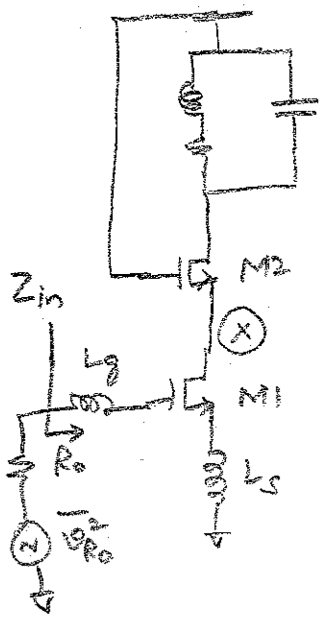
(Adapted from Michael Perrott's notes, lectures 7 & 9, Course 6.976, http://ocw.mit.edu)

Noise performance of source degenerated LNA:



- Assume frequency of operation = ω_0
- Input impedance at ω_0

$$Z_{in} = \underbrace{\frac{g_m L_s}{C_{gs}}} + \underbrace{j\left[\omega_0(L_s+L_g) - \frac{1}{\omega_0 C_{gs}}\right]}_{\text{resonance at } \omega_0}$$

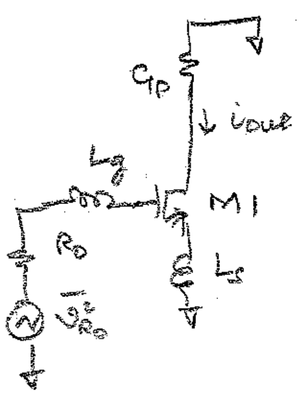
set $\omega_T L_s = R_0$ for power match

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{(L_s+L_g)C_{gs}}}$$

- At ω_0, the load is a pure resistor R_p (or conductance G_p)

- For the present, we will assume that the cascode device does not contribute noise. Of course, this is not accurate; however, as long as the parasitic capacitance C_x is small, M2's noise contribution can be ignored.

- The circuit for noise calculations is then:



→ We have a total of three noise sources:
  - source noise $\overline{v_{Ro}^2}$
  - drain noise of M1 $\overline{i_d^2} = 4kT\gamma g_{do}\Delta f$
  - gate noise of M1 $\overline{i_g^2} = 4kT\delta g_g \Delta f$

→ v_{Ro} is independent and hence uncorrelated to i_d and i_g.

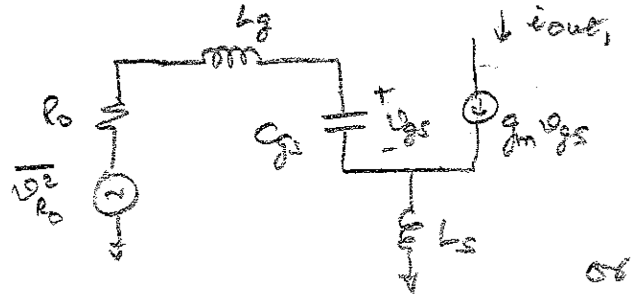
→ i_g and i_d are correlated through a correlation coefficient c which is defined as follows:

$$\overline{i_g i_d^*} = c\sqrt{\overline{i_g^2}\,\overline{i_d^2}}$$

(Van der Ziel: $c = -j0.395$ for long-channel, saturated FET)

→ We shall consider each noise source separately and pay attention to correlation between i_d and i_g when calculating the total mean-square noise current $\overline{i^2_{out,tot}}$

(a) Source noise contribution



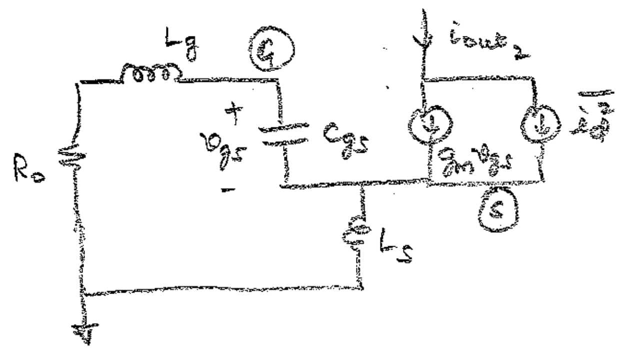
Define $Q = \dfrac{\omega_0 (L_s + L_g)}{\omega_T L_s + R_0} = \dfrac{1}{\omega_0 C_{gs}(R_0 + \omega_T L_s)}$

or $\boxed{Q = \dfrac{\omega_0 (L_s + L_g)}{2R_0} = \dfrac{1}{2\omega_0 C_{gs} R_0}}$

$v_{gs} = Q\, v_{R_0} \quad \Rightarrow \quad i_{out,1} = g_m v_{gs} = Q g_m v_{R_0}$

$$\boxed{i_{out,1} = Q g_m v_{R_0}}$$

(b) Drain noise contribution:



• KCL at S:

$$g_m v_{gs} + i_d + sC_{gs} v_{gs} = \frac{v_s}{sL_s}$$

• KCL at G:

$$\frac{v_{gs} + v_s}{R_0 + sL_g} + sC_{gs} v_{gs} = 0$$

Solve:

$$v_{gs} = -\frac{sL_s i_d}{1 + s(L_s g_m + C_{gs} R_0) + s^2 (L_s + L_g) C_{gs}}$$

At $s = j\omega_0$:

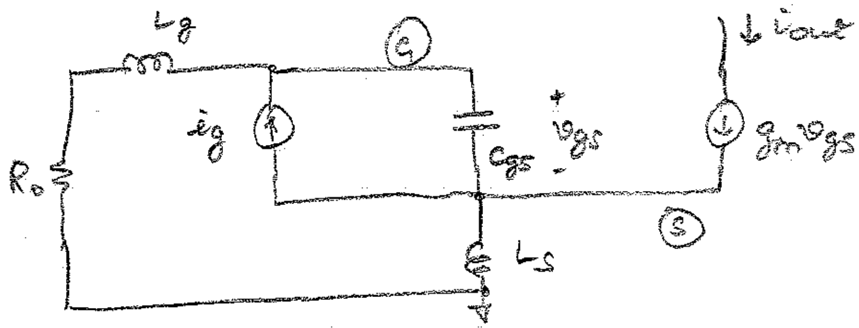
$$v_{gs} = -\frac{j\omega_0 L_s i_d}{\underbrace{\left[1 - \omega_0^2 (L_g + L_s) C_{gs}\right]}_{0 \text{ at } \omega_0} + j\omega_0 \left(\frac{g_m L_s}{C_{gs}} + R_0\right) C_{gs}}$$

$$= -\frac{L_s / C_{gs}\; i_d}{2R_0}$$

$$\Rightarrow i_{out_2} = i_d + g_m v_{gs} = i_d - \frac{g_m L_s / C_{gs}}{2R_o} i_d = i_d - \frac{R_o}{2R_o} i_d$$

$$\Rightarrow \boxed{i_{out_2} = \frac{1}{2} i_d}$$

(c) Gate noise contribution :



KCL at (s) ; $i_g + \frac{v_s}{sL_s} = (g_m + sC_{gs}) v_{gs}$

KCL at (g) : $i_g = sC_{gs} v_{gs} + \frac{v_{gs} + v_s}{R_o + sL_g}$

Solve : $v_{gs} = \frac{R_o + s(L_s + L_g)}{1 + s^2(L_s + L_g)C_{gs} + s(R_o C_{gs} + g_m L_s)}$

At $s = j\omega_0$: $v_{gs} = \frac{R_o + j\omega_0(L_s + L_g)}{\underbrace{[1 - \omega_0^2(L_s + L_g)C_{gs}]}_{0 \text{ at } \omega_0} + s(R_o C_{gs} + g_m L_s)}$

$$= \frac{[1 + j\omega_0(L_s + L_g)/R_o] R_o}{j\omega_0 \left(R_o + \frac{g_m L_s}{C_{gs}}\right) C_{gs}} i_g$$

$$\left.\begin{aligned} Q &= \frac{1}{2\omega_0 R_o C_{gs}} \\ &= \frac{\omega_0(L_s + L_g)}{2R_o} \end{aligned}\right\} = i_g \frac{1 + j2Q}{j} \cdot Q R_o = (2Q - j) \frac{1}{2\omega_0 C_{gs}} i_g$$

$$\Rightarrow \boxed{i_{out_3} = g_m v_{gs} = \frac{1}{2}(2Q - j) \frac{g_m}{\omega_0 C_{gs}} i_g} \Rightarrow \boxed{i_{out_3} = \frac{1}{2}(2Q - j) \frac{\omega_T}{\omega_0} i_g}$$

$$i_{out,tot} = i_{out_1} + i_{out_2} + i_{out_3}$$

$$= Q g_m v_{R_0} + \frac{1}{2} i_d + \frac{1}{2}(2Q - j)\frac{\omega_T}{\omega_0} i_g$$

To find mean-square output noise current:

Note $\overline{|X+Y|^2} = \overline{(X+Y)(X+Y)^*}$

$$= \overline{|X|^2} + \overline{|Y|^2} + \overline{XY^* + X^*Y}$$

$$= \overline{|X|^2} + \overline{|Y|^2} + \overline{2\,\mathrm{Re}(XY^*)}$$

$$\therefore\ \overline{i^2_{out\,tot}} = Q^2 g_m^2 \overline{v^2_{R_0}} + \frac{1}{4}\overline{i_d^2} + \frac{1}{4}|2Q-j|^2\left(\frac{\omega_T}{\omega_0}\right)^2 \overline{i_g^2}$$

$$+ \frac{1}{4}\, 2\,\mathrm{Re}\left\{\overline{i_d^*\,(2Q-j)\frac{\omega_T}{\omega_0}\, i_g}\right\}$$

$$= Q^2 g_m^2 \overline{v^2_{R_0}} + \frac{1}{4}\overline{i_d^2} + \frac{1}{4}(4Q^2+1)\left(\frac{\omega_T}{\omega_0}\right)^2 \overline{i_g^2}$$

$$+ \frac{2}{4}\mathrm{Re}\left\{(2Q-j)\frac{\omega_T}{\omega_0}\,\overline{i_g i_d^*}\right\}$$

$$= Q^2 g_m^2 \overline{v^2_{R_0}} + \frac{1}{4}\overline{i_d^2}\left[1 + (4Q^2+1)\left(\frac{\omega_T}{\omega_0}\right)^2 \frac{\overline{i_g^2}}{\overline{i_d^2}} + 2\,\mathrm{Re}\left\{(2Q-j)\frac{\omega_T}{\omega_0}\underbrace{\frac{\overline{i_g i_d^*}}{\overline{i_d^2}}}\right\}\right]$$

$$\underbrace{\frac{\overline{i_g i_d^*}}{\sqrt{\overline{i_d^2}\,\overline{i_g^2}}}}_{=c!}\cdot\sqrt{\frac{\overline{i_g^2}}{\overline{i_d^2}}}$$

Also, $\dfrac{\overline{i_g^2}}{\overline{i_d^2}} = \dfrac{4kT\delta g_g \Delta f}{4kT\gamma g_{d0}\Delta f} = \dfrac{\delta g_g}{\gamma g_{d0}}$

and $c = -j0.395 = -j|c|$

$$\therefore\ \overline{i^2_{out\,tot}} = Q^2 g_m^2 \overline{v^2_{R_0}} + \frac{1}{4}\overline{i_d^2}\left[1 + (4Q^2+1)\frac{\omega_T^2}{\omega_0^2}\cdot\frac{\delta g_g}{\gamma g_{d0}} - 2\left(\frac{\omega_T}{\omega_0}\right)|c|\sqrt{\frac{\delta g_g}{\gamma g_{d0}}}\right]$$

Finally, substitute $g_g = \dfrac{\omega^2 C_{gs}^2}{5 g_{d0}}$ and $g_m = \omega_T C_{gs}$

$$\Rightarrow \overline{i^2_{out_{tot}}} = Q^2 g_m^2 \overline{v^2_{R_o}} + \frac{1}{4}\overline{i_d^2}\left[1 + (4Q^2+1)\left(\frac{g_m}{g_{d0}}\right)^2 \frac{\delta}{5\gamma} - 2|c|\frac{g_m}{g_{d0}}\sqrt{\frac{\delta}{5\gamma}}\right]$$

Finally, $F = \dfrac{\overline{i^2_{out_{tot}}}}{\overline{i^2_{out,source}}}$

$$\Rightarrow F = 1 + \underbrace{\frac{1}{4Q^2 g_m^2}\frac{\overline{i_d^2}}{\overline{v^2_{R_o}}}}\left[1 + (4Q^2+1)\left(\frac{g_m}{g_{d0}}\right)^2 \frac{\delta}{5\gamma} - 2|c|\frac{g_m}{g_{d0}}\sqrt{\frac{\delta}{5\gamma}}\right]$$

$$\frac{1}{2Q}\cdot\frac{1}{2Q g_m R_o}\cdot\frac{\gamma g_{d0}}{g_m} = \frac{1}{2Q}\left(\frac{\omega_0}{\omega_T}\right)\frac{\gamma g_{d0}}{g_m}$$

$$\Rightarrow F = 1 + \left(\frac{\omega_0}{\omega_T}\right)\cdot\frac{\gamma g_{d0}}{g_m}\cdot\frac{1}{2Q}\left[1 + (4Q^2+1)\left(\frac{g_m}{g_{d0}}\right)^2 \frac{\delta}{5\gamma} - 2|c|\frac{g_m}{g_{d0}}\sqrt{\frac{\delta}{5\gamma}}\right]$$

S_N: scaling coefficient, dependent only on Q and process technology.

- This is a lower bound on the total noise. Parasitic resistances (due to layout, gate resistance, inductor resistances) will add to F. The cascode device noise also adds due to parasitic capacitances at the cascode node.

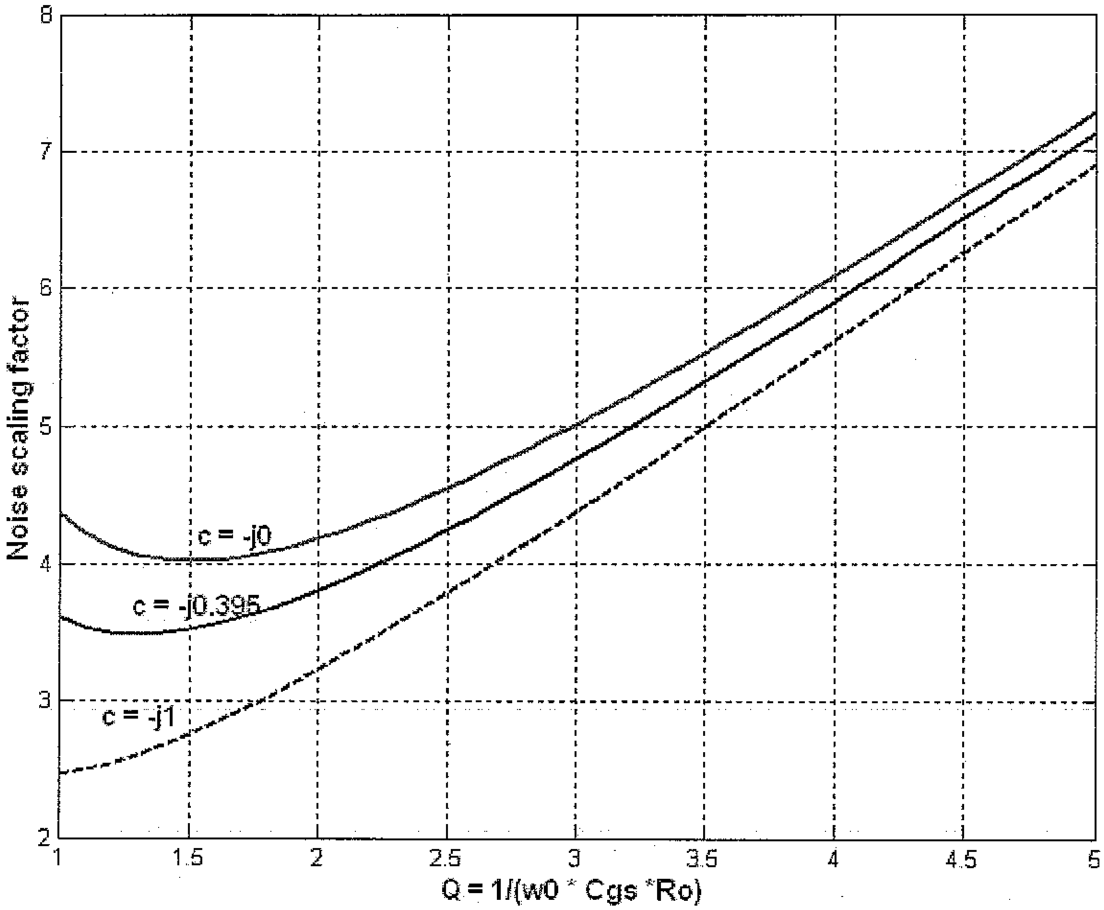
- $F = 1 + \left(\dfrac{\omega_0}{\omega_T}\right) S_N$

  See plot of S_N vs. Q for different c's on the next page. We have assumed the following:

  $$\frac{g_m}{g_{d0}} = \frac{1}{1.8}, \quad \gamma = 3, \quad \delta = 2\gamma = 6$$

- (Long channel approx.) $\omega_t = \dfrac{3}{2}\dfrac{\mu_n}{L^2}(V_{gs} - V_T)$

  $\Rightarrow \omega_t \propto \dfrac{1}{L^2} \rightarrow$ performance improves dramatically with CMOS device scaling.

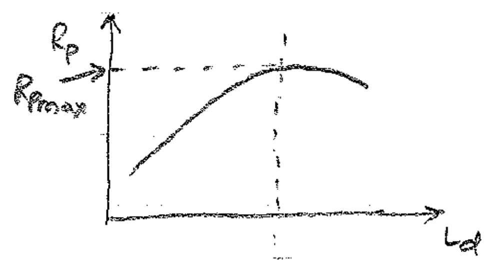


Noise scaling factor
Q = 1/(w0 * Cgs *Ro)
c = -j0
c = -j0.395
c = -j1
8
7
6
5
4
3
2
1
1.5
2
2.5
3
3.5
4
4.5
5

LNA Design procedure :

		eg
Specifications :	Min. voltage gain A_v	> 20dB
	Operating frequency ω_0	5GHz
	source impedance R_0	50Ω
	Max. power dissipation P_{diss}	10mW
	Max. Noise Figure F	2dB
	Min. IIP3 V_{IIP3} (or P_{IIP3})	0dBm
	1dB compression V_{-1dB} (or P_{-1dB})	-10dBm
	Supply voltage V_{dd}	1.4V

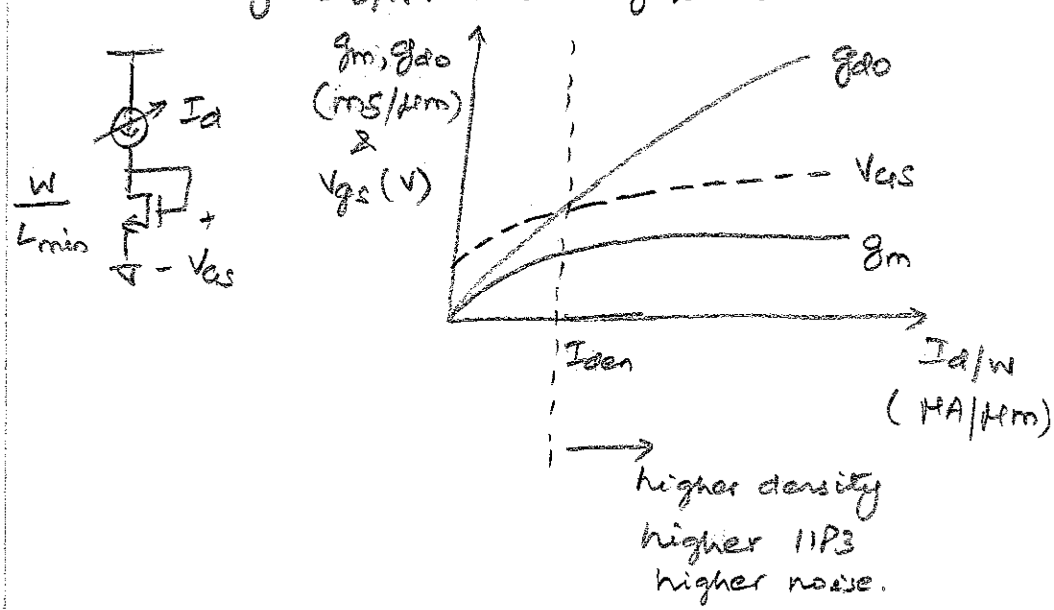
(1) First set R_p. In general, this is highly dependent on the process technology. A plot of R_p vs. L_d (the drain inductance might look like this :



Generally, we would pick an L_d which offers maximum R_p.

(2) From $A_v = G_m R_p$, calculate the effective transconductance G_m. Note that $G_m = Q g_m$.

(3) For a minimum channel-length device, plot (by simulation or otherwise) g_m, g_{do} and V_{GS} as functions of current density I_D/W. One way to do this is :



Another option is to plot these in Matlab using analytical expressions:

- $$I_D = \beta \frac{(V_{GS}-V_T)^2}{1+\eta(V_{GS}-V_T)}$$

- $$g_m = \frac{\beta(V_{GS}-V_T)}{1+\eta(V_{GS}-V_T)}\left[1+\frac{1}{1+\eta(V_{GS}-V_T)}\right]$$
$$= \frac{I_D}{V_{GS}-V_T}\left[1+\frac{1}{\eta(V_{GS}-V_T)}\right]$$

where

$$\beta = \frac{1}{2}\mu_o C_{ox}\frac{W}{L}$$

and

$$\eta = \theta + \frac{\mu_o}{2 v_{sat} L}$$

$$\theta \simeq \frac{10^{-7}}{t_{ox}} V^{-1}.$$

μ_o, C_{ox}, t_{ox}, θ and v_{sat} can be found from the simulation model file.

- $g_{do} = 2\beta(V_{GS}-V_T)$

(4) Pick a current density I_{den} from the plot. We know from analysis that as $I_{den}\uparrow$, $(V_{GS}-V_T)\uparrow$, $F\uparrow$ because $g_{do}/g_m\uparrow$ and IIP3 $\uparrow$. $\Rightarrow$ Noise vs. linearity trade-off

A good compromise is to operate the device in weak-to-moderate inversion i.e., $(V_{GS}-V_T) \sim 100-200$ mV.

Once we know $(V_{GS}-V_T)$ pick off g_m from the graph. Note that this is a normalized g_m (mS/μm)

(5) Under the assumption that the correlation coefficient c is known, find Q for minimum noise scaling factor. Then, use $Q = \frac{1}{2\omega_o C_{gs} R_o}$ to find C_{gs},

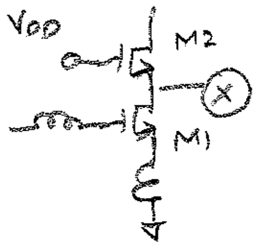
and calculate W from $C_{gs} = \frac{2}{3} C_{ox} W L$

(6) Calculate the g_m: $g_m = W \cdot X$ (normalized g_m)

(7) Find $\omega_T = \dfrac{g_m}{C_{gs}}$ and use $R_s = \omega_T L_s$ to find L_s

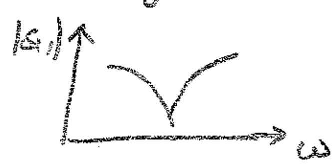
(8) Use $\omega_0^2 = \dfrac{1}{(L_g + L_s) C_{gs}}$ to find L_g.

(9) For the cascode device, set W and L to be the same as that of M1 as a starting point.



Usually, the gate of M2 is tied to the power supply V_{dd}. Since we know the bias current $I_{Bias} = I_{den} \cdot W_1$, and also the size of M1, the bias voltage V_x is now fixed.

At this point, verify stability by simulation.
$|S_{11}|$ must not exhibit "funny" behavior – you should see something like



(10) Simulate to find A_v, F and IIP3 and verify that they meet spec. Also check that P_{diss} is within spec.
If not, iterate steps (4) – (10).