How to Write Fast Code

18-645, spring 2008
2nd Lecture, Jan. 16th

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Technicalities

- **Office hours posted**

- **Whom does this course target?**

- **Remember: Pair up in teams and let me know about project asap (deadline Jan 28th)**

- **Download this tutorial**

Today

- **Problems**

- **Algorithms**

- **Asymptotic analysis: O,Θ,Ω-Notation**

- **Asymptotic analysis of divide-and-conquer algorithms**

- **Standard book: Introduction to Algorithms (2nd edition), Corman, Leiserson, Rivest, Stein, McGraw Hill 2001)**

Problem

- **Problem: Specification of the relationship between a given input and a desired output**

- **Numerical problems: In- and Output are numbers**
  (or lists, vectors, arrays, … of numbers)

- **Examples**
  - Compute the discrete Fourier transform of a given vector x of length n
  - Matrix-matrix multiplication (MMM)
  - Compress an n x n image with a ratio …
  - Sort a given list of integers
  - Multiply by 5, y = 5x, using only additions and shifts
  - Prepare a cheeseburger

Algorithm

- **Algorithm: A precise description of a sequence of steps to solve a given problem.**

- **Numerical problems: These steps involve computation (addition, multiplication, …)**

- **Examples:**
  - Cooley-Tukey fast Fourier transform
  - A description of mat-mat multiplication by definition
  - JPEG encoding
  - Mergesort
  - y = x<<2 + x
  - Algorithms for "food problems:" www.epicurious.com

Tips for Publishing and Presenting

- **If your topic is a new algorithm, you must …**

  - Give a formal problem specification, like:
    Given: ……
    We want to compute …..
    This way the reader knows exactly what problem you address, including all constraints and assumptions

  - Formulate the algorithm in pseudo code or as a sequence of steps, preceded by
    Input: ….
    Output: …

  - Analyze the algorithm, at least asymptotic runtime in O-notation

- **Example: MMM**

Origin of the Word "Algorithm"

- **Mathematician, astronomer and geographer; founder of Algebra (his book: Al'Jabr wa'al'Muqabilah)**

- **Al'Khowârizmî → Algorithm**
  Al'Jabr → Algebra

- **Khowârizm is today the small Soviet city of Khiva**

- **Earlier word Algorism: The process of doing arithmetic using Arabic numerals**
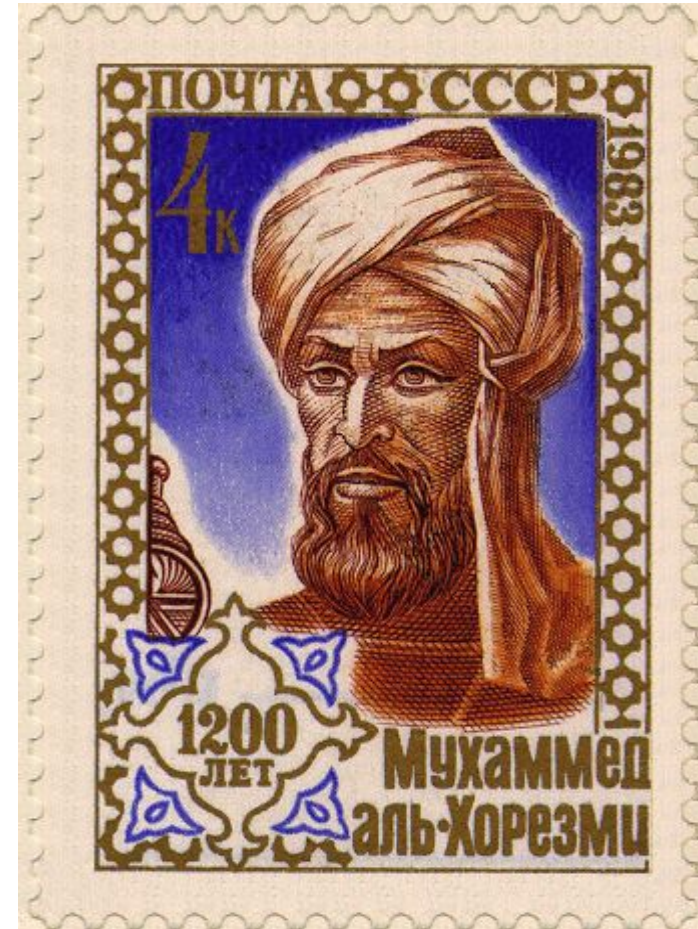
- **Algorithm: since 1957 in Webster Dictionary**

image from http://jeff560.tripod.com/

ПОЧТА CCCP
4к
1983
1200 ЛЕТ
МУХАММЕД аль-Хорезми

source:
http://www.disc-conference.org/disc2000/mirror/khorezmi/

Abu Ja'far Mohammed ibn Mûsâ al'Khowârizmî (c. 825)

Standard (asymptotic) Analysis of Algorithms & Problems

- **Analysis of Algorithms for**
  - Runtime
  - Memory requirement (memory footprint)

- **Runtime analysis of an algorithm:**
  - Count "elementary" steps
    (for numerical algorithms: usually floating point operations)
    dependent on the input size n (more parameters may be necessary)
  - State result in asymptotic O-notation
  - Example: MMM

- **Runtime complexity of a problem =
  Minimum of the runtimes of all possible algorithms**
  - Result also stated in asymptotic O-notation
    (more precisely using Ɵ, explained later)

Complexity is a property of a problem, not of an algorithm

Blackboard

- **O-Notation**

- **How to use**

- **Examples**

Asymptotic Runtime Analysis of Divide-and-Conquer Algorithms

Runtime for problem size *n*   **Cost of conquer step**

Recurrence $T(n) = aT(n/b) + f(n), \quad a \geq 1, b > 1$

a* subproblems of size *n/b

Solution

$$T(n) = \begin{cases} \Theta(n^{\log_b a}), & f(n) = O(n^{\log_b a - \epsilon}), \text{ for some } \epsilon > 0 \\ \Theta(n^{\log_b a} \log(n)), & f(n) = \Theta(n^{\log_b (a)}) \\ \Theta(f(n)), & f(n) = \Omega(n^{\log_b a + \epsilon}), \text{ for some } \epsilon > 0 \end{cases}$$

Stays valid if *n/b* is replaced by its floor or ceiling

Yeah, we need to look at some examples (blackboard):
mat-mat-mult, sorting, searching in sorted list, polynomial mult.