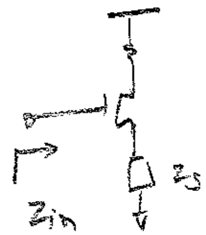
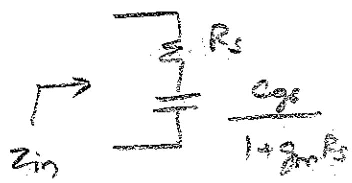
Power match without resistors: Degeneration



General case:

$$Z_{in} = Z_s + \frac{1}{sC_{gs}} + \frac{g_m Z_s}{sC_{gs}}$$

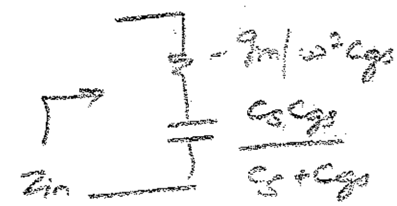
(i) Resistive degeneration: $Z_{in} = R_s + \frac{1}{sC_{gs}}(1 + g_m R_s)$
($Z_s = R_s$)



(ii) capacitive degeneration ($Z_s = \frac{1}{sC_s}$):

$$Z_s = \frac{1}{j\omega C_s} + \frac{1}{j\omega C_{gs}} \underbrace{- \frac{g_m}{\omega^2 C_{gs}}}_{\text{negative real part}} \Rightarrow$$

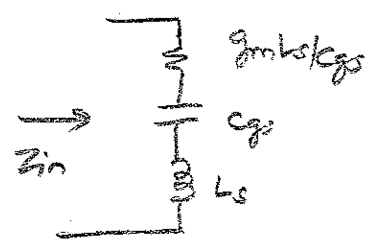
$\Rightarrow$ oscillations possible



(iii) Inductive degeneration ($Z_s = sL_s$):

$$Z_s = j\omega L_s + \frac{1}{j\omega C_{gs}} + \frac{g_m L_s}{C_{gs}}$$

$$= j\omega L_s + \frac{1}{j\omega C_{gs}} + \omega_T L_s$$



($\omega_T = \frac{g_m}{C_{gs}}$ = transition frequency of MOSFET)

$\Rightarrow$ can create a real part in Z_{in} without a resistor

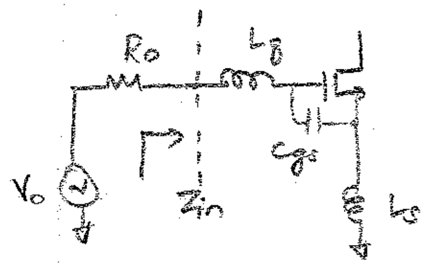
$\rightarrow$ power match without excess noise

To match to a $50\,\Omega$ source, set $\frac{g_m L_s}{C_{gs}} = \omega_T L_s = 50\,\Omega$

The resonant freq. of this circuit is $\omega_0 = \frac{1}{\sqrt{L_s C_{gs}}}$.

$\Rightarrow$ At $\omega = \omega_0$, $Z_{in} = \omega_T L_s \longrightarrow$ purely real

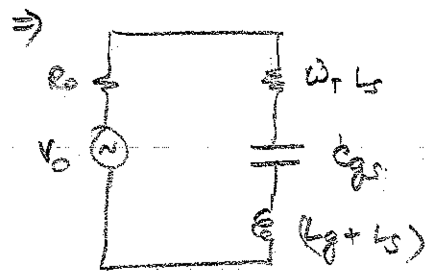
We may want a power match to a real source impedance at some <u>lower</u> frequency. To do this, add a gate tuning inductor:



$$Z_{in} = \frac{g_m L_s}{C_{gs}} + j\left[\omega(L_s + L_g) - \frac{1}{\omega C_{gs}}\right]$$

(a) Want <u>power match</u> $Z_{in} = R_0$ at the resonant frequency

$\Rightarrow$



$$\boxed{\omega_0 = \frac{1}{\sqrt{(L_s + L_g)C_{gs}}}}$$

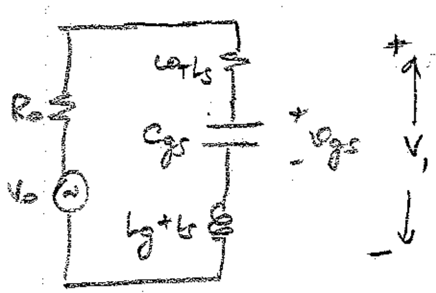
(b) <u>Q of input loop:</u>

$$Q_{in} = \frac{\omega_0(L_g + L_s)}{R_0 + \omega_T L_s} = \frac{1}{\omega_0(R_0 + \omega_T L_s)C_{gs}}$$

$$= \frac{\sqrt{(L_g + L_s)/C_{gs}}}{R_0 + \omega_T L_s}$$

$\sqrt{(L_g + L_s)/C_{gs}}$ $\to$ characteristic impedance

(c) <u>Effective transconductance at resonance</u>



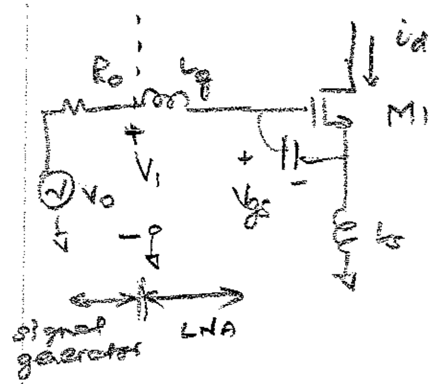
Recall @ resonance $X_c + X_L = 0$

Also $V_c = V_L = Q_{in} V_0$

$\Rightarrow V_{gs} = Q_{in} V_0$

$R_0 = \omega_T L_s \Rightarrow V_1 = \dfrac{\omega_T L_s}{R_0 + \omega_T L_s} V_0 = \dfrac{V_0}{2}$

$\Rightarrow V_{gs} = 2Q_{in} V_1$



$$i_d = g_{m1} v_{gs} \quad (G_m = \text{effective transcond.})$$

$$= (g_{m1} \cdot 2Q_{in}) V_1$$

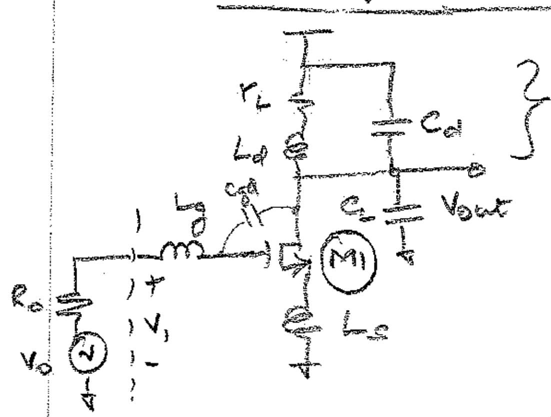
$$\Rightarrow \boxed{G_m = 2Q_{in}\, g_{m1}}$$

Note that we could also define the effective transcond. in terms of V_o: $i_d = G_m' V_o = Q_{in} g_{m1} V_o$

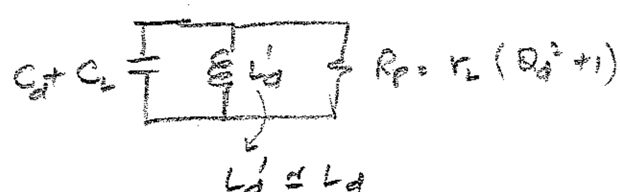
$$\Rightarrow \boxed{G_m' = Q_{in}\, g_{m1} = \frac{G_m}{2}}$$

However, note that V_o is not available to us as a terminal while V_1 is available.

(d) <u>A simple power matched LNA:</u>



$\Rightarrow$ Series-to-parallel transformation



$L_d' \simeq L_d$

Resonance at $\omega_o = \dfrac{1}{\sqrt{L_d(C_d + C_L)}}$

Ignore gate-drain parasitic capacitance C_{gd}

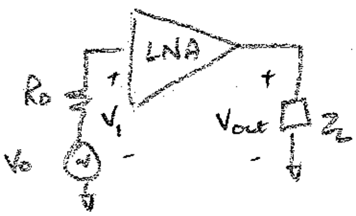
1 Voltage gain @ resonance $\Big\}$ $A_v = \dfrac{V_{out}}{V_1} = \dfrac{G_m V_1 R_p}{V_1} = G_m R_p$

$$\Rightarrow \boxed{A_v = 2Q_{in} g_{m1} R_p}$$

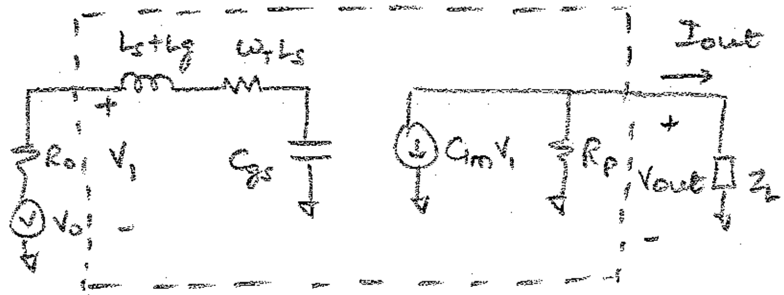
Note that we have neglected the output impedance looking downwards into the drain of M1. This is often justified because we will generally use a cascode along with M1 to improve stability.

(More later)

(2) Transducer power gain G_T:



$\Rightarrow$



$$G_T = \frac{\text{Power delivered to load } (P_{out})}{\text{Power available from source } (P_{avs})}$$

- This is the gain that is measured if we connect a spectrum analyzer or power meter to the output of an LNA that is driven by a signal generator of impedance R_o.

$$P_{avs} = \frac{|V_o|^2}{8R_o} \quad (\text{assume peak phasors}) = \frac{|V_1|^2}{2R_o} \quad (R_p = R_{in})$$

$$P_{out} = \frac{1}{2}\text{Re}\left[V_{out} I_{out}^*\right] = \frac{1}{2}\text{Re}\left[V_{out} \cdot \frac{V_{out}^*}{Z_L^*}\right]$$

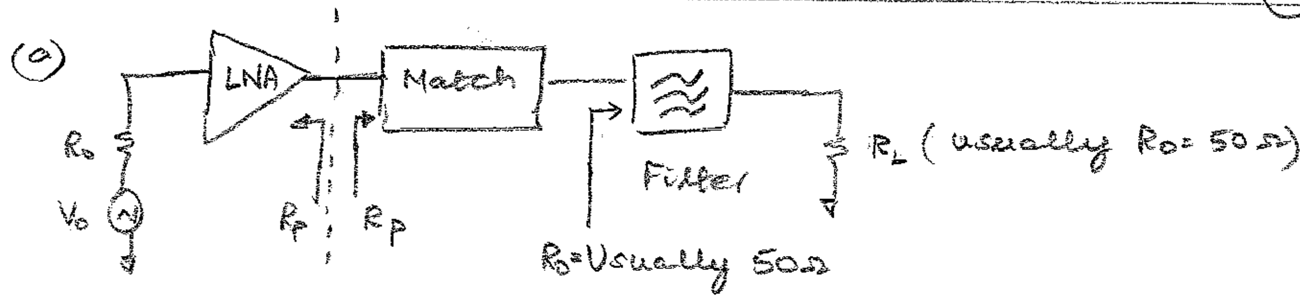
$$= \frac{1}{2}|V_{out}|^2 \,\text{Re}\left(\frac{1}{Z_L^*}\right) \quad \left\{ \text{Note } \text{Re}\left(\frac{1}{Z_L^*}\right) \neq \frac{1}{\text{Re}(Z_L^*)} !! \right\}$$

$$V_{out} = G_m Z_L \| R_p V_1$$

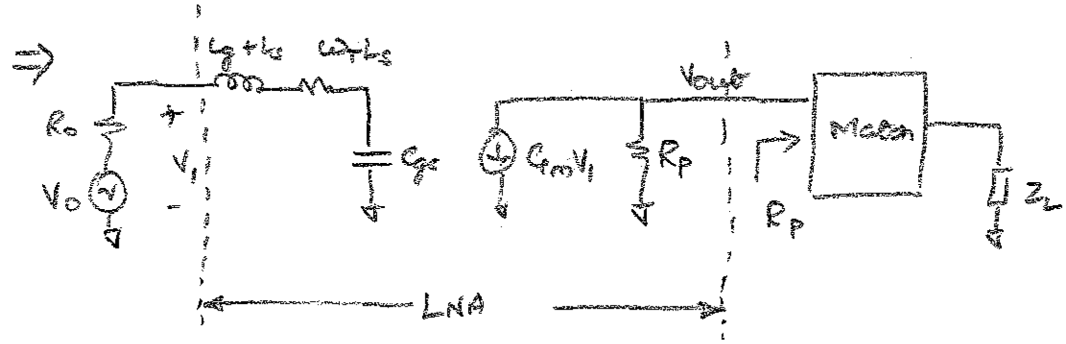
$$\Rightarrow G_T = \frac{\frac{1}{2}\left|G_m Z_L \| R_p\right|^2 |V_1|^2 \,\text{Re}\left(\frac{1}{Z_L^*}\right)}{\frac{1}{2}\frac{|V_1|^2}{R_o}} \Rightarrow \boxed{G_T = \left|G_m Z_L \| R_p\right|^2 \text{Re}\left(\frac{1}{Z_L^*}\right) R_o}$$

Two important cases:
- — LNA drives a filter through an output conjugate matching network
- — LNA drives the gate of a MOSFET (buffer or C.S mixer)

(a)

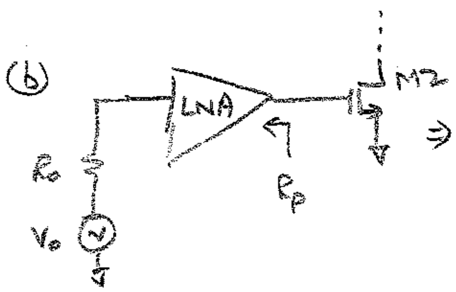


$\Rightarrow$

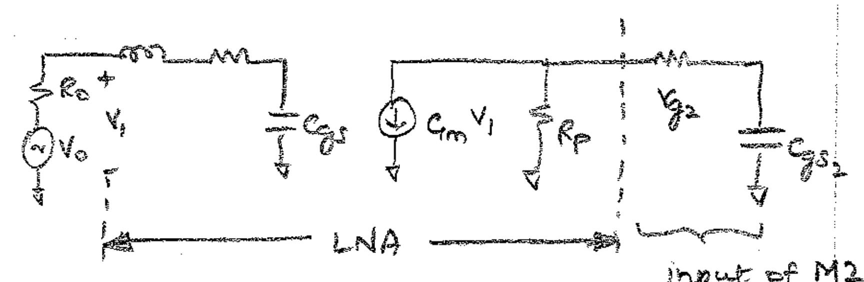


Here, $Z_L = R_p \Rightarrow G_T = \left|\frac{G_m R_p}{2}\right|^2 \mathrm{Re}\left\{\frac{1}{R_p^*}\right\} R_0 = \frac{(G_m R_p)^2 R_0}{4 R_p}$

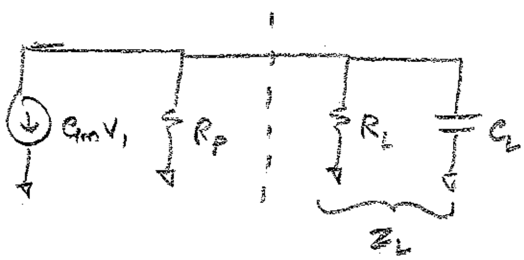
(b)



$\Rightarrow$



$\rightarrow$ Series-parallel tx. on M2:



$R_L = r_{g2}(Q_2^2 + 1) \simeq Q_2^2 r_{g2}$

$C_L = C_{gs_2} \frac{Q_2^2}{Q_2^2 + 1} \simeq C_{gs_2}$

where $Q_2 = \frac{1}{\omega_0 r_{g_2} C_{gs_2}}$

(Note: In practice, we would absorb C_L into the parallel L-C network at the output of the LNA

$\Rightarrow Z_L \simeq R_L = Q_2^2 r_{g_2}$

In a MOSFET with <u>good layout</u>, r_{g_2} is small. Ideally, $r_{g_2} \to 0$ so that $Q_2 \to \infty$

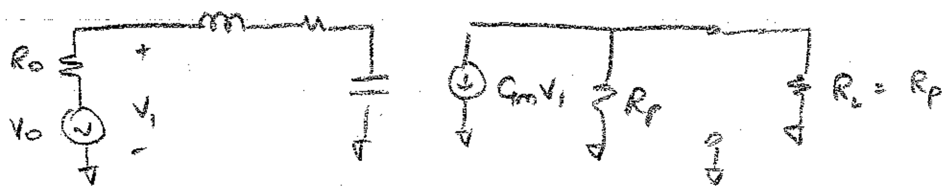
$$\Rightarrow \; G_T = \left(G_m R_p || R_L\right)^2 \cdot \frac{R_0}{R_L} \rightarrow 0 \text{ as } R_L \rightarrow \infty.$$

- Note the gotcha here: If M2 is an ideal MOSFET, its gate would present a purely capacitive input which cannot consume real power. Consequently $P_{out} \rightarrow 0$, and hence $G_T \rightarrow 0$.

In practice, a MOSFET gate can be modeled as purely capacitive only for frequencies below the "unity power-gain frequency f_{max}"

(3) Available power gain G_A:

$$G_A = \frac{\text{Power available from LNA output } P_{av_{LNA}}}{\text{" " " source } P_{avs}}$$

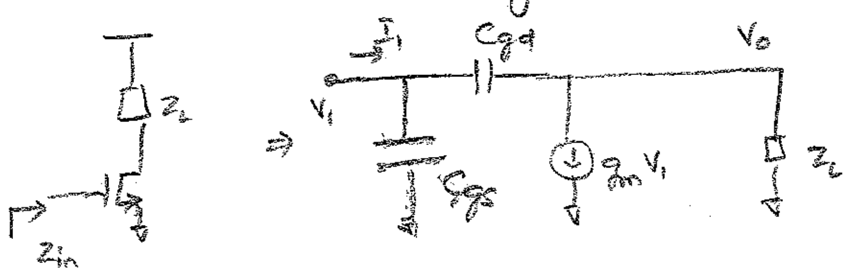


$\Rightarrow$ $P_{av_{LNA}}$ = power consumed by load which is matched to the LNA output.

$\Rightarrow$ same as case (2a).

$$\Rightarrow \; G_A = \frac{(G_m R_p)^2 \cdot R_0}{R_p}$$

Stability: We have ignored the effects of C_{gd} thus far. Since this causes feedback from output to input, it can cause instability:



① $I_1 = sC_{gs}V_1 + sC_{gd}(V_1 - V_o)$

② $g_m V_1 + Y_L V_o = sC_{gd}(V_1 - V_o)$

$$\boxed{Y_L = \frac{I_1}{V_1} = sC_{gs} + sC_{gd}\frac{Y_L + g_m}{Y_L + sC_{gd}}}$$

For parallel RLC tank at low freq., $Z_L \simeq \frac{1}{j\omega L}$

$$\Rightarrow Y_L = \underbrace{-\frac{\omega^2 L C_{gd} g_m}{1 - \omega^2 L C_{gd}}} + j\omega\left[C_{gs} + \frac{C_{gd}}{1 - \omega^2 L C_{gd}}\right]$$

At low frequencies, $Re(Y_L) \simeq -\omega^2 L C_{gd} g_m$

→ negative value whose magnitude gets larger with larger feedback cap. C_{gd}

$\Rightarrow$ can cause instability

Stern stability factor

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{21}||S_{12}|} \qquad \Delta = S_{11}S_{22} - S_{12}S_{21}$$

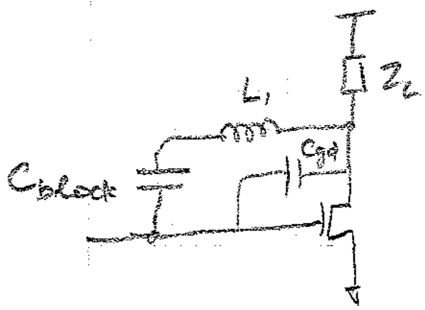
→ If $K > 1$ and $\Delta < 1$ circuit is unconditionally stable.

→ S_{12} is a measure of reverse isolation.

As S_{12} increases, reverse isolation increases & circuit becomes more stable.

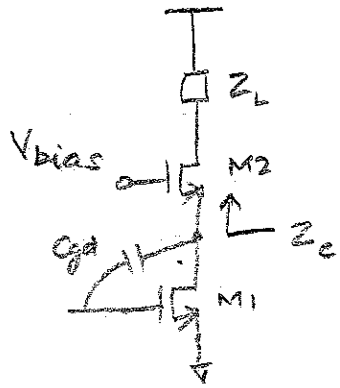
Circuit techniques to increase S_{12}:

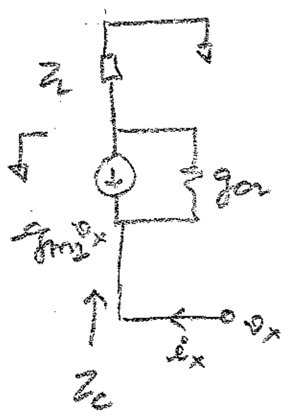
(a) Neutralization



L_1 and C_{gd} resonate at the desired frequency

(b) Cascode





Ideally,

$g_{o2} \to 0$

$\Rightarrow \dfrac{i_x}{v_x} = \dfrac{1}{Z_c} = g_{m2}$

$\Rightarrow Z_c \simeq \dfrac{1}{g_{m2}}$

$$Y_L = sC_{gs1} + sC_{gd1}\frac{g_{m1} + g_{m2}}{g_{m1} + sC_{gd}}$$

$$= \underbrace{\frac{\omega^2 C_{gd}^2 (g_{m1} + g_{m2})}{g_{m2}^2 + \omega^2 C_{gd1}^2}}_{\text{positive at all frequencies.}} + j\omega\left[C_{gs1} + \frac{g_{m2} C_{gd1}(g_{m1} + g_{m2})}{g_{m2}^2 + \omega^2 C_{gd1}^2}\right]$$

- Practically: If we use a short-channel MOSFET for M2, $g_{o2} \gg 0 \Rightarrow$ instability problem can still exist.
  $\Rightarrow$ may want to increase channel length of M2
- Smaller signal swing at output
- Increased parasitic cap. at M2 source, can cause increased NF at high frequencies.