How to Write Fast Code

18-645, spring 2008

11th Lecture, Feb. 20th

Instructor: Markus Püschel

TAs: Srinivas Chellappa (Vas) and Frédéric de Mesmay (Fred)

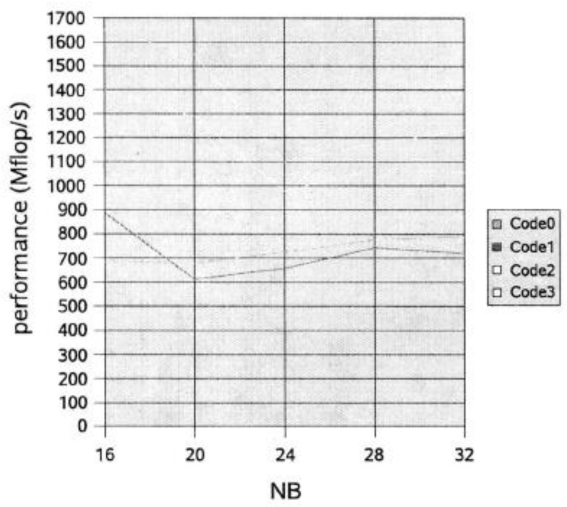
Technicalities

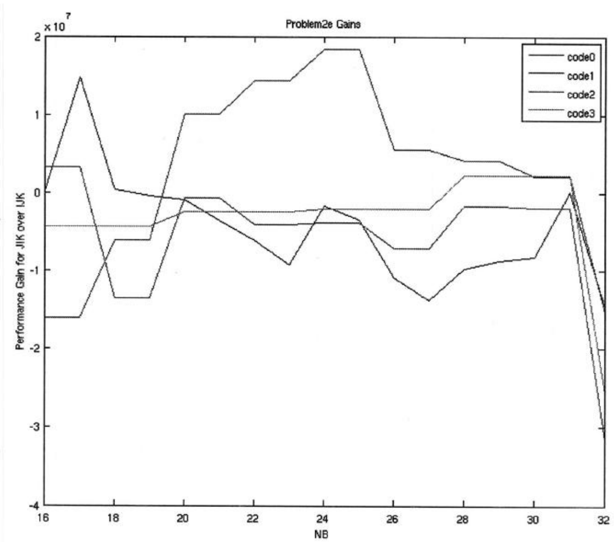
- **HW 2**
  - Grades: $\mu = 82$, $\sigma = 16$, max = 106, min = 39
  - Time: $\mu = 11$, $\sigma = 5$
  - Grades are now in blackboard: please double check
- **HW 2 feedback**

About Plots (and Tables)

- **Above all they have to be readable**
  - If you print out black & white, don't use color (different marker shapes, line styles)
  - Always label axes and put a title
  - Large enough font
  - Proper number format
    (no 10s of zeros, no 10 digits after decimal point, no 2.345E09)
  - Always discuss and analyze plots

Not good:

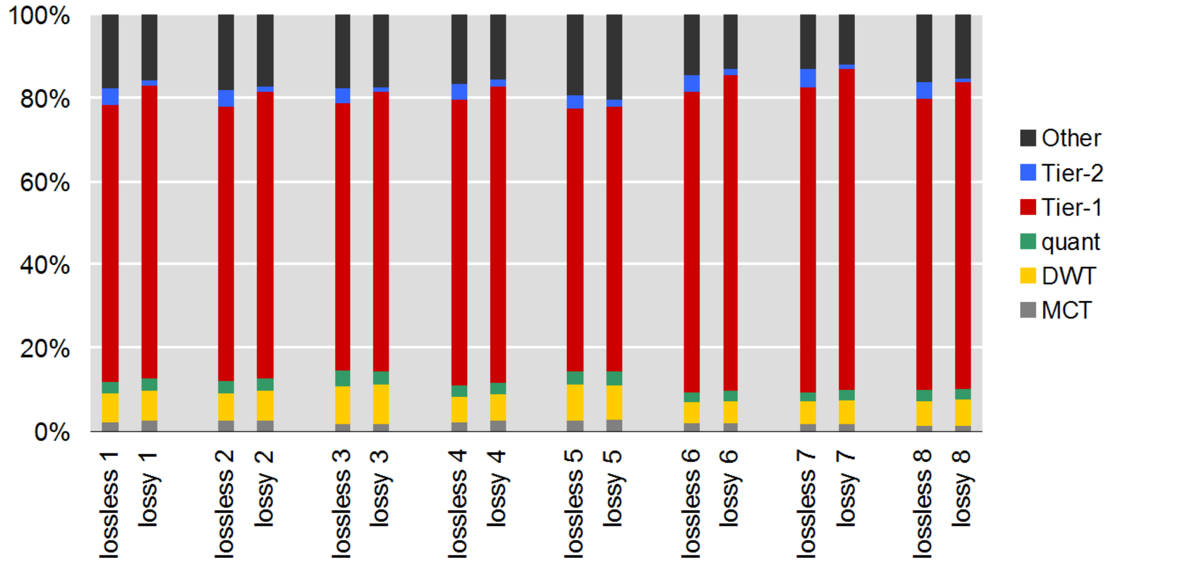




Research Projects

- <u>Projects and supervisors</u>
- **Start thinking about optimization**
  - If your problem is a numerical kernel: try techniques you learned in class, first focus is memory hierarchy
  - If your problem has several steps: determine bottleneck, then start optimizing bottleneck

Example:
Profiling JPEG 2000



Meetings next Monday

Markus	
11 – 11:45	8
11:45 – 12:30	7
1:30 – 2:15	9
2:15 - 3	16
3 – 3:45	14
	12
4:30 – 5:15	6
5:15 – 6	13

Fred	
4:30 – 5:15	1
5:15 – 6	2
6 – 6:45	3

Franz	
1 – 1:45	17
2 – 2:45	11
4:30 – 5:15	5

Vas	
3:45 – 4:30	4
4:30 – 5:15	10
5:15 - 6	15

Today

- Sparse matrix-vector multiplication (MVM)
- Sparsity/Bebop

Sparse MVM

- **y = y + Ax, A sparse but known**
- **Important routine in:**
  - finite element methods
  - PDE solving
  - physical/chemical simulation (e.g., fluid dynamics)
  - linear programming
  - scheduling
  - signal processing (e.g., filters)
  - …
- **In these applications, y = y + Ax is performed many times**
  - justifies one-time tuning effort
- **Fundamental difference between MVM and MMM**
  - blackboard

Storage of Sparse Matrices

- **Standard storage (as 2-D array) inefficient (many zeros are stored)**

- **Several sparse storage formats are available**

- **Explain compressed sparse row (CSR) format (blackboard)**
  - advantage: arrays are accessed consecutively for y = y + Ax
  - disadvantage: inserting elements is costly, no reuse of x

Direct Implementation y = Ax, A in CSR

```
void smvm_1x1( int m, const double* value, const int* col_idx,
               const int* row_start, const double* x, double* y )
{
      int i, jj;

      /* loop over rows */
      for( i = 0; i < m; i++ ) {
            double y_i = y[i];

            /* loop over non-zero elements in row i */
            for( jj = row_start[i]; jj < row_start[i+1];
                 jj++, col_idx++, value++ ) {
                    y_i += value[0] * x[col_idx[0]];
            }
            y[i] = y_i;
      }
}
```

scalar replacement
(only y is reused)

indirect array addressing
(problem for compiler opt.)

Optimizing Sparse MVM

- Sparsity/Bebop

- **Paper used:** Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004 **(can be found on above website)**

Impact of Matrix Sparsity on Performance

- **Adressing overhead (dense MVM vs. dense MVM in CSR):**
  - ~ 2x slower (Mflop/s, example only)
- **Irregular structure**
  - ~ 5x slower (Mflop/s, example only) for "random" sparse matrices
- **Fundamental difference between MVM and sparse MVM (SMVM):**
  - sparse MVM is input **dependent** (sparsity pattern of A)
  - changing the order of computation (blocking) requires changing the data structure (CSR)

Bebop/Sparsity: SMVM Optimizations

- Register blocking

- Cache blocking

Register Blocking

- **Idea: divide SMVM y = y + Ax into micro (dense) MVMs of matrix size r x c**
  - store A in r x c block CSR (r x c BCSR)

- **Explain on blackboard**
  - Advantages:
    - reuse of x and y (as for dense MVM)
    - reduces index overhead
  - Disadvantages:
    - computational overhead (zeros added)
    - storage overhead (for A)

Example: y = Ax in 2 x 2 BCSR

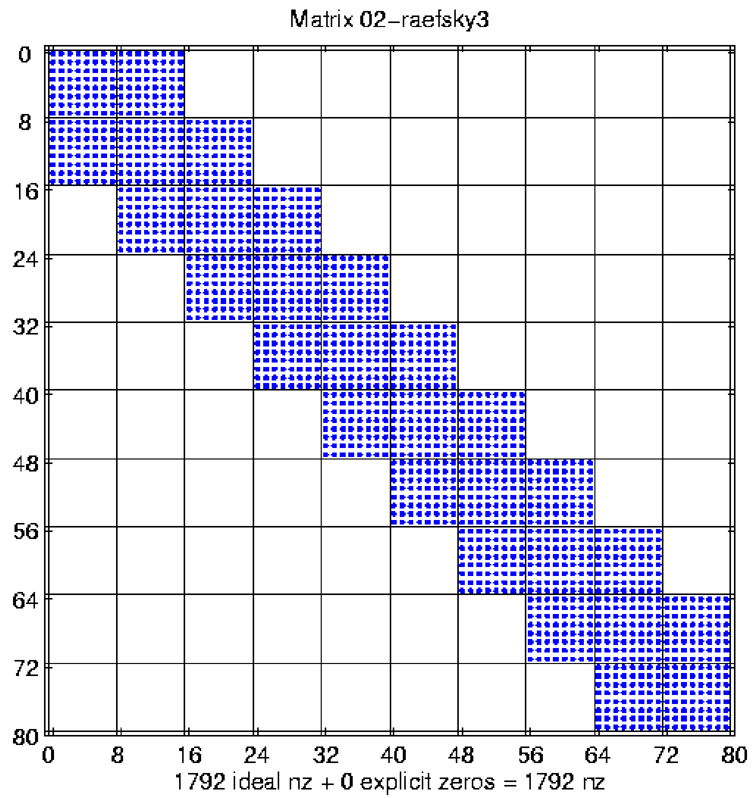
```
void smvm_2x2( int bm, const int *b_row_start, const int *b_col_idx,
               const double *b_value, const double *x, double *y )
{
        int i, jj;

        /* loop over block rows */
        for( i = 0; i < bm; i++, y += 2 ) {
                register double d0 = y[0];
                register double d1 = y[1];

                /* dense micro MVM */
                for( jj = b_row_start[i]; jj < b_row_start[i+1];
                     jj++, b_col_idx++, b_value += 2*2 ) {
                        d0 += b_value[0] * x[b_col_idx[0]+0];
                        d1 += b_value[2] * x[b_col_idx[0]+0];
                        d0 += b_value[1] * x[b_col_idx[0]+1];
                        d1 += b_value[3] * x[b_col_idx[0]+1];
                }
                y[0] = d0;
                y[1] = d1;
        }
}
```

scalar replacement
(y is reused)

Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

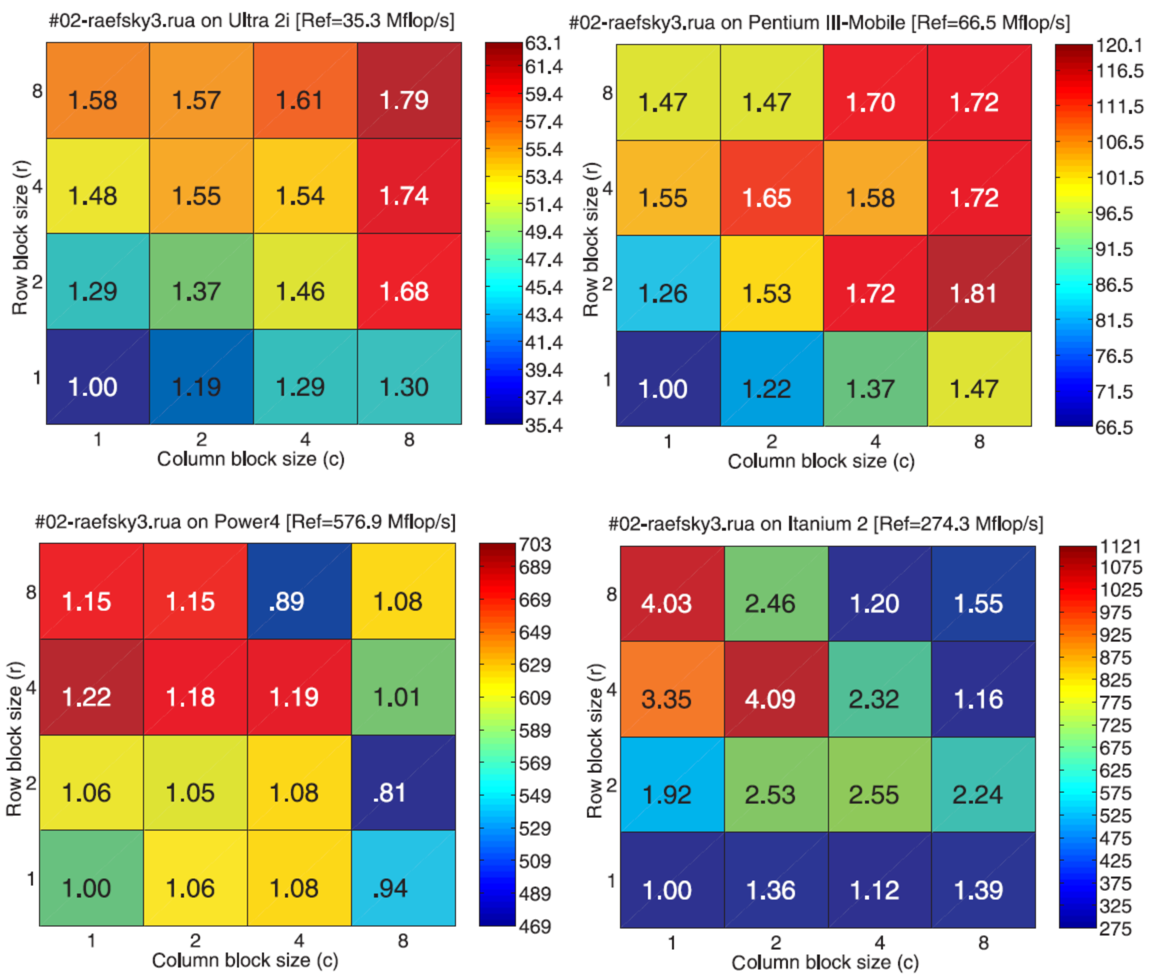
Which Block Size (r x c) is Optimal?

- **Example:** ~ 20,000 x 20,000 matrix with perfect 8 x 8 block structure, 0.33% non-zero entries
- **In this case:**
  no overhead when blocked r x c, with r,c divides 8



source: R. Vuduc, LLNL

Speed-up through r x c Blocking



- **machine dependent**
- **hard to predict**

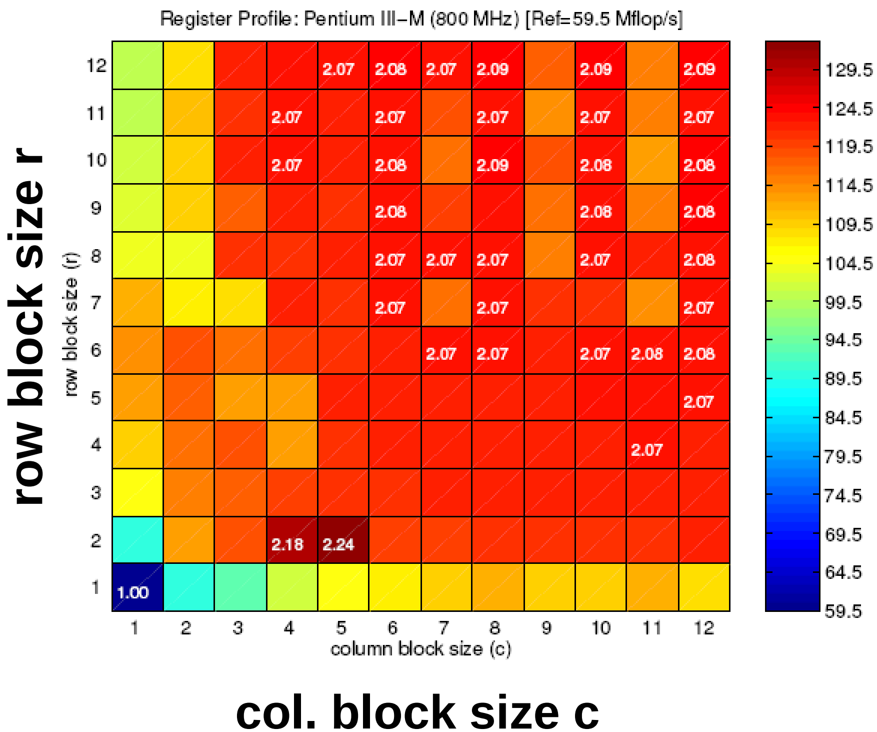
Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

How to Find the Best Blocking for given A?

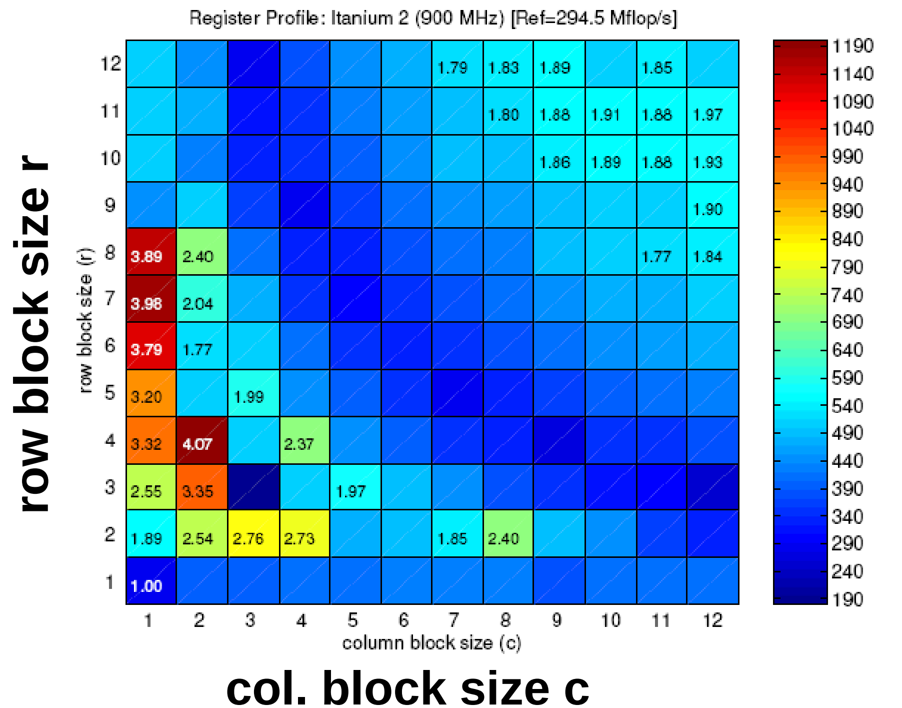
- **Best blocksize hard to predict (see previous slide)**
- **Searching over all r x c (within a range, say 1..12) BCSR expensive**
  - But: conversion of A in CSR to BCSR roughly as expensive as 10 SMVMs
- **Solution: Performance model for given A**
  - blackboard

Gain from Blocking (Dense Matrix in BCSR)

Pentium III



Itanium 2



- **machine dependence**
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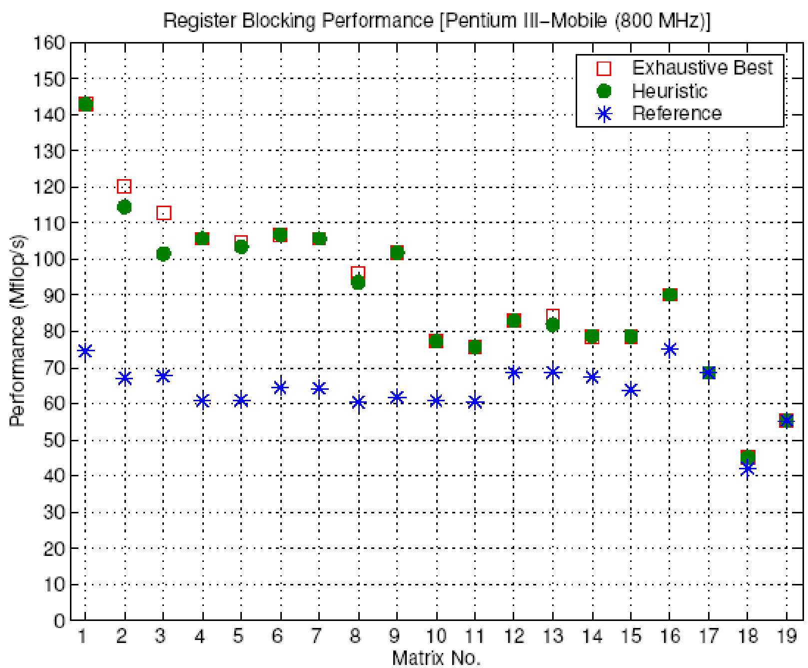
Register Blocking: Experimental results

- Paper applies method to a large set of sparse matrices

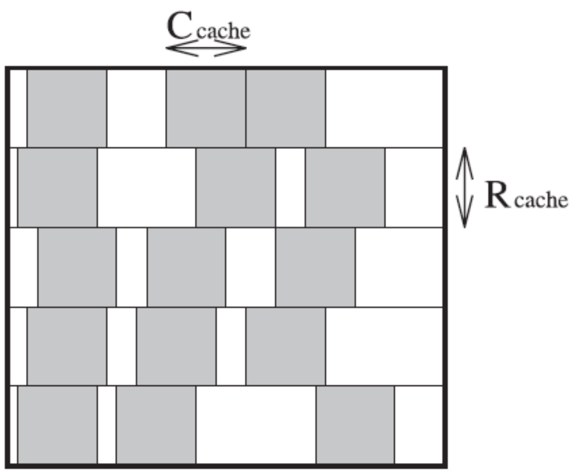
- Performance gains between 1x (no gain) for very unstructured matrices and 4x



Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Cache Blocking

- **Idea: divide sparse matrix into blocks of sparse matrices**

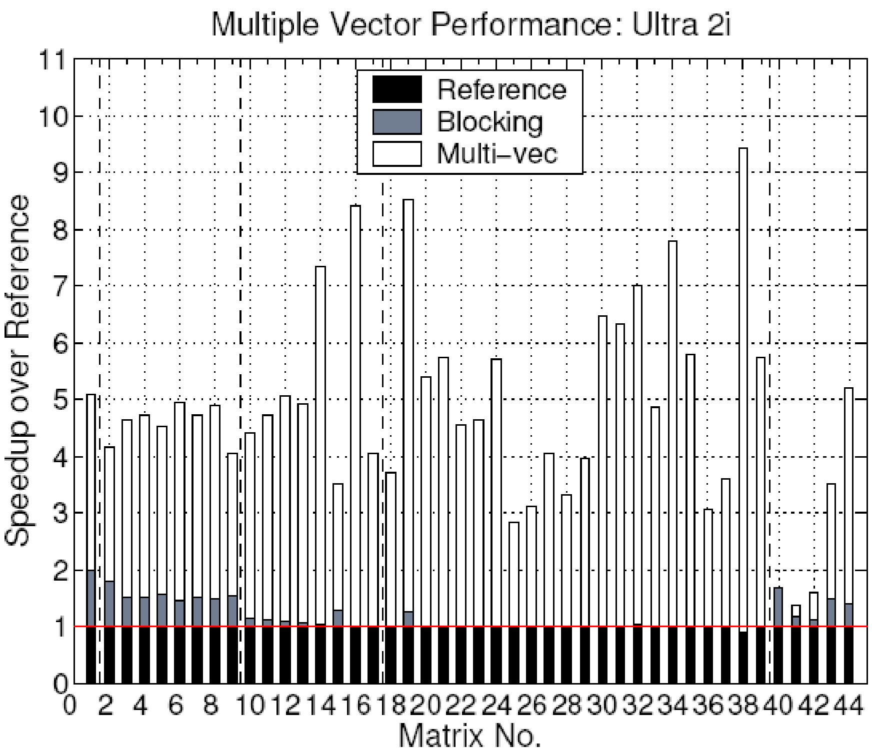


- **Experiments:**
  - requires very large matrices (x and y do not fit into cache)
  - speed-up up to 2.2x, speed-up only for few matrices, with 1 x 1 BCSR

Figure: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Multiple Vector Optimization

- **Blackboard**
- **Experiments: up to 9x speedup for 9 vectors**



Source: Eun-Jin Im, Katherine A. Yelick, Richard Vuduc. *SPARSITY: An Optimization Framework for Sparse Matrix Kernels*, *Int'l Journal of High Performance Comp. App.*, 18(1), pp. 135-158, 2004

Principles in Bebop/Sparsity Optimization

- **Optimization for memory hierarchy = increasing locality**
  - Blocking for registers (micro-MMMs) + change of data structure for A
  - Less important: blocking for cache
  - Optimizations are input dependent (on sparse structure of A)

- **Fast basic blocks for small sizes (micro-MMM):**
  - Loop unrolling (reduce loop overhead)
  - Some scalar replacement (enables better compiler optimization)

- **Search for the fastest over a relevant set of algorithm/implementation alternatives (= r, c)**
  - Use of performance model (versus measuring runtime) to evaluate expected gain

red = different from ATLAS