. Compare to the visualization of the signal model associated with the DFT and note the differences.

(b) *(20 pts)* Determine the Fourier Transform in coordinate-free (Δ) and coordinate (as matrix $\mathcal{F} = \mathcal{P}_{b,\alpha}$) forms.

(c) *(10 pts)* What is the frequency response of $h = h(x) \in \mathcal{A}$ with respect to this signal model?

(d) **Extra-credit** *(20 pts)* Assume $n = 4$ in this exercise. Write a Matlab program that computes the matrix $\phi(h)$ for any $h \in \mathcal{A}$.

Hint: As we did in class, first map the basis b with ϕ.

Now, for a few randomly chosen $h \in \mathcal{A}$ confirm in Matlab that $\mathcal{F}$ in part (b) really diagonalizes $\phi(h)$.

For this exercise, copy-paste the Matlab output into a document and submit with your homework.