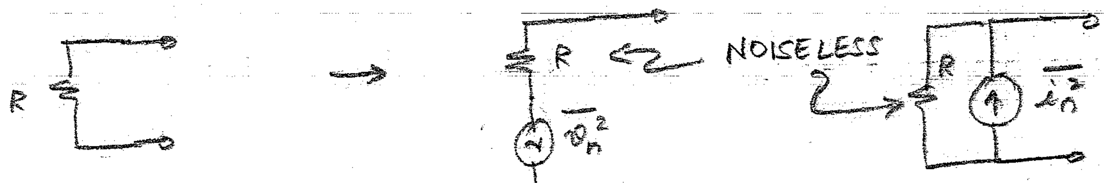
(1) Thermal Noise of Resistors:



Noisy resistor

Thevenin equivalent

Norton equiv.

$v_n(t)$ = random process that characterizes Thevenin noise voltage

$i_n(t)$ = " " " " Norton noise current

Single-sided power spectral densities:

- Mean-square voltage density: $\frac{\overline{v_n^2}}{\Delta f} = 4kTR \quad (V^2/Hz)$
- Mean-square current density: $\frac{\overline{i_n^2}}{\Delta f} = \frac{4kT}{R} \quad (A^2/Hz)$

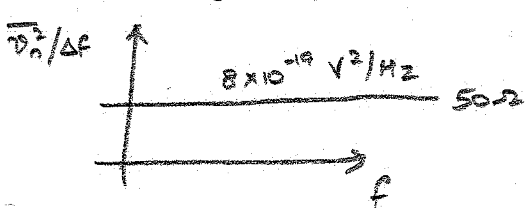
where Δf = measurement BW
k = Boltzmann constant = 1.381×10^{-23} J/K
T = Absolute temperature in K.

Root-mean square densities:

- RMS voltage density: $\frac{\overline{v_n}}{\sqrt{\Delta f}} = \sqrt{4kTR} \quad (V/\sqrt{Hz})$
- RMS current density: $\frac{\overline{i_n}}{\sqrt{\Delta f}} = \sqrt{\frac{4kT}{R}} \quad (A/\sqrt{Hz})$

For $R = 50\Omega$: $\frac{\overline{v_n}}{\Delta f} = 0.9\,nV/\sqrt{Hz}$

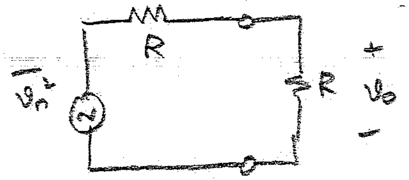
- Note that all mean-square spectral densities are white (i.e., flat over frequency).



- spot noise = mean-square noise density integrated over a 1Hz bandwidth @ frequency of interest

- White Noise ⇒ spot noise independent of frequency.

Available noise power: = maximum noise power that can be delivered to a load.



$\Rightarrow$ conjugate match

Power delivered to load resistor in a 1-Hz BW $\Big\} = \dfrac{(\overline{v_n}/2)^2}{R} = \dfrac{\overline{v_n^2}}{4R}$ (W/Hz)

$$\Rightarrow \boxed{P_{avn} = \frac{4kTR}{4R} \cdot B = kTB \text{ Watts.}}$$

— Available noise power independent of resistance

— proportional to absolute temperature

— proportional to bandwidth.

$\Rightarrow$ For low noise, keep minimum BW and temp. possible.

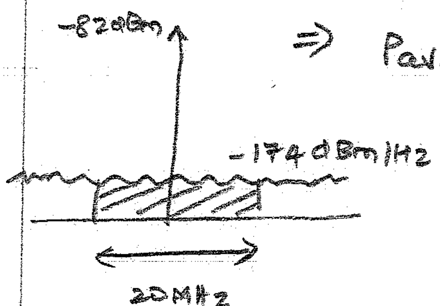
Available noise power per unit BW = kT W/Hz

$$P_{avn}/_{Hz} = 4.143 \times 10^{-21} \text{ W/Hz}$$

$$= -174 \text{ dBm/Hz}$$

Worst case

<u>802.11a</u>: Received signal power = −82 dBm @ 20 MHz BW



$\Rightarrow P_{avn} = 4.143 \times 10^{-21} \dfrac{\text{W}}{\text{Hz}} \times 20\text{MHz} = 8.29 \times 10^{-14}$ W

or $P_{avn} = -100$ dBm

$\Rightarrow$ Max. possible SNR = −82 dBm − (−100 dBm)

= 18 dBm

- This is the best we can do with a 20 MHz BW at room temperature $\Rightarrow$ ideal receiver
- A real receiver will add noise of its own.

• With $P_{avn} = -100$ dBm, the <u>rms</u> noise voltage across a 50Ω resistor is $\overline{v_n} = 2.23\mu$V.

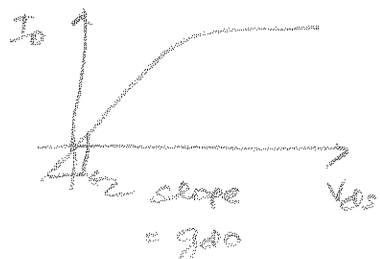
If we applied a sine-wave signal with this amplitude to an oscilloscope, we would <u>barely</u> be able to see it $\Rightarrow$ called <u>MINIMUM DETECTABLE SIGNAL</u>

(2) Noise in MOSFETs :

A) Thermal noise

Case (i) MOSFET as resistor (triode region)

$$I_d = K_n'\left(\frac{W}{L}\right)\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right] \longrightarrow$$



$$g_d = \frac{\partial I_D}{\partial V_{DS}} = K_n'\left(\frac{W}{L}\right)(V_{GS}-V_T-V_{DS})$$

$$\Rightarrow \boxed{g_{d0} = K_n'\left(\frac{W}{L}\right)(V_{GS}-V_T)}$$

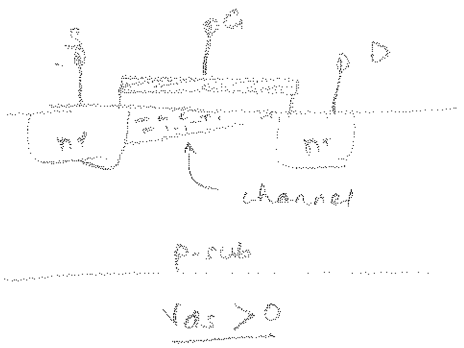
Since MOSFET behaves like a resistor in the triode region it exhibits thermal noise due to Brownian motion of carriers.

$\Rightarrow$

$$\boxed{\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f}$$

γ = empirical parameter
= 1 in this case

Case (ii) Saturated long-channel MOSFET



Cause : When a positive V_{DS} is applied, charge carriers get accelerated by the E-field through the resistive channel.

$\Rightarrow$

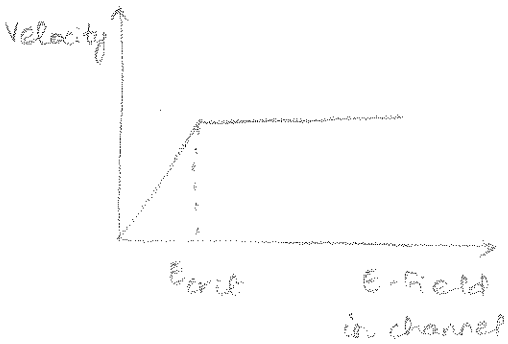
$$\boxed{\begin{array}{l}\overline{i_{nd}^2} = 4kT\gamma g_{d0}\Delta f \\ \gamma = 2/3\end{array}}$$

Recall $I_D = \frac{K_n'}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$

$\Rightarrow$ Transconductance $g_m = \frac{\partial i_D}{\partial V_{GS}} = K_n'\left(\frac{W}{L}\right)(V_{GS}-V_T) = g_{d0}$

$$\Rightarrow \boxed{\overline{i_{nd}^2} = 4kT\cdot\frac{2}{3}g_m\Delta f} \longleftarrow \text{(Common form)}$$

Case (iii) Saturated short-channel MOSFET



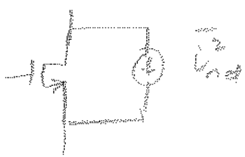
- Charge carriers get accelerated from source to drain by the E-field
- Velocity of the carriers at drain end $v = \begin{cases} \mu E & E < E_{crit} \\ v_{sat} & E > E_{crit} \end{cases}$

→ velocity saturation

– (Excess noise due to HOT CARRIERS)

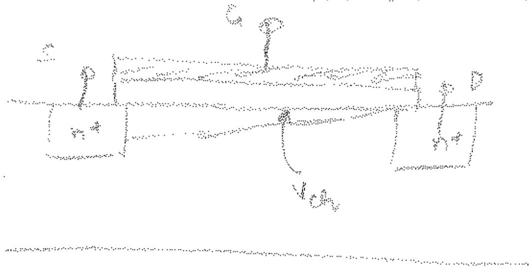
– As E-field is increased beyond E_{crit}, the carrier velocity, and hence the kinetic energy saturates.

⇒ Work done by E-field is dissipated as heat.

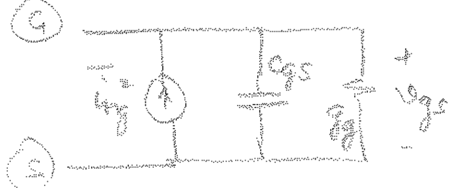


$$\boxed{\begin{gathered}\overline{i_{nd}^2} = 4kT\gamma g_{d0}\,\Delta f \quad A^2/Hz \\ \gamma \simeq 2-3\end{gathered}}$$

(B) Gate-induced noise in MOSFETs



- Because of thermal noise in the drain current, the voltage in the channel fluctuates
- Consequently the gate-channel voltage also fluctuates. Since the gate is capacitive, a <u>noise displacement current</u> flows through the gate.



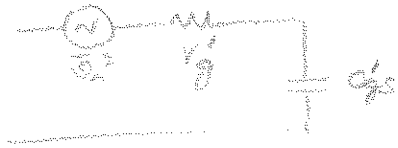
(Van der Ziel Model)

$$\boxed{\begin{gathered}\overline{i_{ng}^2} = 4kT\delta g_g\,\Delta f \\ \delta = 2\gamma \quad \text{for long-channel MOSFETs} \\ g_g = \frac{\omega^2 C_{gs}^2}{5 g_{d0}}\end{gathered}}$$

⇒ Gate-induced noise can be important at high freq. Not an issue at low frequencies

Induced gate noise is not white. To convert to an equivalent white noise model, perform series-parallel conversion (narrowband only!)

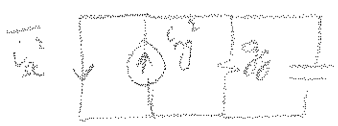
$\Rightarrow$



$$r_g' = \frac{1}{g_g(Q^2+1)} \approx \frac{1}{g_g Q^2} = \frac{1}{g_g\left(\frac{\omega C_{gs}}{g_g}\right)^2} \approx \frac{1}{5g_{d0}}$$

$$\boxed{r_g' = \frac{1}{5g_{d0}}} \qquad \boxed{C_{gs}' \approx C_{gs}}$$

To find $\overline{v_n^2}$, equate short-circuit currents:



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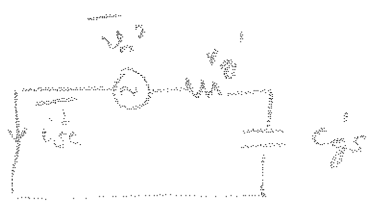


$$i_{sc} = i_{ng} = \frac{v_n}{r_g' + sC_{gs}'} = \frac{v_n}{r_g'}\,\frac{1}{1+\frac{sC_{gs}'}{r_g'}}$$

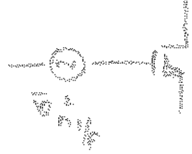
$$\Rightarrow \overline{i_{sc}^2} = \overline{i_g^2} = \frac{\overline{v_n^2}}{r_g'^2}\cdot\frac{1}{1+\frac{\omega^2 C_{gs}'^2}{r_g'^2}} \approx \frac{\overline{v_n^2}}{\omega^2 C_{gs}^2}$$

$$\Rightarrow \boxed{\overline{v_n^2} \approx 4kT\delta r_g' \Delta f}$$

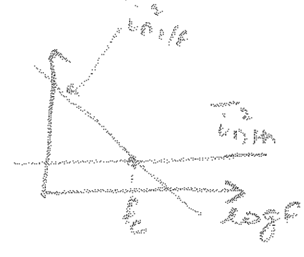
(c) Flicker noise:

$$\overline{i_{n1/f}^2} = \frac{K_{1f}}{f}\cdot\frac{g_m^2}{WLC_{ox}} \quad A^2/Hz$$

$\Rightarrow$



$$\overline{v_{n1/f}^2} = \frac{K_{1f}}{f}\,\frac{1}{WLC_{ox}} \quad V^2/Hz$$



$$f_c \approx \frac{K_{1f}}{4kT} f_T$$

f_T: g_m/C_{gs} = transition freq. of process $\Rightarrow$ faster process $\Rightarrow$ worse 1/f noise for equal area