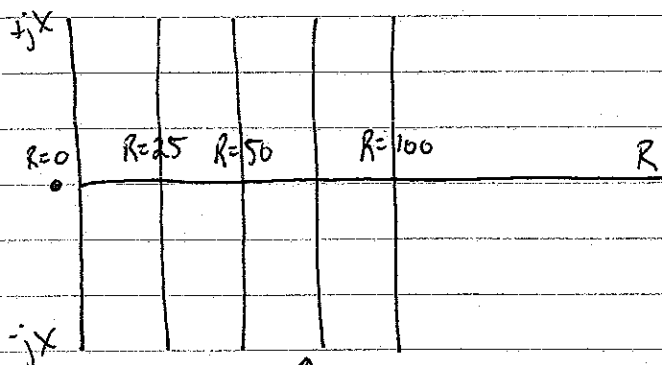
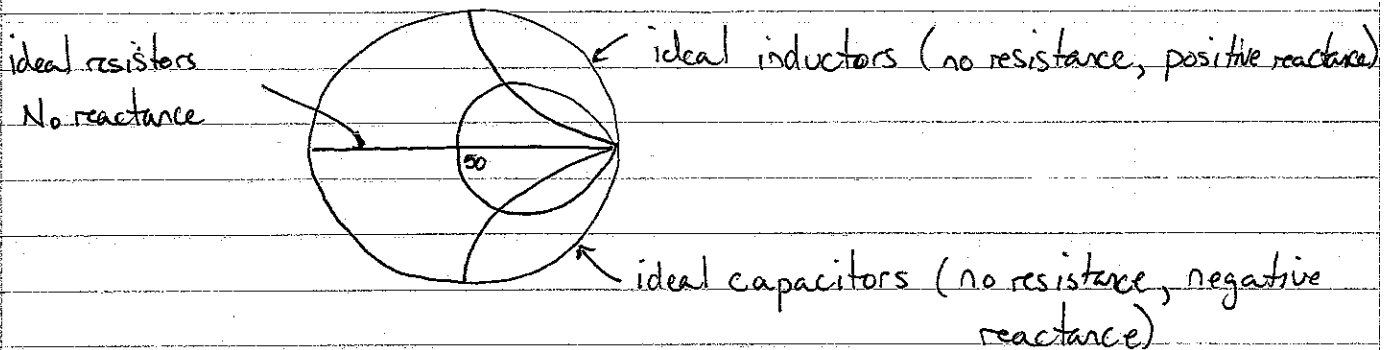


The Impedance chart was developed to map rectangular impedance to polar reflection coefficient

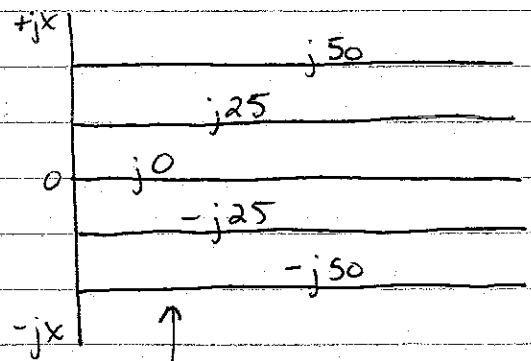
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$Z_0$  is the arbitrary characteristic impedance of the system (typically  $Z_0 = 50\Omega$ )

Impedances can vary from 0 to  $\infty$ , but Reflection coefficient can only vary from 0 to 1



varying reactance  
constant resistance



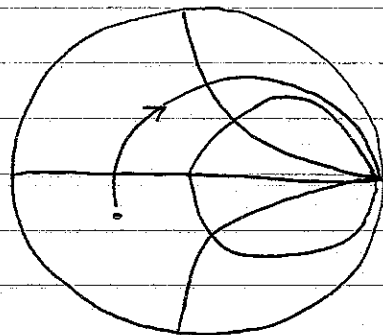
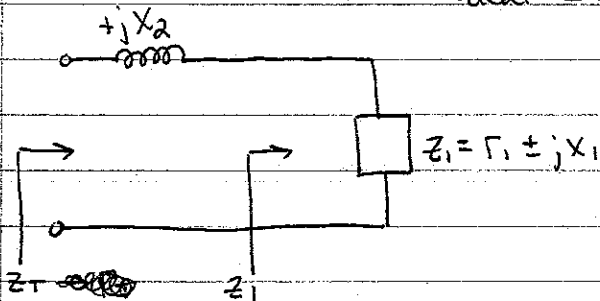
varying resistance  
constant reactance

$$\Gamma = \frac{z - z_0}{z + z_0} = \frac{\frac{z - z_0}{z_0}}{\frac{z + z_0}{z_0}} = \frac{z - 1}{z + 1} \quad z = \frac{z}{z_0}$$

This makes it possible to use the chart with any impedance.

How can the chart be used for impedance matching

series  
Ideal Inductor



$$z_T = z_1 + jx_2 = (r_1 + jx_1) + jx_2$$

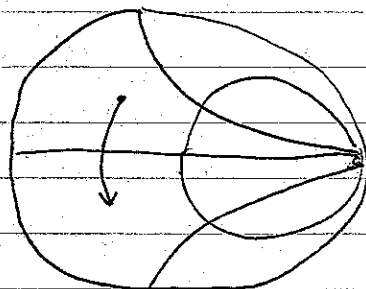
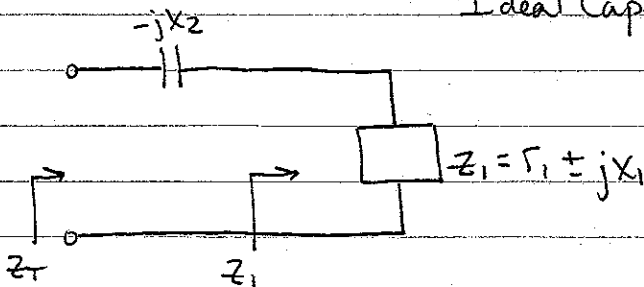
$$z_T = r_1 + j(\pm x_1 + x_2)$$

Addition of an ideal inductor causes <sup>upward</sup> movement on a line of constant resistance towards infinity. As  $L \rightarrow \infty$   $z \rightarrow \infty$

$$\omega L = X_L$$

$$L = \frac{X_L}{\omega} = \frac{x_1 \cdot z_0}{\omega}$$

Series  
Ideal Capacitor



$$z_T = z_1 - jx_2 = r_1 \pm jx_1 - jx_2$$

$$z_T = r_1 + j(\pm x_1 - x_2)$$

Addition of an ideal capacitor causes <sup>downward</sup> movement on a line of constant resistance towards infinity. As  $C \rightarrow 0$   $z \rightarrow \infty$

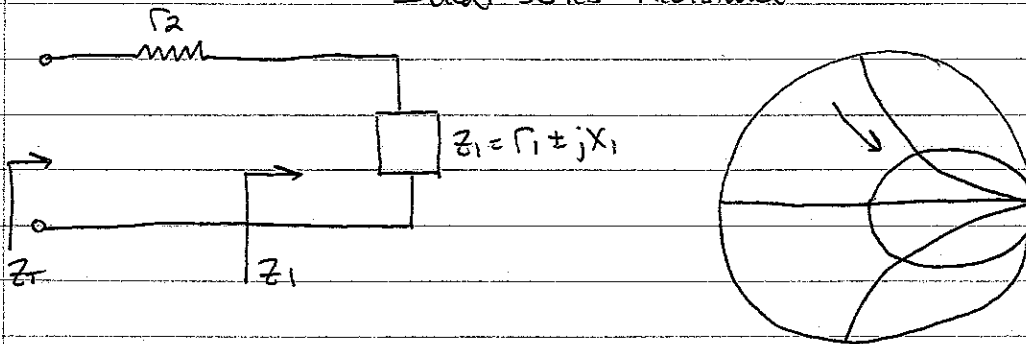
$$\text{As } C \rightarrow 0 \quad z \rightarrow \infty$$

$$\frac{1}{\omega C} = X_c$$

$$C = \frac{1}{\omega X_c} = \frac{1}{\omega X_c \cdot Z_0}$$


---

### Ideal Series Resistance



$$Z_T = R_2 + R_1 \pm jX_1$$

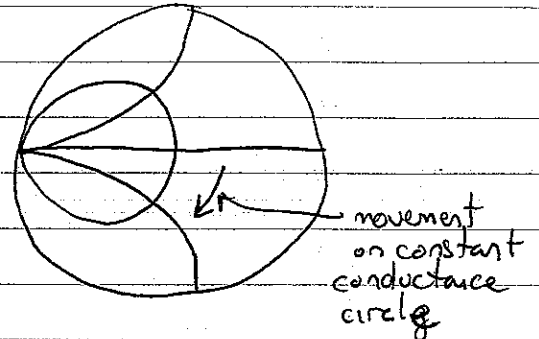
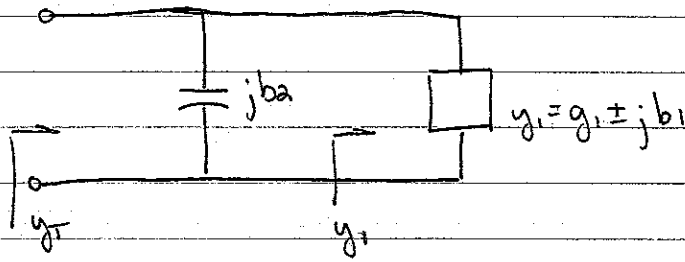
$$Z_T = (R_2 + R_1) \pm jX_1$$

addition of a series resistance  
causes movement on a line of  
constant reactance towards infinity  
As  $R \rightarrow \infty \quad Z \rightarrow \infty$

Admittance Chart development is similar to  
Impedance chart. Chart is rotated by  $180^\circ$  and then  
overlayed on Impedance chart, to form "Immitance" chart

$$\Gamma = \frac{Y - Y_0}{Y + Y_0} = \frac{\frac{Y - Y_0}{Y_0}}{\frac{Y + Y_0}{Y_0}} = \frac{y - 1}{y + 1} \quad y = \frac{Y}{Y_0}$$

## Ideal Parallel Capacitors



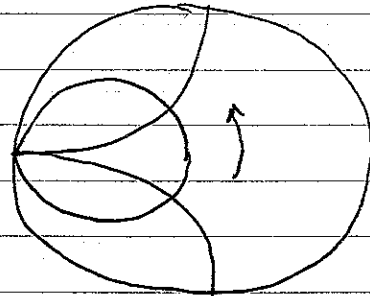
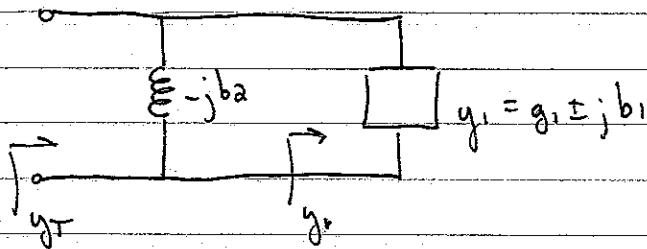
$$y_T = y_1 + j b_1 = (g_1 \pm j b_1) + j b_2$$

$$y_T = g_1 + j(\pm b_1 + b_2)$$

$$\omega C = B \quad C = \frac{B}{\omega}$$

Addition of parallel capacitor causes downward movement on chart toward  $y = \infty$  ( $C \rightarrow \infty \quad y \rightarrow \infty$ )

## Ideal Parallel Inductors

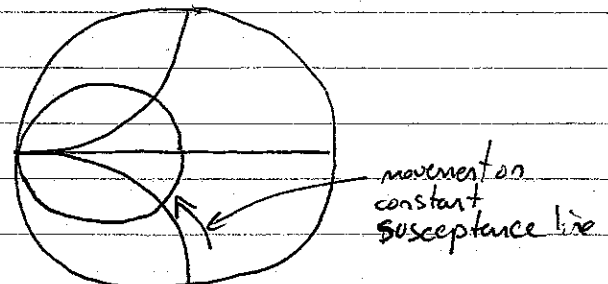
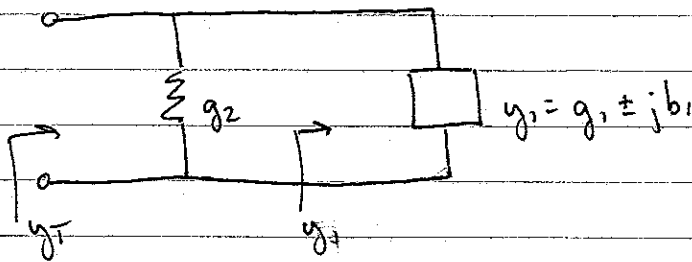


$$y_T = y_1 - j b_2 = g_1 \pm j b_1 - j b_2$$

$$y_T = g_1 + j(\pm b_1 - b_2)$$

Addition of parallel inductance causes upward movement on chart toward  $y = \infty$  ( $L \rightarrow 0 \quad y \rightarrow \infty$ )

## Ideal Parallel Resistors



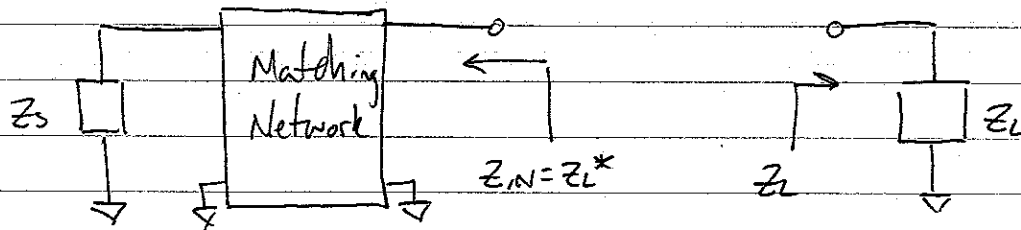
$$y_T = g_1 \pm j b_1 + g_2$$

$$y_T = (g_1 + g_2) \pm j b_1$$

Addition of parallel resistance causes ~~upward~~ movement on chart toward  $y = 0$

$$R \rightarrow \infty \quad y \rightarrow 0$$

Matching objective: transform an impedance from a source to the complex conjugate of the load.



Why complex conjugate: For maximum power transfer, the real parts of impedance must be equal, and ~~the~~ the voltage must be in phase with current.

What does this mean?

Match always occurs from an impedance to the complex conjugate of another impedance

$$Z_s \rightarrow Z_L^* \quad \text{or} \quad Z_L \rightarrow Z_s^*$$

• Constant  $Q$  ~~circle~~ curves

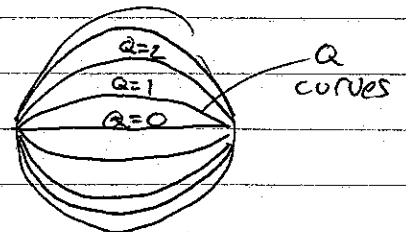
$$Q = \frac{X}{R}$$

a  $Q$  curve is defined for a series of Impedance points where ratio between  $X$  and  $R$  is steady.

i.e.  $Q = 5$  when  $X = 5R$

$Q = 4$  when  $X = 4R$

$Q = 0$  when  $X = 0$

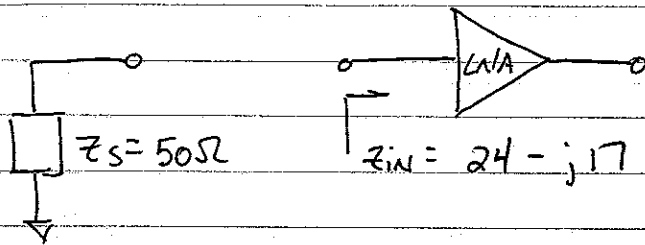


## Matching Procedure:

- 1.) Normalize source and load impedance to  $Z_0$
- 2.) Plot impedance of starting point and complex conjugate of ending point (i.e.  $Z_s$  and  $Z_L^*$ )
- 3.) Choose topology
- 4.) Follow rules for adding passive components to the network. (i.e. shunt components move on constant conduct. circle.)
- 5.) Calculate component values.

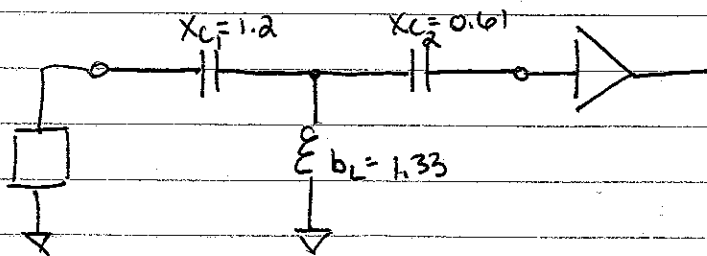
### Example

Smith chart on next page: Match @ 2.4 GHz



Use high pass topology:

plot  $Z_s = 1$       plot  $Z_{in}^* = 0.48 + j0.34$



$$C_1 = 1.1 \text{ pF}$$

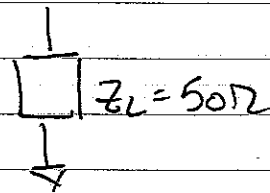
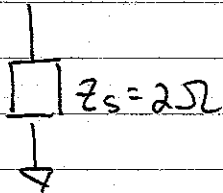
$$L = 2.49 \text{ nH}$$

$$C_2 = 2.17 \text{ pF}$$

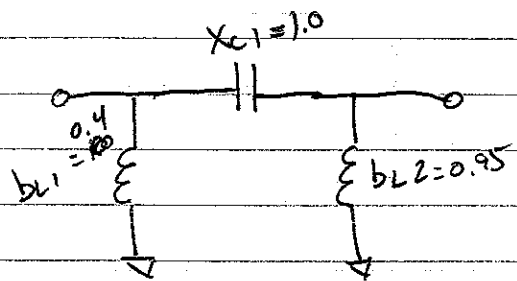
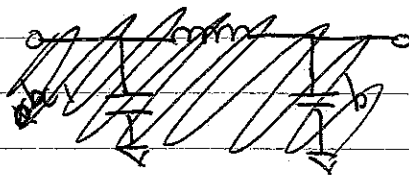
$$C_p F = \frac{3.183}{f_{GHz} \cdot X_c} = \frac{3.183 \text{ bc}}{f_{GHz}}$$

$$L_n H = \frac{7.96 \cdot X_L}{f_{GHz}} = \frac{7.96}{f_{GHz} \cdot Z_0}$$

# Class HW 1 examples



High pass  $\pi$  model match:



Normalize to  $10 \Omega$

$$Z_s = 0.25 \Omega$$

$$Z_L^* = 5 \Omega$$

$$L_{nH} = \frac{159}{19 \cdot 0.95} = 0.881 \text{ nH}$$

$$C_{pF} = 8.44 \text{ pF}$$

$$b_L = \frac{.159 \cdot Z_0}{f_{GHz} \cdot L_{nH}}$$

$$X_c = \frac{159 / Z_0}{f_{GHz} \cdot C_{pF}}$$

$$C_{pF} = \frac{15.9}{19 \cdot 1.0} = 8.44 \text{ pF}$$

NAME	TITLE <b>www.besserassociates.com</b>	DWG. NO.
SMITH® CHART FORM ZY-01-A	ANALOG INSTRUMENTS COMPANY, NEW PROVIDENCE, N.J. 07974	DATE

NORMALIZED IMPEDANCE AND ADMITTANCE COORDINATES

