Algorithms and Computation in Signal Processing

special topic course 18-799B
spring 2005
14th Lecture Feb. 24, 2005

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Course Evaluation

- Email sent out today by Suzie Laurich-McIntyre
- Please fill out (is anonymous)

Midterm

- **What to learn:**
  - Understand O, Ω, Θ
  - DFT properties explained in class
  - Know the most important complexities
  - Solve a recurrence using generating functions
  - Be able to analyze the cost of a recursive transform algorithm given in terms of tensor products etc.

Feedback for 2nd Assignment

Plotting Graphs

- **Almost always include data points on a line graph made up of discrete data**
  - Without data points, a dip in the graph could have been because of one single deviated value, or because of multiple values
  - Curve of the line is arbitrarily decided by the plotting program
- **Ensure that a line graph begins at an appropriate value (and not zero, unless that is an actual data value)**

Plotting Graphs

- **Discuss and analyze:**
  - A plot by itself is usually of little value: What really matters is a meaningful discussion and analysis of the plot. Eg:
    - Why is a curve on the plot shaped a certain way?
    - What factors (apparent or hidden) influence or could potentially influence the plot
    - How would the extrapolated graph look (esp. important for MFLOPS plots)
    - What significance do the global maximas and minimas have?
  - At the very least, discuss and speculate

Plotting Graphs

- **Use the correct kind of graph to illustrate your data:**
  - Trends: line graph
  - Bars: values across categories
  - Pie charts: Contributions to a total value
- **MFLOPS for this assignment: do not use bar graphs!**
- **Similarly, use a table when appropriate**

Presenting data

- If there is a significant amount of variance in your data, either execute adequate iterations to get a meaningful mean, and/or choose to also present a measure of variance like standard deviation

Experiments

- **If conducting a new experiment, (or deviating from the question in any manner):**
  - First, clearly and explicitly present the objective or hypothesis
  - Next, present the experiment and how it verifies the hypothesis

Measuring time

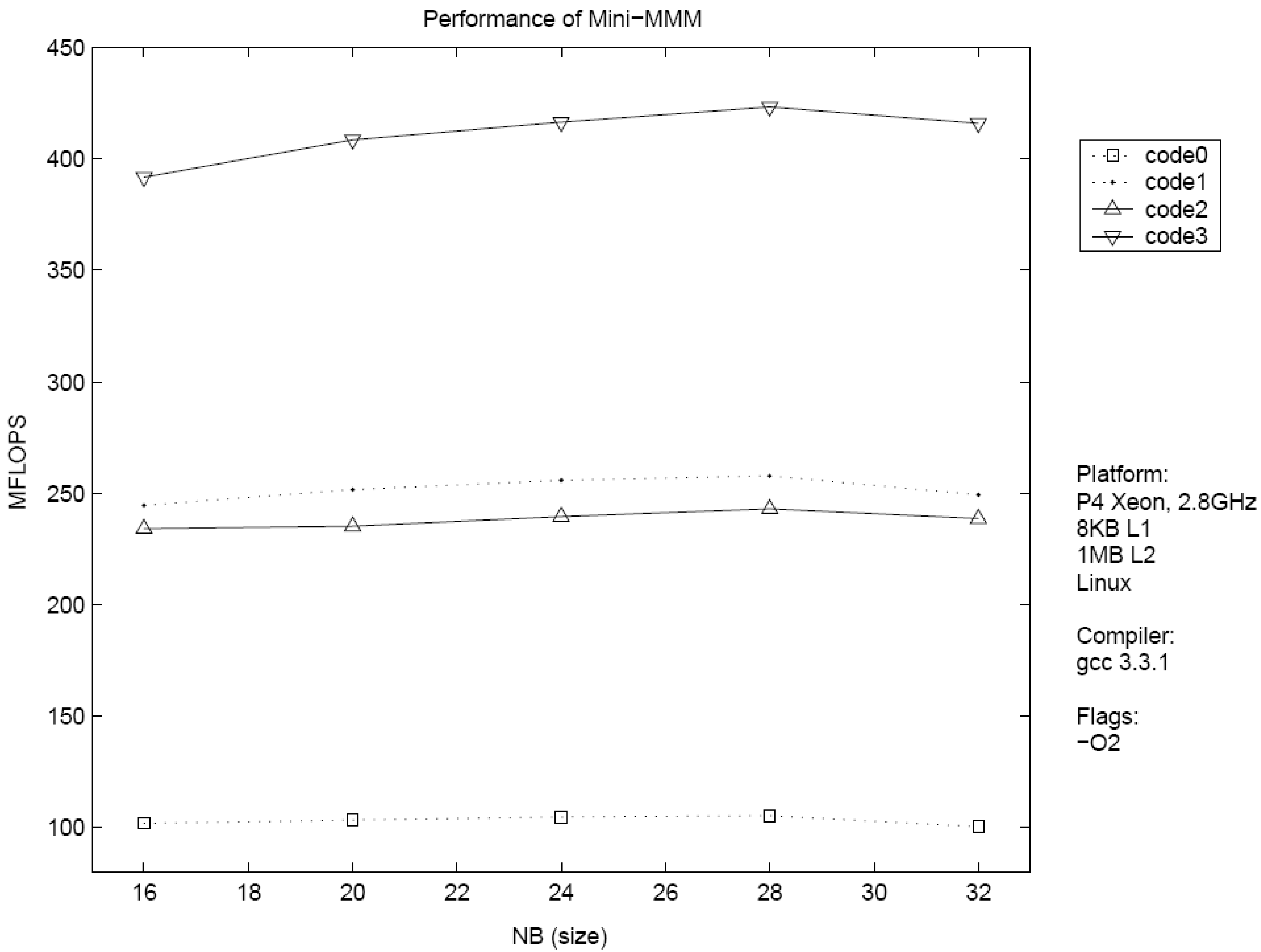
- Understand the difference between wall clock time, user time, system time etc. This is important!

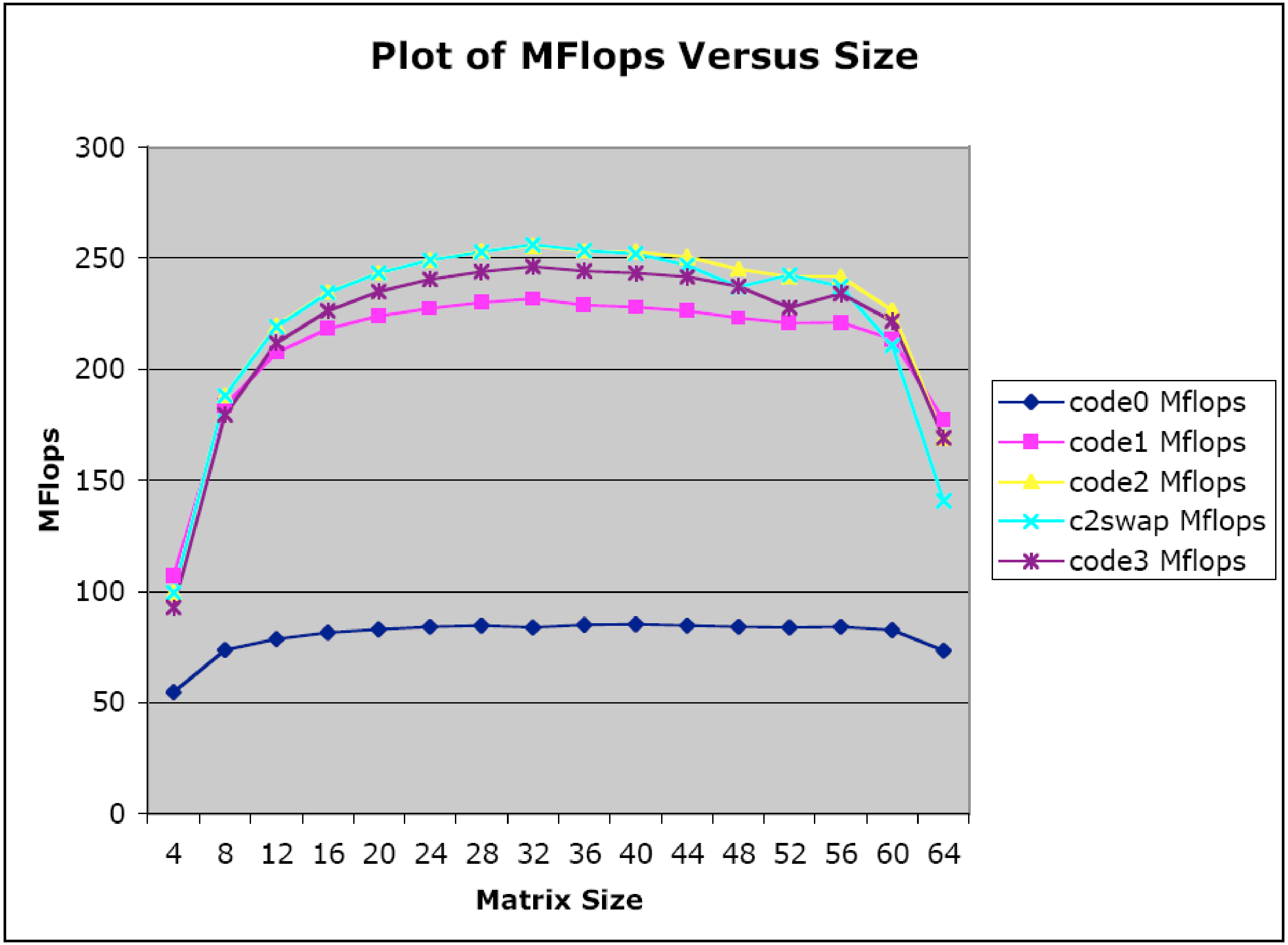
FLOPS and Peak performance calculation

- MFLOPS: Includes only FP +, *
- Does not include loads/stores (or you have to adjust peak performance)

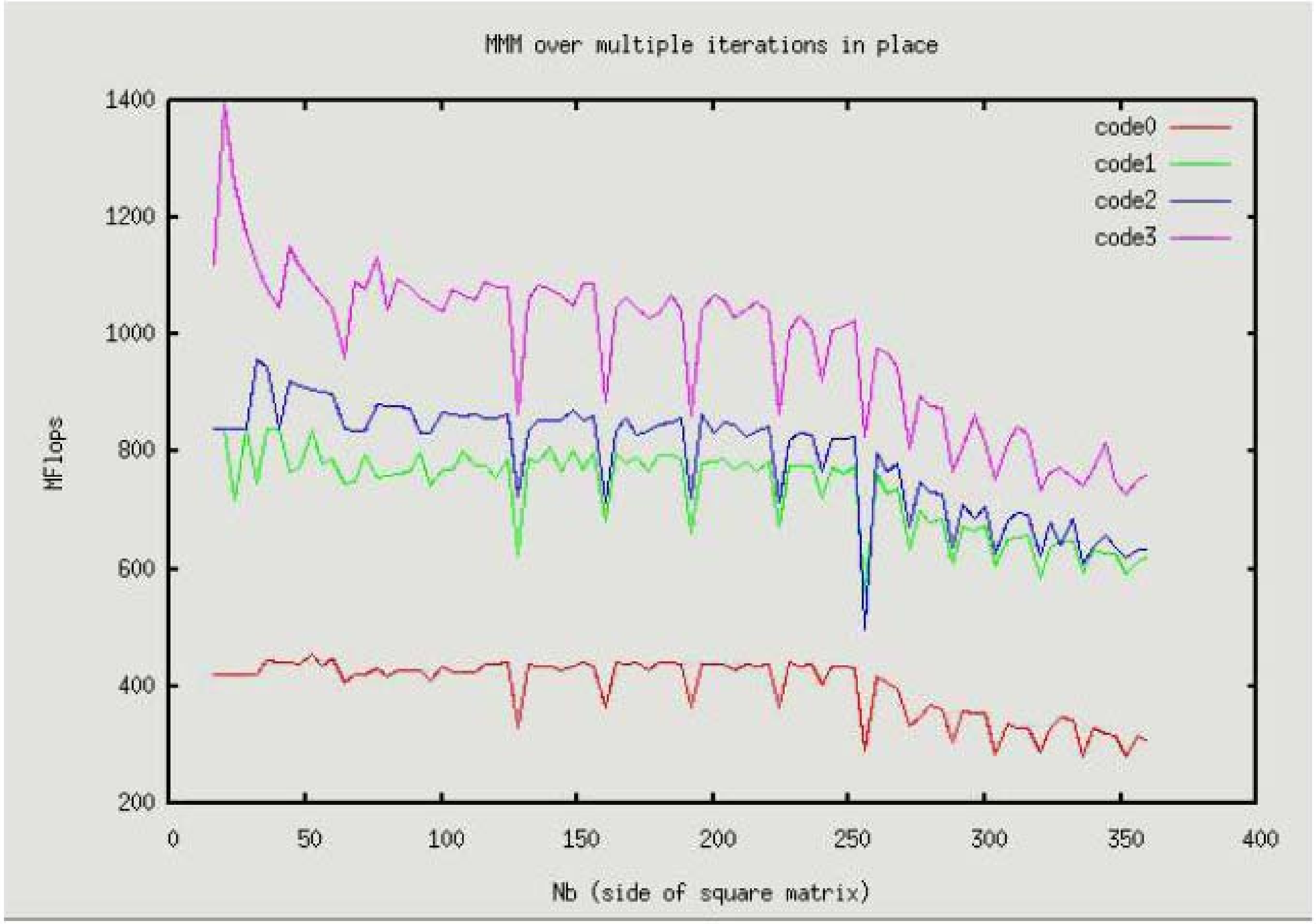
- Peak performance: Do not assume this is the same as the clock frequency. This value depends on the computer and needs to be found out.

Some Plots from the 2nd Assignment

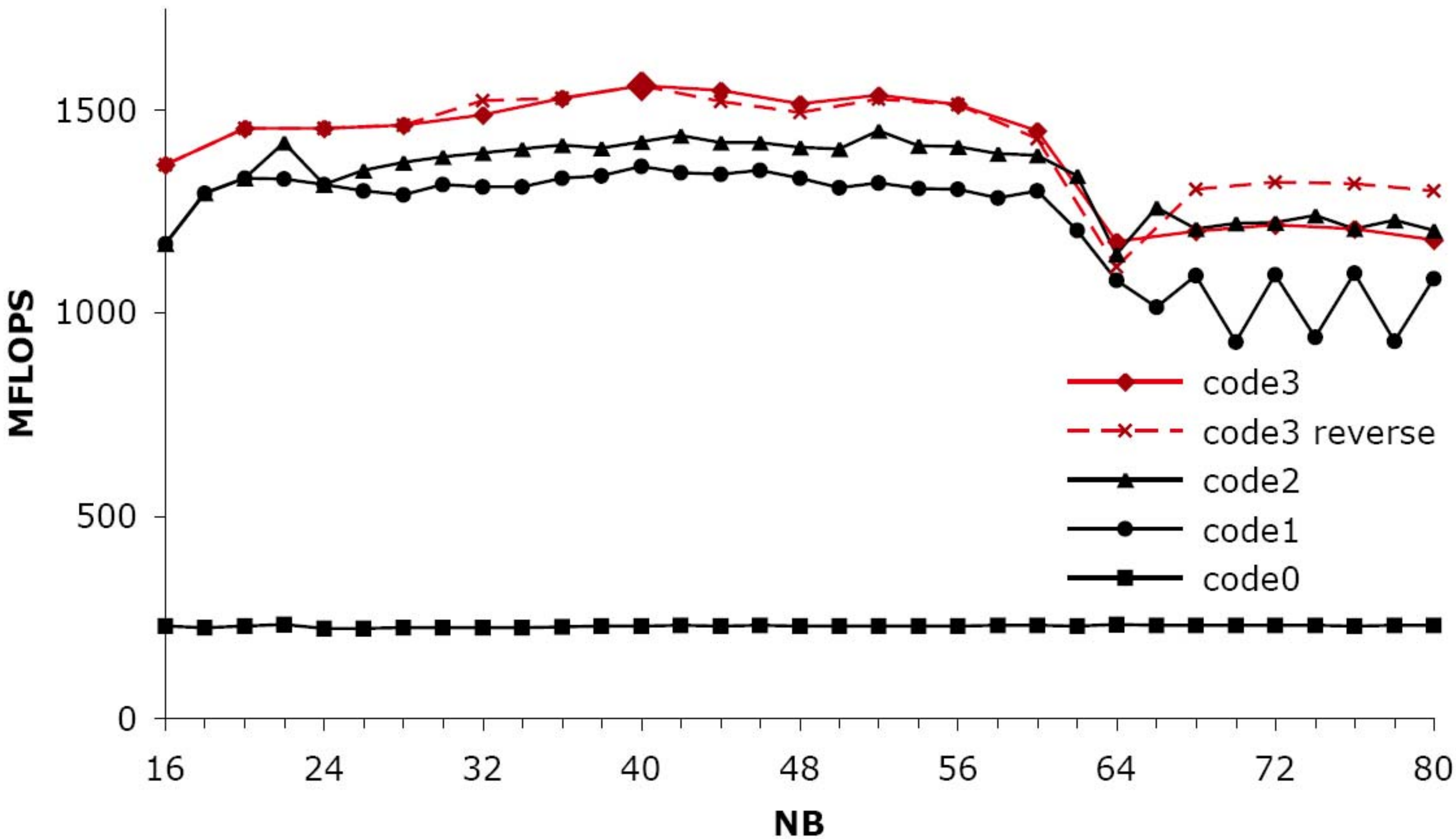

Performance of Mini-MMM
MFLOPS
450
400
350
300
250
200
150
100
16
18
20
22
24
26
28
30
32
NB (size)
code0
code1
code2
code3
Platform:
P4 Xeon, 2.8GHz
8KB L1
1MB L2
Linux
Compiler:
gcc 3.3.1
Flags:
−O2



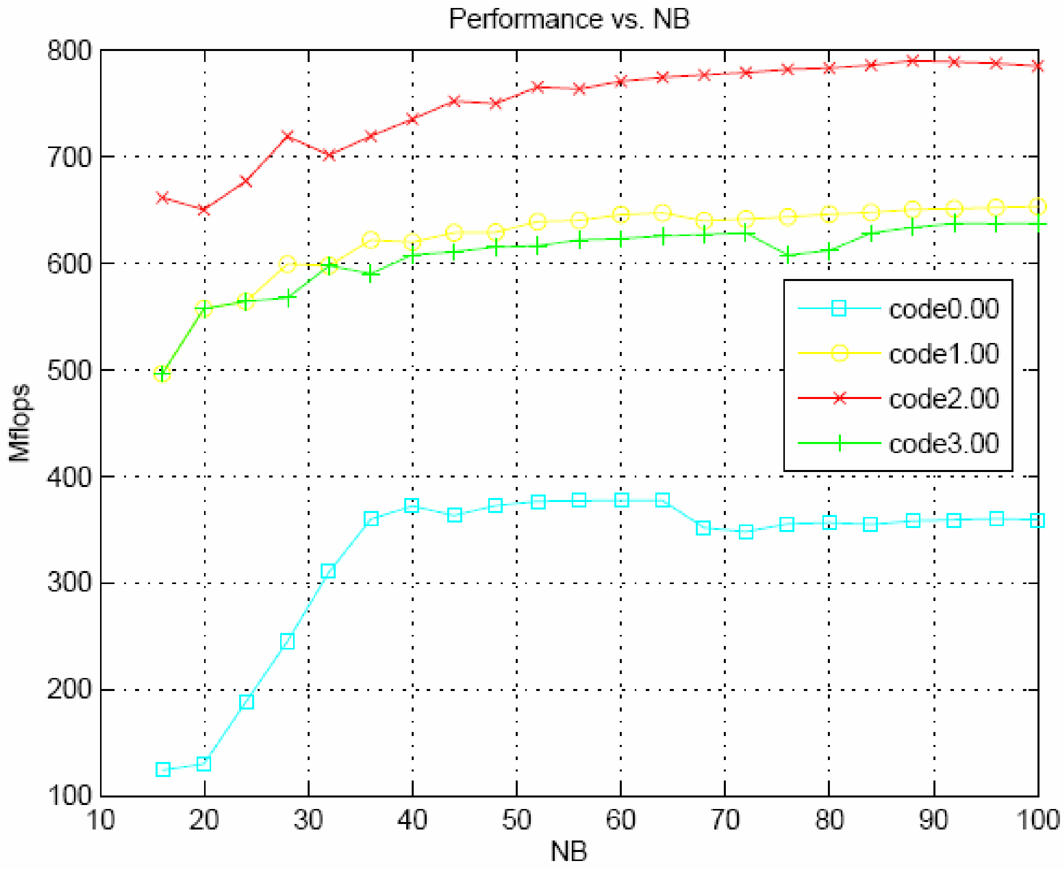
- CPU: PowerPC 750 (“G3”) / 400 MHz /32K L1-I,D caches / 1MB of L2 cache
- c2swap = i,j loops swapped



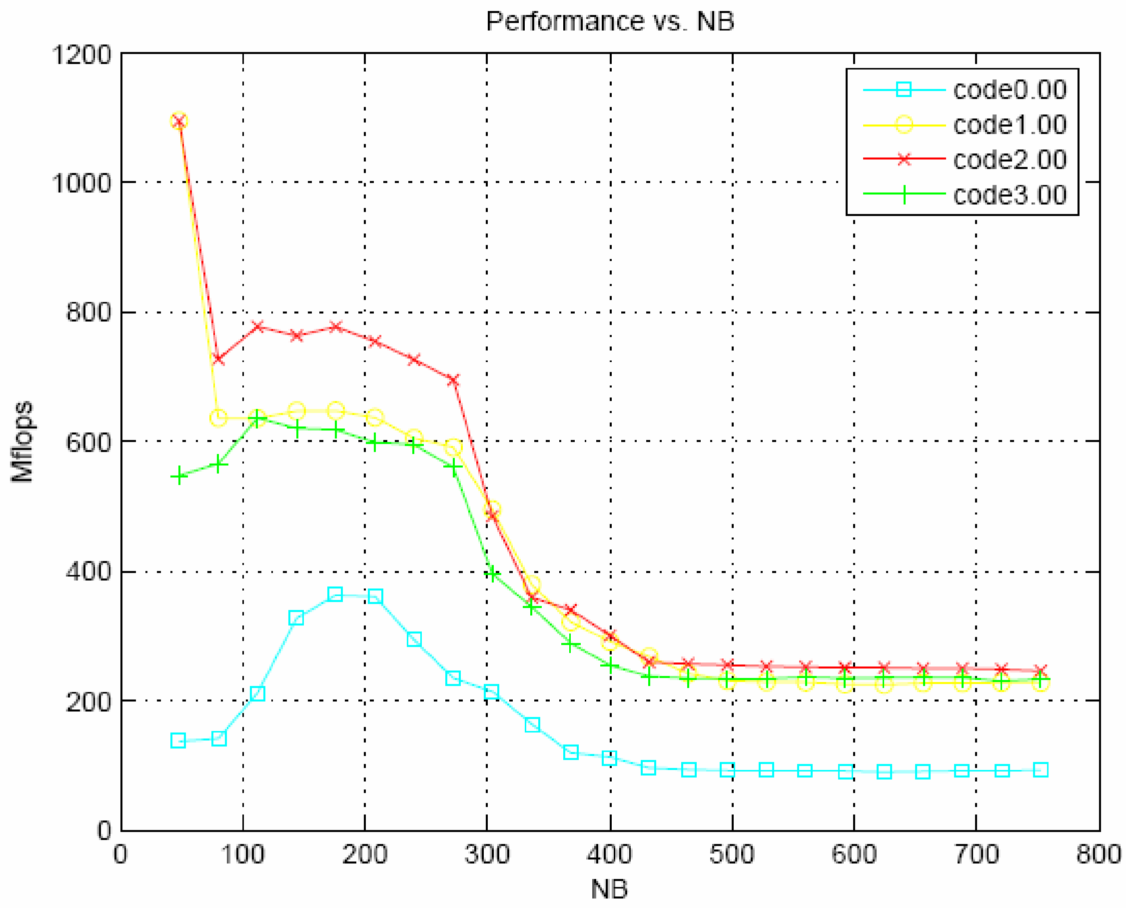
- CPU: Pentium M 2GHz / 1Gb / gcc 3.3.49



- **PowerBook G4 / Freescale PowerPC MPC7447A CPU at 1.5 GHz**
- **Code3 reverse: loop order from ijk to jik**



- Pentium M / 1600 MHz / 32k L1 D,I caches / 1MB L2 cache



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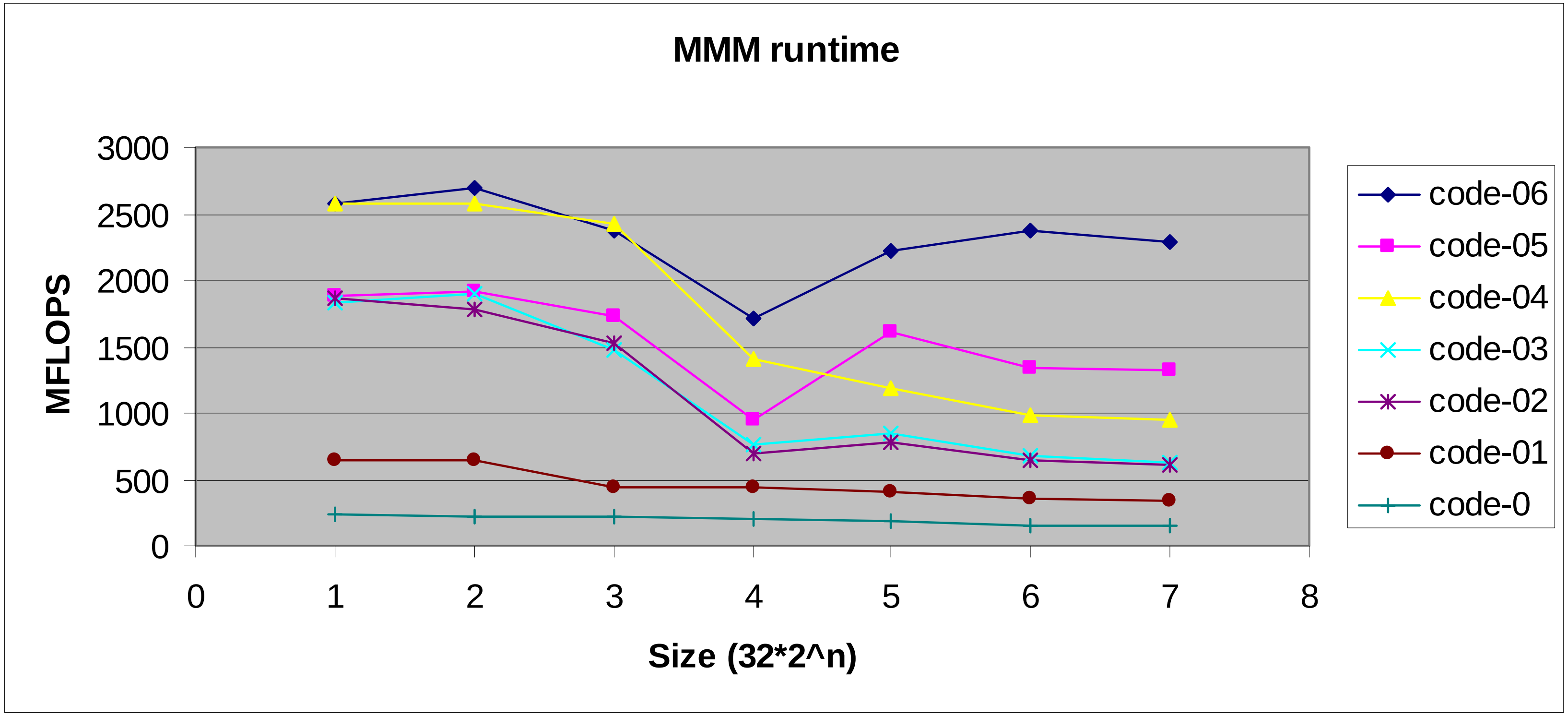
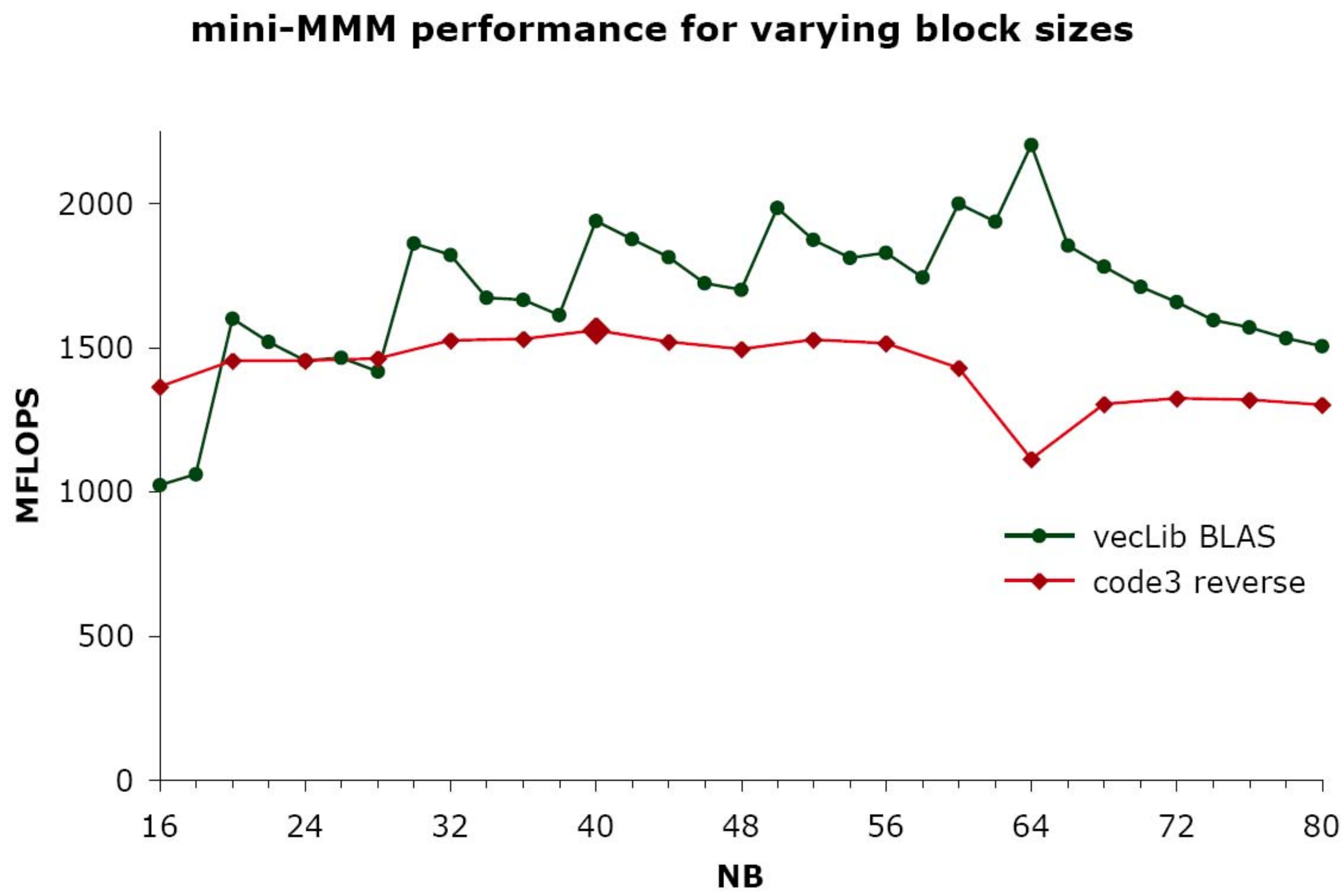


- Pentium 4 / ICC

- **Code-00 : Triple loop naïve implementation**
- **Code-01 : Block for Register**
- **Code-02 : Block for Register Unroll 2**
- **Code-03 : Block for Register Unroll 4**
- **Code-04 : Block for Register + SSE**
- **Code-05 : Block for Register + Block for L1 Cache**
- **Code-06 : Block for Register + Block for L1 Cache + SSE**



- PowerBook G4 / Freescale PowerPC MPC7447A CPU at 1.5 GHz
- Code3 reverse: code3+jik loop order

FFT Summary

FFT Algorithm Summary

- There is not just one FFT (Cooley-Tukey, Rader, etc.)
- Even if only Cooley-Tukey FFT is considered there are many ways of recursing (similar cost, but different dataflow)
- Several complexity results for the DFT are available.
  If c is bounded, then $L_c(DFT_n) = \Theta(n \log(n))$

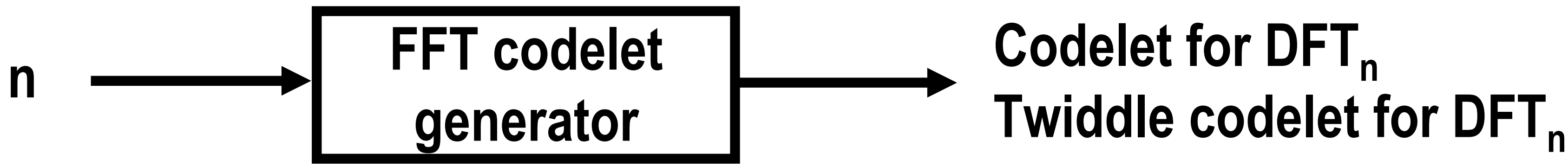
The FFT codelet generator in FFTW

M. Frigo, "A Fast Fourier Transform Compiler," Proc. PLDI 1999 *link*

FFTW homepage *link*

Basic Block Optimizations for FFTs

- **Problem:** as in MMM, we do not want to recurse all the way down. Infrastructure destroys performance.
- **Solution:** Unrolled code for small size (<= 64)
- Optimization for these blocks is much harder than the micro/mini MMMs in MMM
- Again, compilers don't do a good job on unrolled code
- **Solution:** Code generator/optimizer for small sizes



Codelet Generator: Details

- **DAG: directed acyclic graph**
  - Represents a DFT algorithm (the dataflow)
  - Nodes: load, store, adds, mults by constant
- **Give example on blackboard**

DAG Generator

- Knows FFTs: Cooley-Tukey, split-radix, Good-Thomas, Rader, represented in sum notation

$$y_{n_2 j_1 + j_2} = \sum_{k_1=0}^{n_1-1} \left(\omega_n^{j_2 k_1}\right) \left(\sum_{k_2=0}^{n_2-1} x_{n_1 k_2 + k_1} \omega_{n_2}^{j_2 k_2}\right) \omega_{n_1}^{j_1 k_1}$$

- For given n, suitable FFTs are recursively applied to yield n (real) expression trees for $y_0, \ldots, y_{n-1}$
- Trees are fused to an (unoptimized) DAG

Simplifier

- **Applies: algebraic transformations, common subexpression elimination (CSE), DFT-specific optimizations**

- **Algebraic transformations**
  - Simplify mults by 0, 1, -1
  - Distributivity law: kx + ky = k(x + y), kx + lx = (k + l)x
    May destroy common subexpressions and thus increase op count!
  - Canonicalization: (x-y), (y-x) to (x-y), -(x-y)

- **CSE: standard**
  - E.g., two occurrences of 2x+y: assign new temporary variable

Simplifier (cont'd)

- **DFT-specific optimizations**
  - All numeric constants are made positive
  - Reason: constants need to be loaded into registers, too
  - CSE on the transposed DAG (Blackboard)

Scheduler

- Determines in which sequence the DAG is unparsed to C (topological sort of the DAG)
  Goal: minimizer register spills

- If C register are available, then a 2-power FFT needs at least Ω(nlog(n)/C) register spills [1]

- Scheduler achieves this (asymptotic) bound independent of C

- Explain on blackboard

[1] Hong and Kung: "I/O Complexity: The red-blue pebbling game," Proc. ACM Symp. Theor. Comp. pp. 326-333, 1981