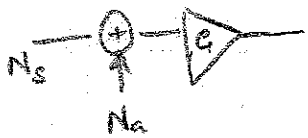
Noise figure - another definition



Consider an amplifier with input-referred noise power N_a and available power gain G.

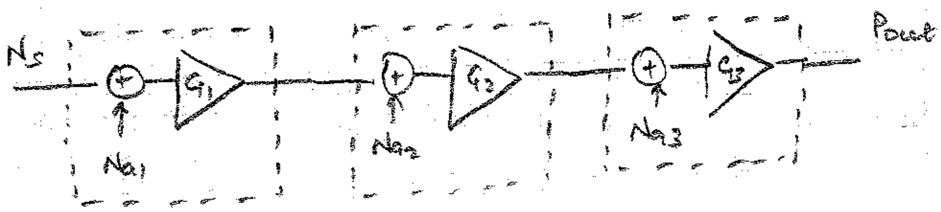
$$\text{Noise factor } F = \frac{\text{Total noise power}}{\text{Noise power from source}} = \frac{N_a + N_s}{N_s} = 1 + \frac{N_a}{N_s}$$

Let S = signal power ; Input SNR = $SNR_{in} = \frac{S}{N_s}$

$$\text{Output SNR, } SNR_{out} = \frac{GS}{G(N_a + N_s)}$$

$$\Rightarrow \frac{SNR_{in}}{SNR_{out}} = \frac{S/N_s}{GS/G(N_a+N_s)} = \frac{N_a + N_s}{N_s} = F \Rightarrow \boxed{F = \frac{SNR_{in}}{SNR_{out}}}$$

Friis equation: Cascaded Noise figure (simple derivation, assume all impedances equal)



Total output noise power

$$P_{out\,tot} = \{[(N_s + N_{a1})\, G_1 + N_{a2}]\, G_2 + N_{a3}\}\, G_3$$

Noise power due to source

$$P_{out_s} = N_s \cdot G_1 \cdot G_2 \cdot G_3$$

$$\Rightarrow F_{tot} = \frac{P_{out\,tot}}{P_{out_s}} = 1 + \frac{N_{a1}}{N_s} + \frac{N_{a2}}{N_s G_1} + \frac{N_{a3}}{N_s \cdot G_1 G_2}$$

$$\boxed{F_{tot} = 1 + (F_1 - 1) + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2}}$$

Sensitivity: Minimum signal power level that can be detected with adequate SNR.

$$F = \frac{SNR_{in}}{SNR_{out}} = \frac{P_{sig}/KTB}{SNR_{out}}$$

$$\Rightarrow \quad P_{sig_{min}} = F \cdot SNR_{out_{min}} \cdot KTB$$

$$\Rightarrow \quad P_{sig_{min}}(dBm) = NF(dB) + SNR_{min}(dB) + 10\log_{10}\frac{KTB}{1mW}$$

$$= NF(dB) + SNR_{min}(dB) - 174\,dBm/Hz + 10\log_{10} B$$

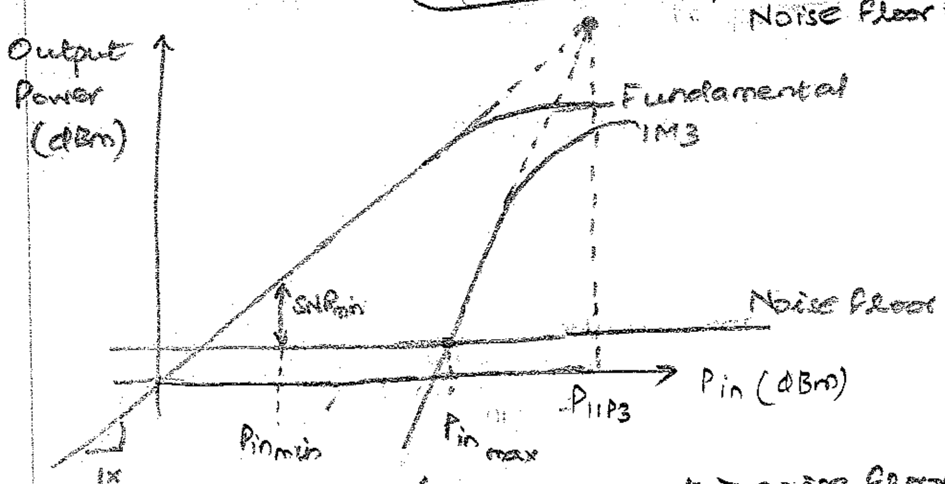
Spurious - free dynamic range:

Sensitivity = $P_{in,min}$ = minimum detectable signal with acceptable SNR

P_{inmax} = maximum signal that can be input to receiver without undesired products at the output

= input signal level at which input-referred IM3 product becomes greater than input-referred noise floor

$$\rightarrow \quad P_{in\,min} = \underbrace{-174\,dBm/Hz + 10\log_{10}B + NF}_{\text{Noise floor} = R} + SNR_{min} = R + SNR_{min}$$



(IM3 component > noise floor)

Recall $P_{IIP3} = P_{in} + \dfrac{P_{out} - P_{OIM3}}{2}$ $\quad$ $P_{out} = P_{in} + G$, $\quad P_{OIM3} = P_{IIM3} + G$ ← input-referred IM3 product

$$= P_{in} + \frac{P_{in} - P_{IIM3}}{2} = \frac{3P_{in} - P_{IIM3}}{2}$$

$$\Rightarrow \quad P_{in} = \frac{2P_{IIP3} + P_{IIM3}}{3}$$

At $P_{in} = P_{inmax}$, $P_{IIM3} = R$ (noise floor)

$$\Rightarrow P_{inmax} = \frac{2P_{IIP3} + R}{3}$$

$$\Rightarrow SFDR = P_{inmax} - P_{inmin} = \left(\frac{2P_{IIP3} + R}{3}\right) - (R + SNR_{min})$$

$$\Rightarrow \boxed{SFDR = \frac{2(P_{IIP3} - R)}{3} - SNR_{min}} \quad (dB)$$

Where Noise floor $R = \left(-174\, dBm/_{Hz} + 10\log_{10} B + NF\right)$ dBm

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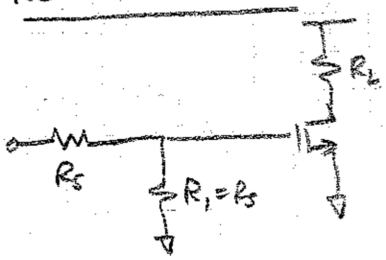
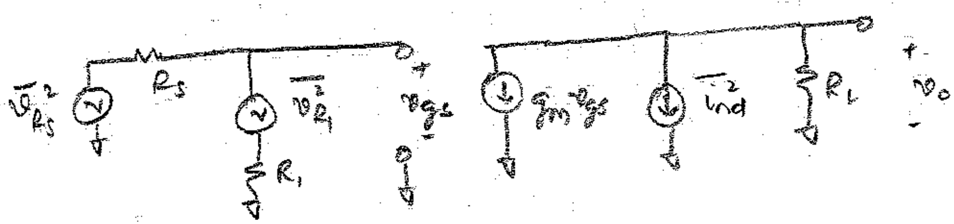
<u>Low-noise amplifiers</u> – <u>Input matching</u>

1) <u>Resistor match</u>:



- Broadband match
- attenuates signal
- R_1 adds noise
- Estimate noise factor F:
  (ignore 1/f and induced gate noise)



$$v_o = \left[\frac{R_1}{R_1 + R_S}(v_{R_S} + v_{R_1})\, g_m + i_{nd}\right] R_L$$

$$\Rightarrow \overline{v_{o\,tot}^2} = \left[\left(\frac{R_1}{R_1 + R_S}\right)^2 \left(\overline{v_{R_S}^2} + \overline{v_{R_1}^2}\right) g_m^2 + \overline{i_{nd}^2}\right] R_L^2$$

$$\overline{v_{o_S}^2} = \left(\frac{R_1}{R_1 + R_S}\right)^2 \overline{v_{R_S}^2}\, g_m^2\, R_L^2$$

$$\Rightarrow F = 1 + \frac{\overline{v_{R_1}^2}}{\overline{v_{R_S}^2}} + \left(\frac{R_1 + R_S}{R_1}\right)^2 \cdot \frac{1}{g_m^2} \cdot \frac{\overline{i_{nd}^2}}{\overline{v_{R_S}^2}}$$

$R_1 = R_s$, $\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f$ and $\overline{v_{R_s}^2} = 4kTR_s\Delta f$

$$\Rightarrow F = 1 + \frac{R_1}{R_s} + \frac{4}{g_m^2} \cdot \frac{4kT\gamma g_{d0}\Delta f}{4kTR_s\Delta f}$$

$$\Rightarrow F = 2 + \frac{4}{g_m R_s} \cdot \frac{\gamma g_{d0}}{g_m}$$

Define $\alpha = \dfrac{g_m}{g_{d0}}$

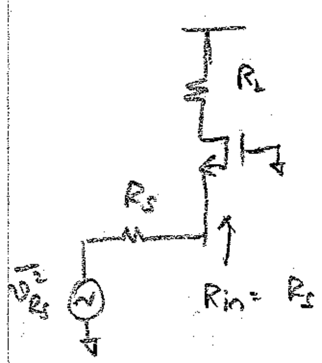
$$\Rightarrow \boxed{F = 2 + \frac{4}{g_m R_s}\frac{\gamma}{\alpha}}$$

Long-channel $\Rightarrow \alpha \simeq 1$

Typically, $\dfrac{\gamma}{\alpha} \simeq \dfrac{1}{1.8}$ due to short-channel effects

$\Rightarrow NF = 10\log_{10}(2 + \cdots) > 3dB$!

(2) Common-gate stage:

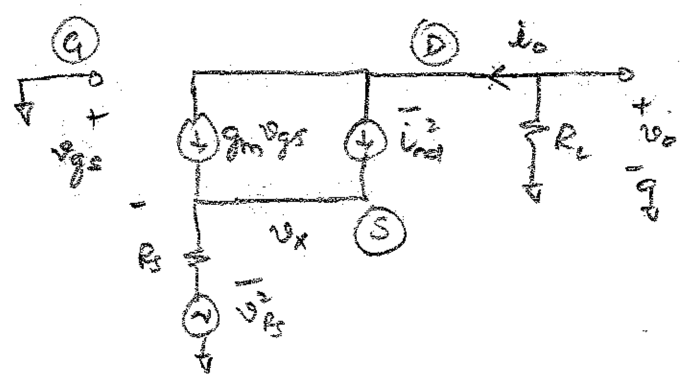


→ Broadband match

→ No input attenuation

→ no extra noise sources

→ Estimate F due to thermal noise only:



$$i_o = \frac{v_x - v_{R_s}}{R_s} = -g_m v_x + i_{nd}$$

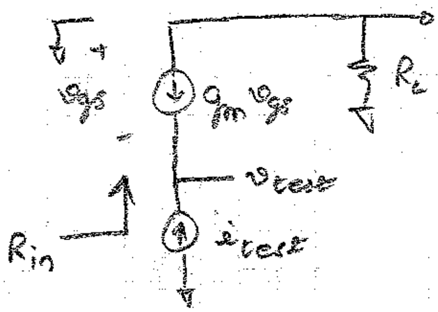
$$\Rightarrow v_x = i_o R_s + v_{R_s}$$

$$\therefore \quad i_o = -g_m(v_{R_s} + i_o R_s) + i_{nd}$$

$$\Rightarrow i_o(1 + g_m R_s) = -g_m v_{R_s} + i_{nd}$$

$$\Rightarrow i_o = -\frac{g_m}{1 + g_m R_s} v_{R_s} + \frac{i_{nd}}{1 + g_m R_s}$$

Calculate input impedance for power match:



$$i_{test} = -g_m v_{gs} = g_m v_{test}$$

$$\Rightarrow R_{in} = \frac{v_{test}}{i_{test}} = \frac{1}{g_m}$$

$\Rightarrow$ Set $R_{in} = R_s = 1/g_m$ for power match

$$i_o = -\frac{g_m}{1+g_m R_s} v_{R_s} + \frac{i_{nd}}{1+g_m R_s}$$

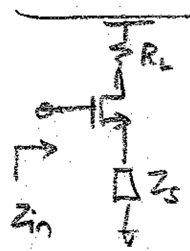
$$= -\frac{v_{R_s}}{2R_s} + \frac{i_{nd}}{2}$$

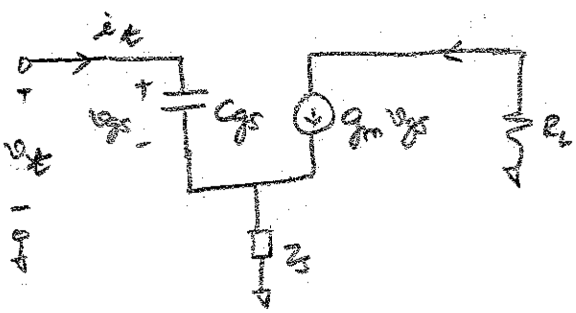
$$\Rightarrow F = \frac{\overline{i^2_{o,tot}}}{\overline{i^2_{o,s}}} = \frac{\dfrac{\overline{v^2_{R_s}}}{4R_s^2} + \dfrac{\overline{i^2_{nd}}}{4}}{\dfrac{\overline{v^2_{R_s}}}{4R_s^2}} = 1 + R_s^2 \frac{\overline{i^2_{nd}}}{\overline{v^2_{R_s}}}$$

$$\Rightarrow F = 1 + R_s^2 \cdot \frac{4kT\gamma g_{d0}\Delta f}{4kTR_s\Delta f} = 1 + \gamma g_{d0} R_s = 1 + \frac{\gamma g_{d0}}{g_m}$$

$$\Rightarrow \boxed{F = 1 + \frac{\gamma}{\alpha}}$$ ← Long-channel lower bound $= 2.2\text{dB}$

3) Obtain resistive input without resistor or C-G input:





$$v_{gs} = \frac{i_t}{sC_{gs}}, \quad \frac{v_t - v_{gs}}{Z_s} = i_t + g_m v_{gs}$$

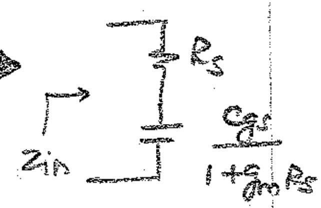
$$\Rightarrow \quad v_t = i_t Z_s + (1 + g_m Z_s) v_{gs}$$

$$= i_t Z_s + (1 + g_m Z_s) \cdot \frac{i_t}{sC_{gs}}$$

$$\Rightarrow \quad \boxed{Z_{in} = \frac{v_t}{i_t} = Z_s + \frac{1}{sC_{gs}} + \frac{g_m Z_s}{sC_{gs}}}$$

(a) $Z_s = R_s$ (resistor degeneration)

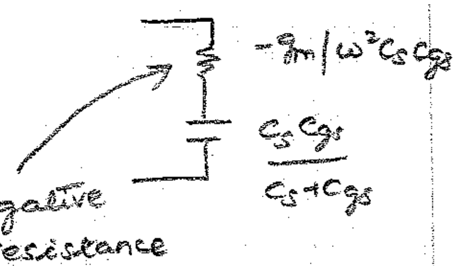
$$Z_{in} = R_s + \frac{g_m R_s}{sC_{gs}} + \frac{1}{sC_{gs}} = R_s + \frac{1 + g_m R_s}{sC_{gs}} \Rightarrow$$



(b) Capacitive degeneration $Z_s = \frac{1}{sC_s}$

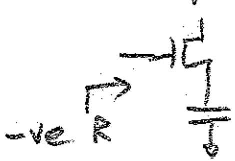
$$\Rightarrow \quad Z_{in} = \frac{1}{sC_s} + \frac{1}{sC_{gs}} + \frac{g_m}{sC_{gs}} \cdot \frac{1}{sC_s}$$

$$= \frac{1}{sC_s}\left(1 + \frac{C_s}{C_{gs}}\right) - \frac{g_m}{\omega^2 C_s C_{gs}} \quad \Rightarrow$$



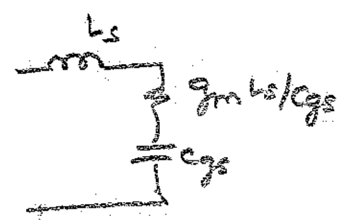
→ Used in oscillators

→ Beware of parasitic cap. in amplifiers → can oscillate



(c) <u>Inductive degeneration</u>: $Z_s = sL_s$

$$\Rightarrow \quad Z_{in} = sL_s + \frac{1}{sC_{gs}} + \frac{g_m L_s}{C_{gs}} \longrightarrow$$



→ we have created a real part in Z_{in} <u>without</u> a resistor

⇒ Low-noise power match.

Set $\underbrace{\frac{g_m L_s}{C_{gs}}}_{\omega_T L_s} \approx 50\Omega$ (R_{source})