18-645/SP08: How to Write Fast Code

Assignment 8

Due Date: Fr Mar 28 6:00pm

http://www.ece.cmu.edu/~pueschel/teaching/18-645-CMU-spring08/course.html

Submission instructions: Your submission for this assignment will include two parts.

Part 1: Writeup. The first part will be a file that contains a writeup. If you use other programs (such as MS-Word) to create your assignment, convert them to PDF (google for 'pdfcreator' for a free conversion program). Name your file '18645-assign8-userid.pdf' where 'userid' is your andrew user id. The .pdf file must include all plots and figures. Do not put the .pdf file in a zip or tar archive - attach it separately. Send it along with part 1b (see below) to <schellap+18645-assign8@andrew.cmu.edu>. *In addition to the electronic copy, you must also submit a print-out of your pdf to the TAs at PH-B10 or to Carol Patterson at PH-B15.*

Part 2: Source code. The second part will consist of your source code.

Place all your files (including timer, verifier, Makefiles etc. - do not archive/zip them) in the following AFS directory (already created for you):

/afs/ece/class/ece645/submit/assign8/<yourandrewid>

Refer to Homework 2 for more information about copying files to AFS.

Exercises: Note that this is a shorter homework (70 points + extra only). You should spend the remaining time on your research project.

The Walsh-Hadamard transform (WHT) is a discrete signal transform related to the DFT but with simpler structure and simpler algorithms. It is used in signal processing and in communications.

The WHT is defined only for 2-power input sizes $n = 2^k$, and given by the matrix

$$\mathrm{WHT}_{2^k} = \underbrace{\mathrm{DFT}_2 \otimes \mathrm{DFT}_2 \otimes \ldots \otimes \mathrm{DFT}_2}_{k \text{ factors}}.$$

1. *(10 pts)* How many entries of the WHT matrix are zeros and why?

2. *(30 pts)* The WHT can of an input vector can be computed iteratively or recursively using the following formulas:

$$\begin{aligned} \mathrm{WHT}_{2^k} &= \prod_{i=0}^{k-1} (I_{2^{k-i-1}} \otimes \mathrm{DFT}_2 \otimes I_{2^i}) \quad \text{(iterative)} \\ \mathrm{WHT}_{2^k} &= (\mathrm{DFT}_2 \otimes I_{2^{k-1}})(I_2 \otimes \mathrm{WHT}_{2^{k-1}}) \quad \text{(recursive)} \end{aligned}$$

   In the midterm exam you saw the code for the recursive WHT and computed its arithmetic cost (look it up). The iterative WHT has the exact same cost.

   Implement both algorithms (for the recursive WHT, use the code in the midterm exam). The iterative WHT should be a triple loop. The recursive code needs in addition an implementation of the base case $\mathrm{WHT}_2 = \mathrm{DFT}_2$. Create a performance plot (Mflop/s versus size) for sizes 2^1–2^{20} (you may go higher as long as the computation finished within seconds). Discuss the plot.

3. *(25 pts)* Create an unrolled "codelet" for the WHT of size 4 (the number of operations should be 8).

   Now implement a recursive radix-4 implementations of the WHT based on

$$\mathrm{WHT}_{2^k} = (\mathrm{WHT}_4 \otimes I_{2^{k-2}})(I_4 \otimes \mathrm{WHT}_{2^{k-2}}).$$

   In this implementation, the left WHT of size 4 should be your unrolled codelet (which then has to handle strided input data); the right WHTs are recursive calls. Further, you may need one step with a different radix to handle all input sizes.

Measure the performance of this implementation, again for 2^1–2^{20} and add it to the previous plot (three lines total). Discuss the plot.

4. *(5 pts)* How many hours did you spend total on the above problems?

5. *(up to 30 extra pts)* Do one of the following:

   - extend the implementation to include the higher radices 8 and 16 and use a dynamic programming search to find the best recursion (radix-2, -4, -8, or -16) in each step independently. Report performance and which radices where found and discuss.
   - Completely vectorize (2-way or 4-way) the radix-4 implementation and aim for a speedup. Report the performance and discuss.

6. You should spend the remaining time on your research project. Write a short summary (at most 1/2 page) describing what you did and update the plot if appropriate (and then, of course, briefly discuss the plot).