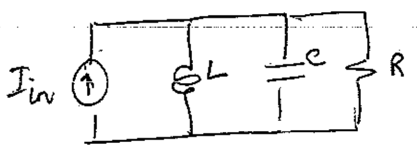
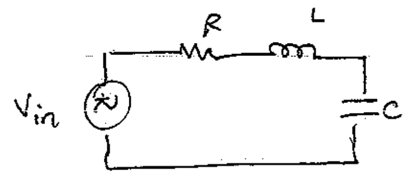
Parallel RLC tank



Series RLC tank



Resonant Frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$

characteristic Impedance $Z_0 = \sqrt{\dfrac{L}{C}}$

$$Q = \omega \frac{\text{Energy stored}}{\text{Power Dissipated}}$$

Parallel:

$$Q = \frac{R}{\sqrt{L/C}} = \frac{R}{\omega_0 L} = \omega_0 RC = \frac{\omega_0}{\Delta\omega_{-3dB}}$$

At $\underline{\omega = \omega_0}$ $|Z_c| = |Z_L| = \sqrt{\dfrac{L}{C}}$

$|I_c| = |I_L| = Q|I_{in}|$

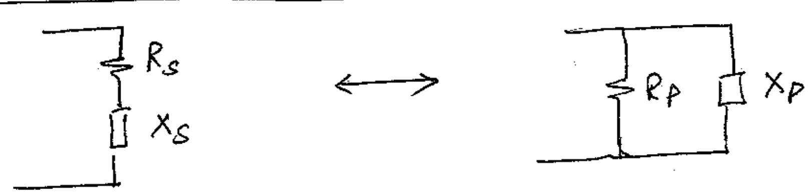
Series:

$$Q = \frac{\sqrt{L/C}}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC} = \frac{\omega_0}{\Delta\omega_{-3dB}}$$

$|Z_c| = |Z_L| = \sqrt{\dfrac{L}{C}}$

$|V_c| = |V_L| = Q|V_{in}|$

Series – Parallel conversions (Narrowband only)

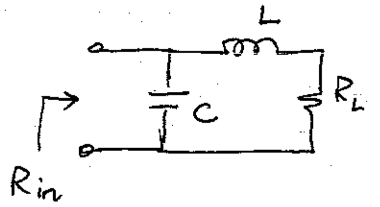


$$Q_s = Q_p = Q$$

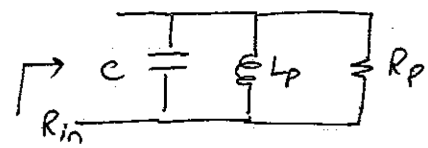
$$R_p = R_s(1 + Q^2)$$

$$X_p = X_s\left(\frac{Q^2+1}{Q^2}\right)$$

Lowpass L-match (Upward transformation)



$\longrightarrow$



At $\underline{\omega = \omega_0}$

1) $R_p = R_{in} = R_L(Q^2+1) \Rightarrow Q = \sqrt{\dfrac{R_{in}}{R_L} - 1}$

2) $Q_p = Q = \dfrac{R_p}{\omega_0 L_p} \Rightarrow L_p = \dfrac{R_{in}}{Q\omega_0}$

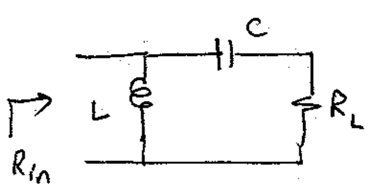
3) $L_p = \left(\dfrac{Q^2+1}{Q^2}\right) L \Rightarrow L = \left(\dfrac{Q^2}{Q^2+1}\right) L_p$

4) $\omega_0 = \dfrac{1}{\sqrt{L_p C}} \Rightarrow C = \dfrac{1}{\omega_0^2 L_p}$

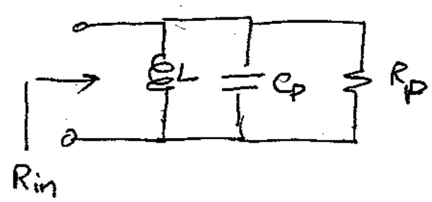
MNEMONIC

Add "something" in series with load
$\Rightarrow$ upward transform.

Highpass upward transformation :



$\Rightarrow$



At $\underline{\omega = \omega_0}$

1) $R_p = R_L(Q^2+1) \Rightarrow Q = \sqrt{\dfrac{R_p}{R_L} - 1}$

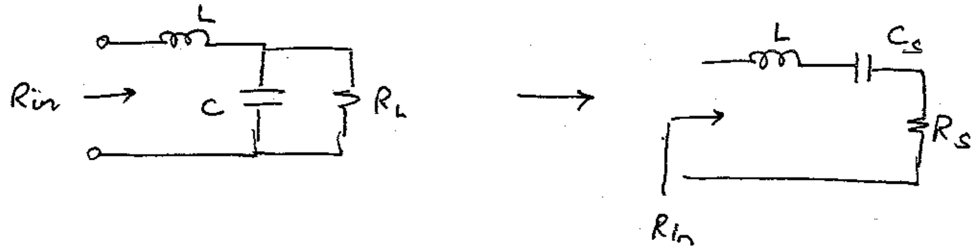
2) $Q = \omega_0 R_p C_p \Rightarrow C_p = \dfrac{Q}{\omega_0 R_p}$

3) $C_p = \dfrac{Q^2}{Q^2+1} C \Rightarrow C = \dfrac{Q^2+1}{Q^2} C_p$

4) $\omega_0 = \dfrac{1}{\sqrt{L C_p}} \Rightarrow L = \dfrac{1}{\omega_0^2 C_p}$

L-match Downward transformation

(a) Lowpass:



At $\omega = \omega_0$

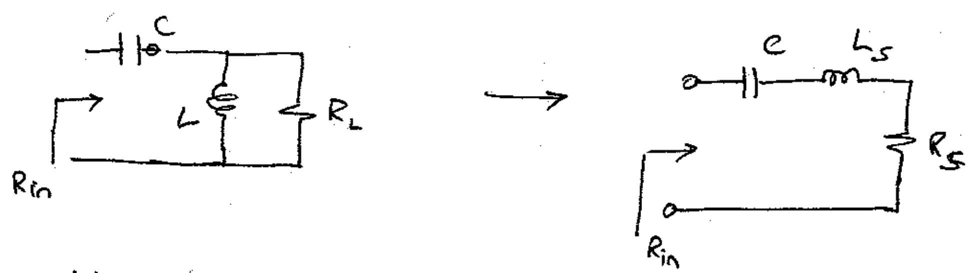
1) $R_s = R_{in} = \dfrac{R_L}{Q^2+1} \Rightarrow Q = \sqrt{\dfrac{R_L}{R_{in}} - 1}$

2) $Q = \dfrac{1}{\omega_0 R_s C_s} \Rightarrow C_s = \dfrac{1}{\omega_0 R_{in} Q}$

3) $C_s = \left(\dfrac{Q^2+1}{Q^2}\right) C \Rightarrow C = \left(\dfrac{Q^2}{Q^2+1}\right) C_s$

4) $\omega_0 = \dfrac{1}{\sqrt{LC_s}} \Rightarrow L = \dfrac{1}{\omega_0^2 C_s}$

(b) Highpass:



At $\omega = \omega_0$

1) $R_s = R_{in} = \dfrac{R_L}{Q^2+1} \Rightarrow Q = \sqrt{\dfrac{R_L}{R_{in}} - 1}$

2) $Q = \dfrac{\omega_0 L_s}{R_s} \Rightarrow L_s = \dfrac{Q R_s}{\omega_0}$

3) $L_s = \dfrac{Q^2}{Q^2+1} L \Rightarrow L = \left(\dfrac{Q^2+1}{Q^2}\right) L_s$

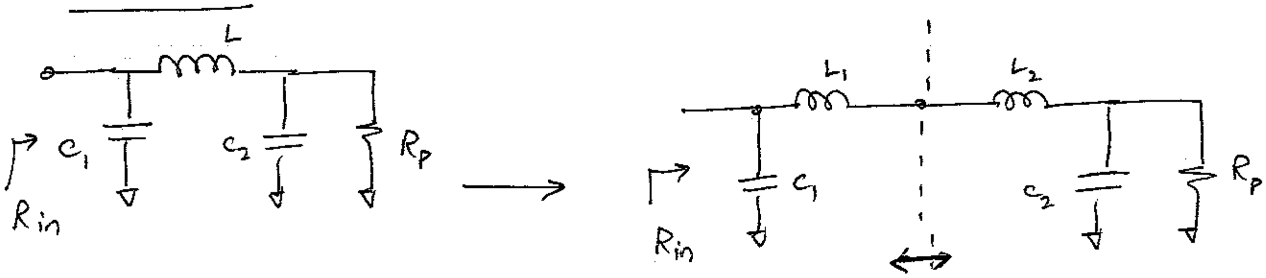
4) $\omega_0 = \dfrac{1}{\sqrt{L_s C}} \Rightarrow C = \dfrac{1}{\omega_0^2 L_s}$

Limitations of L-matching networks

- Only 2 degrees of freedom L & C
- We know R_{in}, R_L $\Rightarrow$ Q is fixed. (usually the case)
- Among ω_0, $\frac{R_{in}}{R_L}$ and Q, we can specify only 2 of 3 parameters

To add a 3rd degree of freedom, add one extra element
- π-match
- T-match

π-Match :



R_I @ ω_0

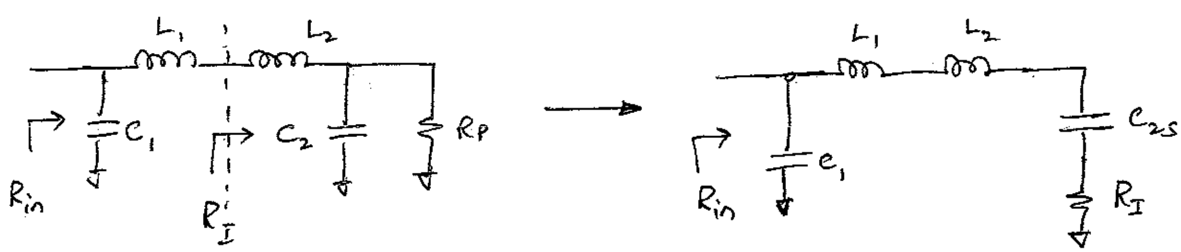
Decompose into to L-match networks

upward transformation

downward transform.

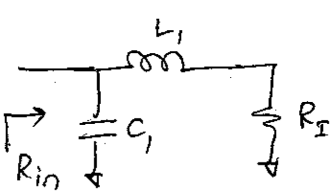
R_I = image/intermediate resistance.

⇒ For the right-hand section:



$$Q_{right} = \sqrt{\frac{R_P}{R_I} - 1} = \frac{\omega_0 L_2}{R_I}$$

⇒ Left-hand section:



$$\Rightarrow \quad Q = \frac{\omega_0 L_1}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1}$$

Overall $Q = Q_{left} + Q_{right}$

$$\Rightarrow Q = \frac{\omega_0 L_1}{R_I} + \frac{\omega_0 L_2}{R_I} = \sqrt{\frac{R_{in}}{R_I} - 1} + \sqrt{\frac{R_P}{R_I} - 1}$$

__Design procedure__ — Given R_P, R_{in}, Q and ω_0

- Use the above equation for Q and solve for R_I (iterative)
- Design the right-hand section as a downward L-match from R_P to R_I
- Design the left-hand section as an upward match from R_I to R_{in}

__Example__: Match $R_p = 200\Omega$ to $R_{in} = 50\Omega$ @ $\omega_0 = 5GHz$ with $Q = 10$

$$Q = 10 = \sqrt{\frac{50}{R_I} - 1} + \sqrt{\frac{200}{R_I} - 1}$$

Iterate: Start with $R_I = 10\Omega \Rightarrow Q = 6.35\ (\neq 10)$

$R_I = 5\Omega \Rightarrow Q = 9.24$

$\vdots$

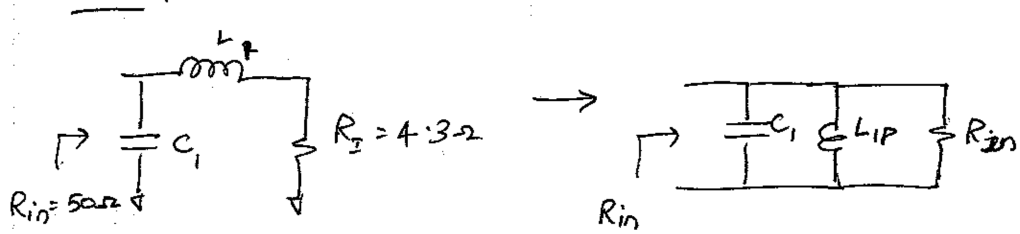
$R_I = 4.3\Omega$.

$$Q_{right} = \sqrt{\frac{R_P}{R_I} - 1} = 6.75$$

$$Q_{left} = \sqrt{\frac{R_{in}}{R_I} - 1} = 3.26$$

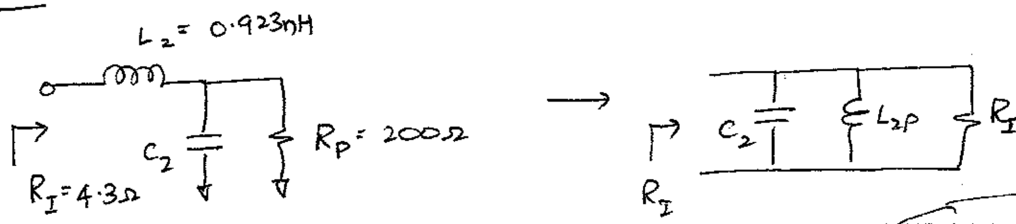
$$\left.\begin{aligned} Q_{right} &= \frac{\omega_0 L_2}{R_I} \Rightarrow L_2 = 0.923nH \\ Q_{left} &= \frac{\omega_0 L_1}{R_I} \Rightarrow L_1 = 0.446nH \end{aligned}\right\} \boxed{L = 1.37nH}$$

__LHS__:



$$L_{1P} = \left(\frac{Q_{left}^2+1}{Q_{left}^2}\right) L_1 = 0.488\,nH \Rightarrow \boxed{C_1 = \frac{1}{\omega_0^2 L_{1P}} = 2.07\,pF}$$

RHS:

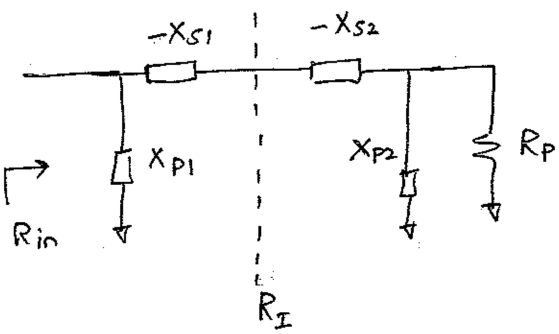


$$L_{2P} = L_2\left(\frac{Q_{right}^2+1}{Q_{right}^2}\right) = 0.943\,nH$$

$$\Rightarrow \boxed{C_2 = \frac{1}{\omega_0^2 L_{2P}} = 1.074\,pF}$$

Advantage of lowpass pi-network
→ shunt caps. to ground
⇒ absorb parasitics

Generalized π-matching network:



General idea:

- Once we use $Q = \sqrt{\frac{R_P}{R_I}-1} + \sqrt{\frac{R_{in}}{R_I}-1}$ to find R_I, the problem boils down to the design of two separate L-match networks.
- For each L-match, we only require that the two reactive elements be of opposite sign.
- Four combinations:

RHS	LHS
LP	LP
LP	HP
HP	LP
HP	HP

A practical note: In many amateur radio designer resources, you will find a different INACCURATE procedure for Π-match design.

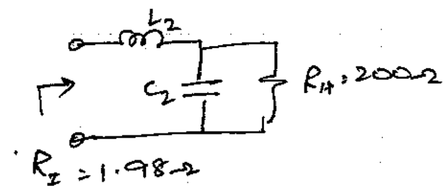
Here, Q is defined as $Q = \sqrt{\frac{R_H}{R_I} - 1}$ where $R_H = \max(R_{in}, R_P)$

Using this definition,
For our previous example where $R_P = 200\Omega$ & $R_{in} = 50\Omega$,

$$Q = \sqrt{\frac{200}{R_I} - 1} = 10 \Rightarrow R_I = 1.98\Omega.$$

Since the right-hand section contains R_H (the larger value),

$$Q_{right} = Q = \omega_0 R_P C_2$$

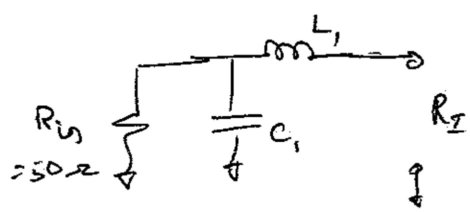


$$\Rightarrow \boxed{C_2 = 1.59pF}$$

and $L_2 = \dfrac{1}{\omega_0^2 \left(\frac{Q_{right}^2 + 1}{Q_{right}^2}\right) C_2}$

$$\boxed{L_2 = 0.63nH.}$$

For the left-hand section, calculate Q as you would in an upward L-match:

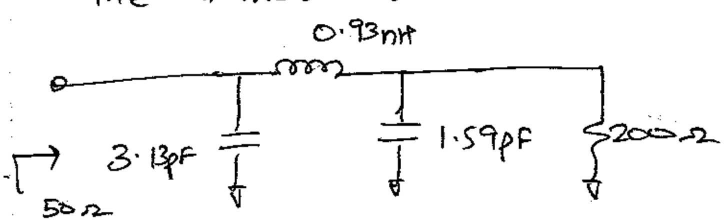


$$Q_{left} = \sqrt{\frac{R_{in}}{R_I} - 1} = 4.92$$

$$\Rightarrow Q_{left} = \frac{\omega_0 L_1}{R_I}$$

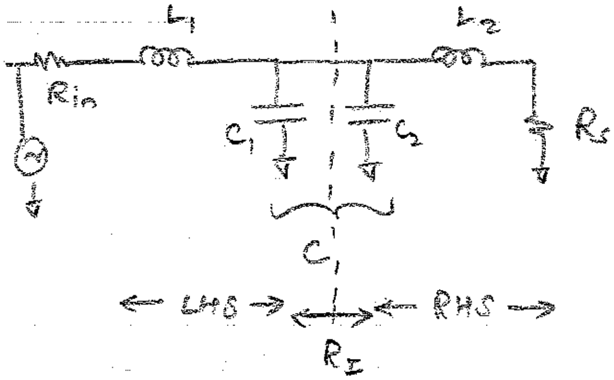
$$\Rightarrow \boxed{L_1 = 0.31nH}$$
and $\boxed{C_1 = 3.13pF}$

The Π-match is then:



Notice how different these values are from the ones that we calculated previously.

T-match

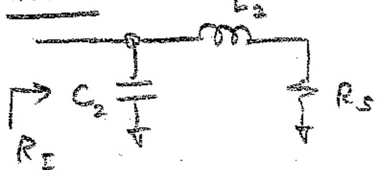


- Usually, we are given R_s, R_{in}, ω_0 and Q
- Split the center cap. C into C_1 and C_2, so that we have two L-match sections

- RHS → transforms R_s <u>UP</u> to R_I (the image resistance)

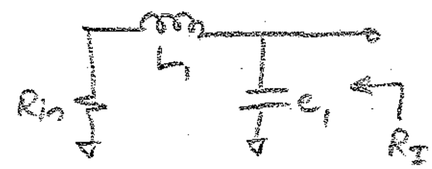
  LHS → transforms R_I <u>DOWN</u> to R_{in}
- What is R_I ?

<u>RHS:</u>



$$Q_{right} = \omega_0 R_I C_2 = \sqrt{\frac{R_I}{R_s} - 1}$$

<u>LHS:</u>



$$Q_{left} = \omega_0 R_I C_1 = \sqrt{\frac{R_I}{R_{in}} - 1}$$

Overall $Q = Q_{left} + Q_{right}$ (remember Q is specified to us)

- Find R_I by solving

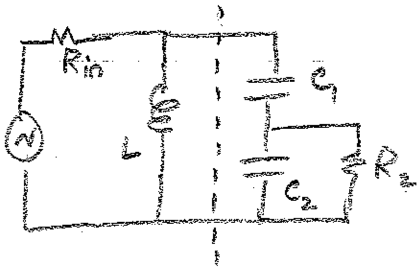
$$Q = \sqrt{\frac{R_I}{R_s} - 1} + \sqrt{\frac{R_I}{R_{in}} - 1}$$

(usually solved iteratively)

- Use $Q = \omega_0 R_I (C_1 + C_2) \Rightarrow C = C_1 + C_2 = \dfrac{Q}{\omega_0 R_I}$
- Find $Q_{right} = \sqrt{\dfrac{R_I}{R_s} - 1}$ and $Q_{left} = \sqrt{\dfrac{R_I}{R_{in}} - 1}$
- $Q_{right} = \dfrac{\omega_0 L_2}{R_s}$ and $Q_{left} = \omega_0 L_1 / R_{in}$

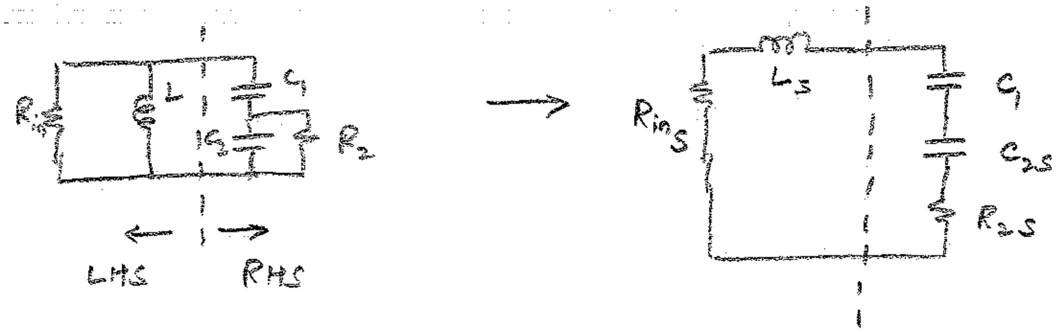
$$\Rightarrow L_2 = \frac{Q_{right} R_s}{\omega_0} \qquad \Rightarrow L_1 = \frac{Q_{left} R_{in}}{\omega_0}$$

Tapped - capacitor match:



- As in π- and T-matching networks, we can set $\frac{R_{in}}{R_2}$ (the transformation ratio), Q and ω_0 independently.

- To design, we again use series-parallel transformations:



LHS Overall $Q = \frac{R_{in}}{\omega_0 L}$ $\Rightarrow$ $\boxed{L = \frac{R_{in}}{\omega_0 Q}}$

$$R_{in_S} = \frac{R_{in}}{Q^2+1} \quad \text{and} \quad L_S = \frac{Q^2}{Q^2+1} L$$

RHS: $R_{2S} = \frac{R_2}{Q_2^2+1}$ (Q_2 = quality factor of R_2-C_2 parallel network)

For match, we want $R_{in_S} = R_{2S}$

$$\Rightarrow \frac{R_{in}}{Q^2+1} = \frac{R_2}{Q_2^2+1} \Rightarrow \boxed{Q_2 = \sqrt{\frac{R_2}{R_{in}}(Q^2+1) - 1}}$$

But $Q_2 = \omega_0 R_2 C_2$

$$\Rightarrow \boxed{C_2 = \frac{Q_2}{\omega_0 R_2}}$$

To find C_1: the equivalent cap. of the C_1 - C_{2S} series combination

is $C_{eq} = \frac{C_1 C_{2S}}{C_1 + C_{2S}}$

But $Q = \frac{1}{\omega_0 R_{2S} C_{eq}}$ $\Rightarrow$ After some algebra $\boxed{C_1 = \frac{C_2(Q_2^2+1)}{Q Q_2 - Q_2^2}}$