Electrical & Computer ENGINEERING

How to Write Fast Code

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Technicalities

- **Homework 4:**
  - Is no homework
  - Get research project started
  - Already posted
- **Tasks: For your chosen problem**
  - Straightforward, correct implementation
  - Cost measure definition and cost analysis
  - Performance plot, percentage of peak
- **Scalar replacement**

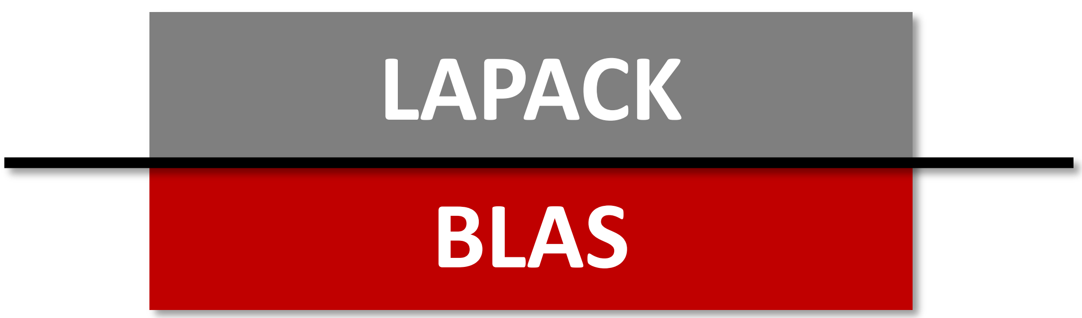
Today

- **Linear algebra algorithms and optimization**
  - Solving linear systems (Gauss elimination)
  - Matrix inversion
  - Determinant

Reminder: LAPACK

- Implements linear algebra algorithms
- Implemented on top of BLAS using BLAS 3 as much as possible (by "blocking" the algorithms)

Linear system solving
Matrix inversion
Singular value decomposition
... and more



BLAS 1: vector-vector ops
BLAS 2: matrix-vector ops
BLAS 3: matrix-matrix ops

Example: Linear Systems and Related

- Solving linear systems
- PLU factorization
- Matrix inversion
- Determinant

Complexity

- **Source:** **Buergisser, Clausen, Shokrollahi "Algebraic Complexity Theory," Springer 1997, pp. 426**

- **Definition:** **P(n), n > 0, a sequence of problems (n = problem size), complexity measure = number of adds + mults, then**

$$w(P) = \inf(\, g \mid complexity(P(n)) = O(n^g)\,)$$

- **Problems:**
  - MMM(n): multiplying two n x n matrices
  - MInv(n): inverting an n x n matrix
  - PLU(n): computing PLU factorization of an n x n matrix
  - Det(n): computing the determinant of an n x n matrix

Complexity Results

- **Example (we had that before): $2 \leq \omega(\mathrm{MMM}(n)) < 2.38$**

- **Theorem:**
  $\omega(\mathrm{MMM}(n)) = \omega(\mathrm{MInv}(n)) = \omega(\mathrm{PLU}(n)) = \omega(\mathrm{Det}(n))$

- **Cost of usual implementations:**
  - $\mathrm{MMM}(n) = 2n^3 + O(n^2)$
  - $\mathrm{MInv}(n) = 8/3\ n^3 + O(n^2)$
  - $\mathrm{PLU}(n) = 2/3\ n^3 + O(n^2)$
  - $\mathrm{Det}(n) = 2/3\ n^3 + O(n^2)$

How it's Implemented

- **Blackboard**