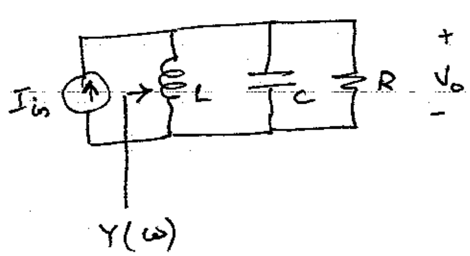
Agenda:

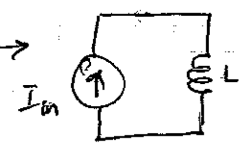
- Passive RLC networks (review)
- impedance transformations
- power matching networks

} Ref: Lee, PP 74-87

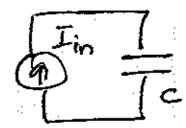
RLC networks:



"low frequencies" ⇒ inductor dominates admittance →



"High" Frequencies ⇒ capacitor dominates admittance

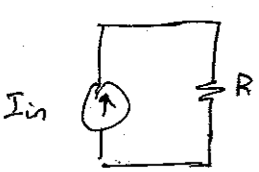


~~What separates "high" and "low" frequencies?~~ ⇒ resonance frequency

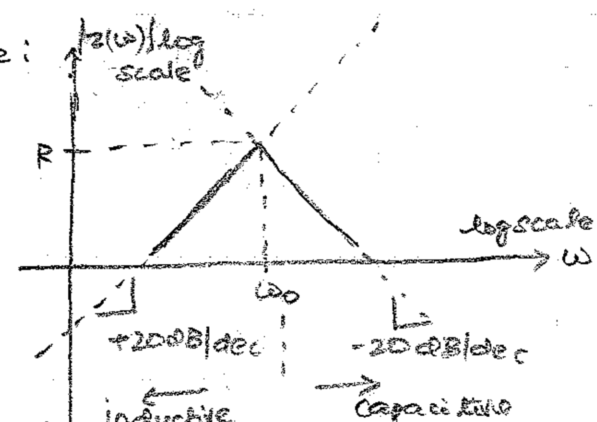
Admittance: $Y(\omega) = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$

At resonance: $\omega_0 C = \frac{1}{\omega_0 L} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$

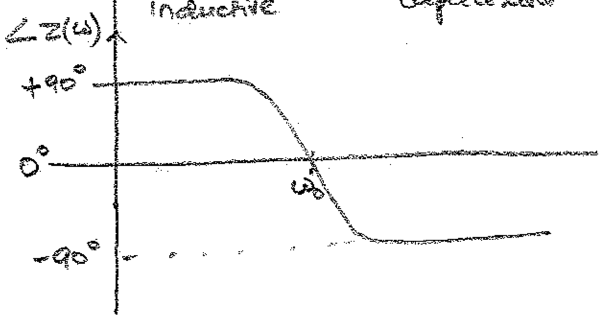
Equivalent circuit @ resonance:



Purely resistive



Define $Z(\omega)$: impedance of parallel RLC tank

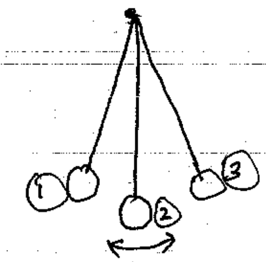


- At resonance
  Magnitude $|Z(\omega_0)| = R$
  Phase $\measuredangle Z(\omega_0) = 0°$

Quality Factor Q

- Measure of how well a system stores energy

Analogy of pendulum:



At position 2 : pendulum has only kinetic energy & no potential energy

At 1 and 3 : pendulum has only potential energy, no kinetic energy

For a lossless pendulum : KE + PE = constant for ever
→ no dissipation

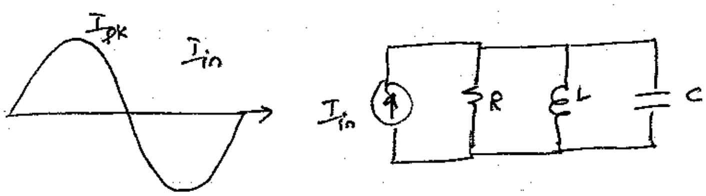
Lossy pendulum : Energy dissipates until pendulum stops oscillating.

⇒ Quality is high if energy is dissipated very slowly.

For RLC circuits (or any circuit or system for that matter)

$$Q = \omega \frac{\text{Energy stored}}{\text{Average power dissipated}}$$

RLC tank → energy sloshes between L and C with a constant sum at resonance.



When current reaches I_{pk}, voltage across capacitor (at resonance)

$$V_c = I_{pk} \cdot R$$

⇒ energy in capacitor reaches a peak at this instant & the energy in the inductor at this instant is zero.

⇒ Peak energy stored $E_{pk} = \frac{1}{2} C V_{pk}^2 = \frac{1}{2} C (I_{pk} R)^2$

Also, at resonance, all the current from the current source flows into the resistor

$\Rightarrow$ Average power dissipated $P_{avg} = \frac{1}{2} I_{pk}^2 R$

$$\Rightarrow \quad Q = \frac{\omega_0 \cdot E_{pk}}{P_{avg}} = \frac{1}{\sqrt{LC}} \cdot \frac{1}{2} C (I_{pk} R)^2 \cdot \frac{1}{\frac{1}{2} I_{pk}^2 R} = \frac{R}{\sqrt{L/C}}$$

$\sqrt{\frac{L}{C}} \rightarrow$ called characteristic impedance (analogy with transmission lines)

$$Q = \frac{R}{\sqrt{L/C}} \rightarrow \quad \text{As } R \rightarrow \infty,\ Q \rightarrow \infty$$

$\Rightarrow$ intuitively correct

<u>At resonance,</u>

$$\left.\begin{aligned} |Z_C| &= \frac{1}{\omega_0 C} = \frac{\sqrt{LC}}{C} = \sqrt{\frac{L}{C}} \\ |Z_L| &= \omega_0 L = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}} \end{aligned}\right\} \Rightarrow \text{characteristic impedance}$$

Three forms of Q for parallel RLC networks:

$$Q = \frac{R}{\sqrt{L/C}}$$

$$Q = \frac{R}{|Z_L|} = \frac{R}{\omega_0 L}$$

$$Q = \frac{R}{|Z_L|} = \omega_0 RC$$

<u>Branch currents at resonance:</u>

$$\left.\begin{aligned} |I_L| &= \frac{|I_{in}| R}{\omega_0 L} = Q |I_{in}| \\ |I_C| &= |I_{in}| R . \omega_0 C = Q |I_{in}| \end{aligned}\right\}$$

<u>Beware</u> $\rightarrow$ for very high Q we have very large branch currents.

$\rightarrow$ To say that the inductor and capacitor "cancel" at resonance is dangerously incorrect!

Relationship between bandwidth and Q

→ What happens @ frequencies near resonance?

$\omega = \omega_0 + \Delta\omega$

$$Y(\omega) = G + j\left(\omega C - \frac{1}{\omega L}\right) = G + j\left[(\omega_0+\Delta\omega)C - \frac{1}{(\omega_0+\Delta\omega)L}\right]$$

$$= G + \frac{j}{(\omega_0+\Delta\omega)L}\left[(\omega_0+\Delta\omega)^2 CL - 1\right]$$

$$= G + \frac{j}{\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 L}\left[\left\{\omega_0^2 + 2\omega_0\Delta\omega + (\Delta\omega)^2\right\} LC - 1\right]$$

$$\xrightarrow{\omega_0^2 LC = 1} \quad = G + \frac{j}{\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 L}\left[2\omega_0\Delta\omega LC + (\Delta\omega)^2 LC\right]$$

$$= G + \frac{j\,\Delta\omega LC}{\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 L}\left[2\omega_0 + \Delta\omega\right]$$

$$= G + \frac{j\,\Delta\omega . LC}{\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 L}\left[2 + \frac{\Delta\omega}{\omega_0}\right].\omega_0$$

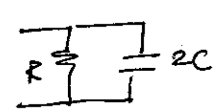
Use $\frac{1}{1+x} \simeq 1-x$ for $x \ll 1$

$$= G + j\Delta\omega . C\left(1 - \frac{\Delta\omega}{\omega_0}\right)\left(2 + \frac{\Delta\omega}{\omega_0}\right)$$

$$= G + j\Delta\omega C\left[2 \underbrace{- \frac{\Delta\omega}{\omega_0} - \left(\frac{\Delta\omega}{\omega_0}\right)^2}_{\text{neglect} \ll 1}\right]$$

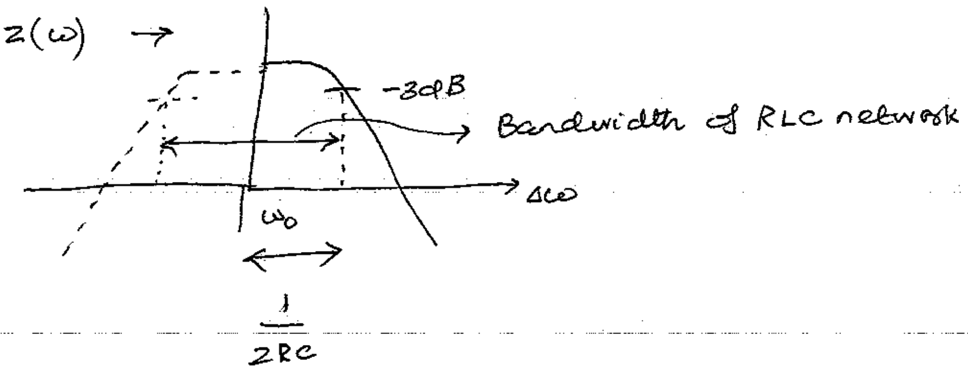
$$\Rightarrow \boxed{Y(\omega) \simeq G + j2\Delta\omega C}$$

⇒ Equivalent circuit @ resonance →



→ ~~At~~ Near resonance, we can replace the original circuit with this equivalent circuit, and replace absolute frequency by frequency offset with respect to ω_0.

$\Rightarrow$ $Z(\omega) \rightarrow$

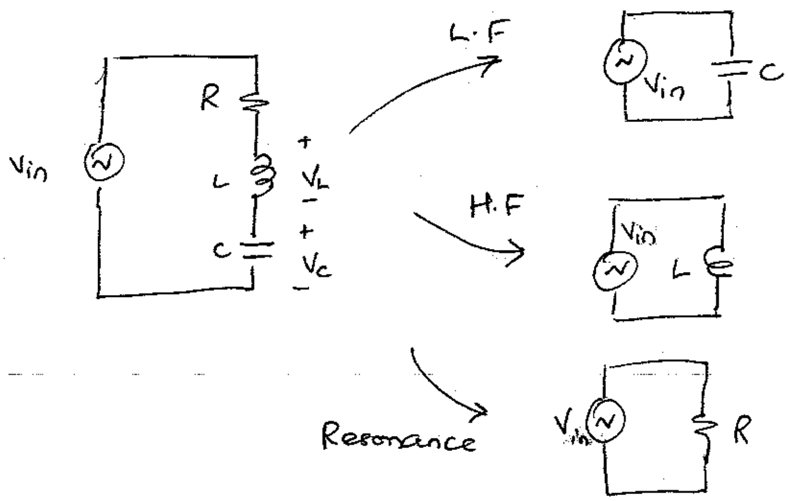


$\Rightarrow$ BW of RLC network $= 2 \cdot \frac{1}{2RC} = \frac{1}{RC}$

$\rightarrow$ Normalize w.r.t $\omega_0 \rightarrow \frac{\Delta\omega}{\omega_0} = \frac{1/RC}{1/\sqrt{LC}} = \frac{1}{R} \cdot \sqrt{\frac{L}{C}}$

$\Rightarrow \boxed{\frac{\omega_0}{\Delta\omega} = \frac{R}{\sqrt{L/C}} = Q}$ $\Rightarrow$ higher the Q, narrower the frequency.

Series RLC network:



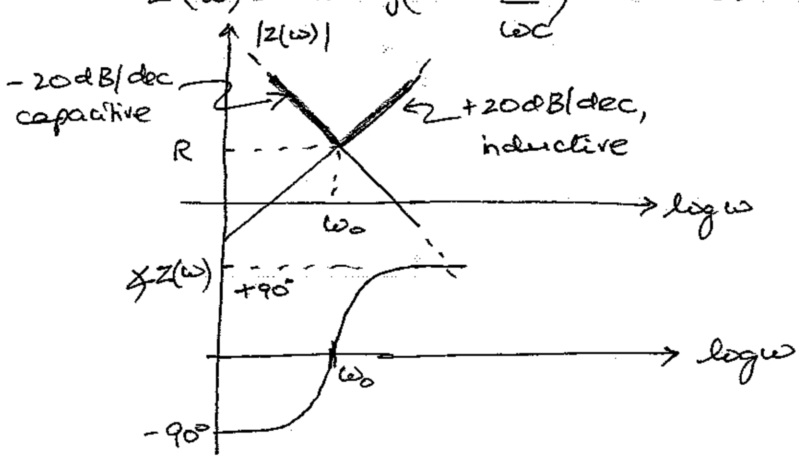
L.F: Capacitance dominates the impedance

H.F: Inductance dominates impedance

Resonance: Purely resistive

At resonance - $V_C + V_L = 0$

$Z(\omega) = R + j\left(\omega L - \frac{1}{\omega C}\right) \rightarrow$ resonance @ $\omega_0 = \frac{1}{\sqrt{LC}}$



$$Q = \omega \cdot \frac{\text{Energy stored}}{\text{Avg. power dissipated}}$$

At resonance, when voltage reaches its peak, the current is $I_{pk} = \frac{V_{pk}}{R}$ $\Rightarrow$ peak magnetic energy in the inductor at this instant

& zero capacitive energy at that instant

$$\Rightarrow \quad E_{pk} = \frac{1}{2} L I_{pk}^2 = \frac{1}{2} L \left(\frac{V_{pk}}{R}\right)^2$$

Avg. power dissipated $P_{avg} = \frac{I_{pk}^2 R}{2} = \frac{V_{pk}^2}{2R}$

$$\Rightarrow \quad Q = \omega_0 \cdot \frac{\frac{1}{2}(V_{pk}/R)^2 L}{\frac{1}{2} V_{pk}^2 / R} = \frac{\omega_0 L}{R} = \frac{\sqrt{L/C}}{R}$$

$\sqrt{L/C}$ $\rightarrow$ characteristic impedance

$$\boxed{Q = \frac{\sqrt{L/C}}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}}$$

$Q \rightarrow \infty$ as $R \rightarrow 0$ intuitively correct

Branch voltages at resonance: $|V_L| = |I_{in}| \cdot \omega_0 L = \frac{|V_{in}|}{R}\omega_0 L = Q|V_{in}|$

$$|V_C| = |I_{in}| \cdot \frac{1}{\omega_0 C} = \frac{|V_{in}|}{\omega_0 RC} = Q|V_{in}|$$

$\Rightarrow$ High branch voltages at resonance $\rightarrow$ breakdown

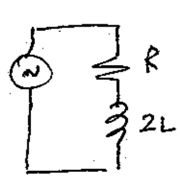
$\rightarrow$ useful for voltage amplification in LNA's.

Bandwidth ~~calc~~ near resonance

For $\omega = \omega_0 + \Delta\omega$ for $\Delta\omega \ll \omega_0$:

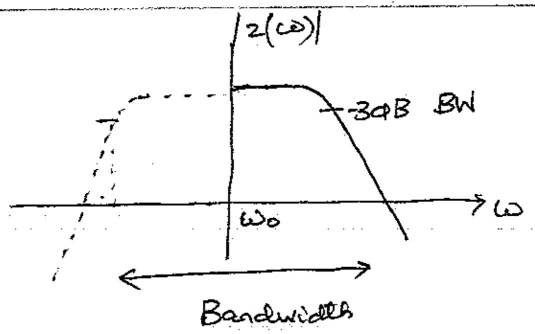
$$Z(\omega) \simeq R + j2\Delta\omega L \quad \text{(PROVE THIS!)}$$

$\Rightarrow$



$\rightarrow$ Equivalent circuit near resonance (replace absolute frequency by offset frequency w.r.t ω_0)

3dB BW of equivalent circuit $= \frac{R}{2L}$



$BW = 2\omega_{3dB} = R/L$

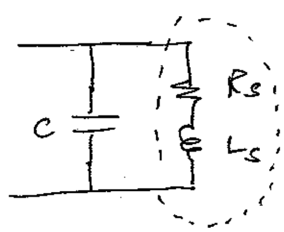
$$\text{Normalized BW} = \frac{BW}{\omega_0} = \frac{R/L}{1/\sqrt{LC}} = \frac{R}{\sqrt{L/C}} = \frac{1}{Q}$$

$$\Rightarrow Q = \frac{\omega_0}{BW} \quad \text{or} \quad \boxed{BW = \frac{\omega_0}{Q}}$$

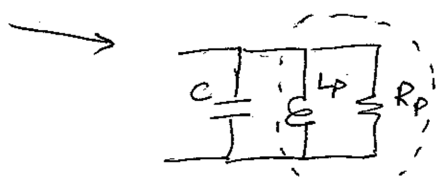
Series-parallel transformations:

- Ubiquitous in RF design
- Can we use what we know about pure-series or pure-parallel RLC networks to analyze networks that are not pure series or parallel?
- On-chip RF circuits → inductors are a lot more lossy than capacitors

LC tank with a lossy inductor:



Convert to a pure parallel RLC



Equate impedances at resonance:

$$R_s + j\omega_0 L_s = R_p \| j\omega_0 L_p = \frac{R_p\, j\omega_0 L_p}{R_p + j\omega_0 L_p} \cdot \frac{R_p - j\omega_0 L_p}{R_p - j\omega_0 L_p}$$

$$= \frac{(\omega_0 L_p)^2 R_p + j\omega_0 L_p R_p^2}{R_p^2 + (\omega_0 L_p)^2}$$

Equate real and imaginary parts:

$$R_s = \frac{(\omega_0 L_p)^2 R_p}{R_p^2 + (\omega_0 L_p)^2} \qquad \omega_0 L_s = \frac{\omega_0 L_p \cdot R_p^2}{R_p^2 + (\omega_0 L_p)^2}$$

Note $Q = \dfrac{\omega_0 L_s}{R_s} = \dfrac{R_p}{\omega_0 L_p}$ (Q's must be the same after transformation if we want the circuits to be identical)

$$R_s = \frac{R_p}{1 + \left(\frac{R_p}{\omega_0 L_p}\right)^2}$$

$$\Rightarrow R_s = \frac{R_p}{Q^2+1}$$

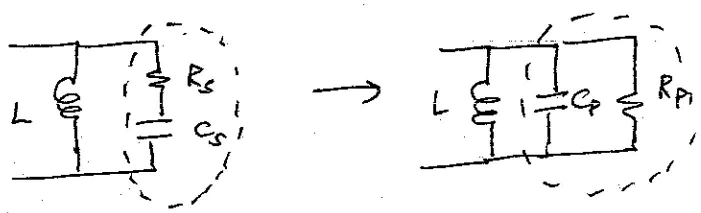
$$\Rightarrow \boxed{R_p = (Q^2+1) R_s}$$

$$\omega_0 L_s = \frac{R_p^2/(\omega_0 L_p)^2 \, (\omega_0 L_p)}{1 + \left(\frac{R_p}{\omega_0 L_p}\right)^2}$$

$$\Rightarrow L_s = \frac{L_p \cdot Q^2}{Q^2+1}$$

$$\boxed{L_p = \left(\frac{Q^2+1}{Q^2}\right) L_p}$$

Similarly, to transform series-RC to parallel-RC :



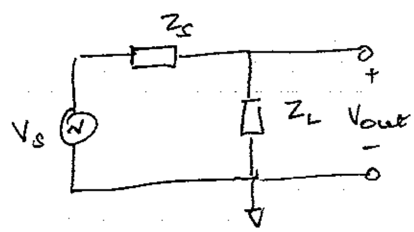
$$R_p = R_s (Q^2+1) \qquad C_p = \frac{Q^2}{Q^2+1} C_s$$

Generally :

$$\boxed{\begin{aligned} R_p &= R_s (Q^2+1) \\ X_p &= X_s \left(\frac{Q^2+1}{Q^2}\right) \end{aligned}} \qquad Q = \frac{X_s}{R_s} = \frac{R_p}{X_p}$$

Note that these transformations holds only in a narrow band centered about ω_0.

The Maximum power transfer theorem:



Suppose that we have a source with a source impedance Z_S, at what value of load impedance will maximum (real) power be delivered to the load?

$\rightarrow$ $\boxed{Z_L = Z_S^*}$ (complex conjugate)

or $\begin{cases} R_L = R_S \\ X_L = -X_S \end{cases}$ $\rightarrow$ called a conjugate match. Other types of "matches" are reflection matching and noise matching.

Derivation (good refresher in basics!)

Power delivered to load $P_L = \frac{1}{2} Re[V_L I_L^*]$ (peak phasors!)

$$V_L = \frac{Z_L}{Z_L + Z_S} V_S \quad , \quad I_L = \frac{V_S}{Z_L + Z_S}$$

$$\therefore P_L = \frac{1}{2} Re\left[\frac{Z_L}{Z_L+Z_S} V_S \cdot \frac{V_S^*}{(Z_L+Z_S)^*} \right] = \frac{|V_S|^2}{2|Z_L+Z_S|^2} \cdot Re(Z_L)$$

$$P_L = \frac{|V_S|^2 R_L}{2|Z_L+Z_S|^2} = \frac{|V_S|^2 R_L}{2[(R_L+R_S)^2 + (X_L+X_S)^2]}$$

Note that the source impedance is fixed & we are allowed to vary only the load impedance.

For maximum power, $\frac{\partial P_L}{\partial X_L} = 0$ and $\frac{\partial P_L}{\partial R_L} = 0$

$$\frac{\partial P_L}{\partial X_L} = \frac{|V_S|^2}{2} \cdot \frac{(R_L)[-2(X_L+X_S)]}{[(R_L+R_S)^2 + (X_L+X_S)^2]^2} = 0 \Rightarrow X_L + X_S = 0$$

or $\boxed{X_L = -X_S}$

$$\frac{\partial P_L}{\partial R_L} = \frac{|V_s|^2}{2}\left[\frac{1}{(R_L+R_s)^2+(X_L+X_s)^2} - \frac{2R_L(R_L+R_s)}{\{(R_L+R_s)^2+(X_L+X_s)^2\}^2}\right] = 0$$

$$\Rightarrow \quad (R_L+R_s)^2 + \underbrace{(X_L+X_s)^2}_{\substack{=0 \\ \text{since } X_L=-X_s}} = 2R_L(R_L+R_s)$$

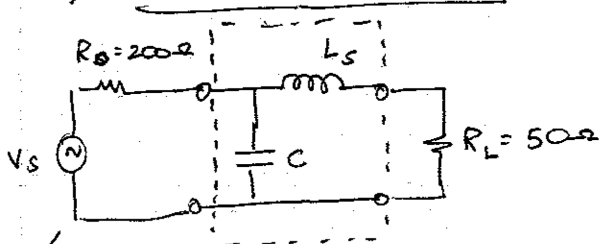
$$\therefore \quad R_L + R_s = 2R_L \quad \text{or} \quad \boxed{R_L = R_s}$$

The power delivered to the load when $Z_L = Z_s^*$ is defined as the "available" power from a source.

$$\boxed{P_{avs} = P_{L_{max}} = \frac{|V_s|^2}{8R_s}}$$

RLC Networks as impedance transformers:

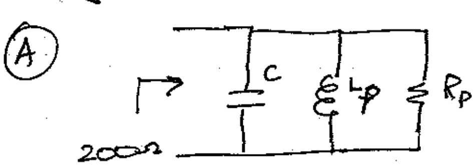
1) Upward transformation:



L-match network

- Find L_s and C to match the load to the source at 5GHz

- Series-to-parallel transformation:



Suppose L_p & C resonate at 5GHz,

then $R_p =$ ~~50~~ 200Ω @ 5GHz

$$R_p = R_L(Q^2+1)$$

$$R_L = 50\Omega, \quad R_p = 200\Omega \Rightarrow Q = \sqrt{\frac{R_P}{R_L} - 1}$$

$$\text{or } Q = \sqrt{3} = 1.732$$

$$\text{But } Q = \frac{\omega_0 L_s}{R_L} = \frac{R_P}{\omega_0 L_P} \Rightarrow L_s = \frac{Q \cdot R_L}{\omega_0} = \frac{\sqrt{3}\cdot 50}{2\pi \cdot 5\text{GHz}}$$

$$\text{or } \boxed{L_s = 2.75\text{nH}}$$

$$\therefore \quad L_p = \frac{R_p}{\omega_0 Q} = \frac{200}{2\pi \cdot 5GHz \sqrt{3}} = 3.68 nH$$

(Alternatively, we can use the formula $L_p = \frac{L_s(Q^2+1)}{Q^2}$)

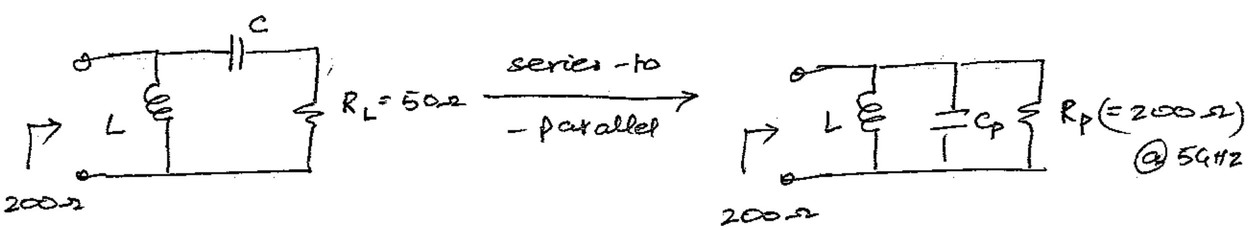
Finally, L_p & C must resonate at 5GHz

$$\Rightarrow \quad \omega_0 = \frac{1}{\sqrt{L_p C}} = \frac{1}{\sqrt{3.66nH \cdot C}} = 2\pi \cdot 5GHz$$

$$\text{or} \quad \boxed{C = 276.83\, fF}$$

This network has a low pass response from the source to the load. We can design another network with a high pass response :

(B)



$$R_p = R_L(Q^2+1) \Rightarrow Q = \sqrt{\frac{R_p}{R_L} - 1} = \sqrt{3}$$

$$Q = \frac{1}{\omega_0 R_L C} \Rightarrow C = \frac{1}{\omega_0 R_L Q} = \frac{1}{2\pi \cdot 5GHz \cdot 50 \cdot \sqrt{3}}$$

$$\text{or} \quad \boxed{C = 367.55 fF}$$

$$C_p = C \cdot \frac{Q^2}{Q^2+1} = 275.66 fF$$

We want L and C_p to reach resonance @ 5GHz

$$\Rightarrow \boxed{L = \frac{1}{\omega_0^2 C_p} = 3.67 nH}$$

Which network to choose (A) or (B) ?

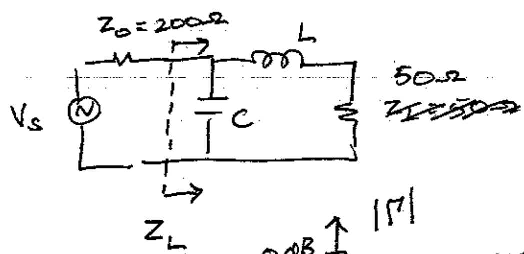
- High-pass vs. Low pass response
- Usually want the network with the smallest inductor (larger inductors have more loss and area)
- Parasitics

— Bandwidth: Match exact only at 5GHz. At other frequencies, we will have an impedance mismatch.

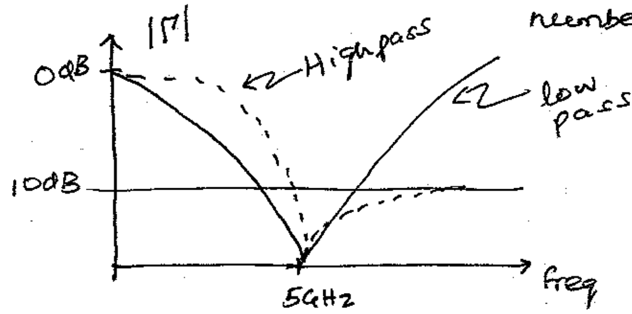
Basic microwave theory: ~~if~~ an impedance mismatch causes power to be "reflected" back to the source

Mismatch quantified by reflection coefficient (= S11 for a 1-port network)

$$S_{11} = \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$



Rule-of-thumb $|\Gamma| \sim -10dB$ is generally a good number.

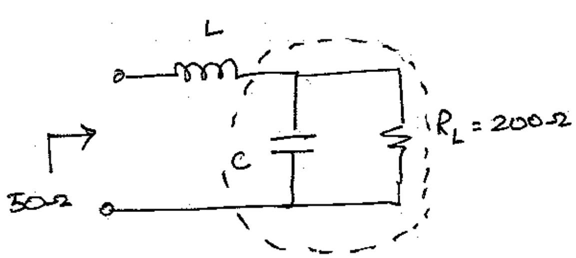


Note: For a mismatch $|\Gamma|$ between the load & source impedances, the power delivered to the load is

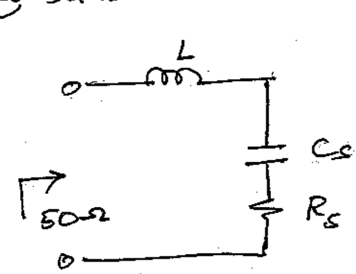
$P_L = P_{av}(1 - |\Gamma|^2)$ when source impedance is real.

↑ "available power" from source.

2) Downward transformation Match @ 5GHz



Parallel -to-series transformation



• If we resonate L & C_s @ 5GHz, then $R_s = 50\Omega$ @ 5GHz

$$R_s = \frac{R_P}{Q^2+1} \Rightarrow Q = \sqrt{\frac{R_P}{R_s} - 1} = \sqrt{3}$$

$$Q = \frac{1}{\omega_0 R_s C_s}$$

$$\Rightarrow \boxed{C_s = \frac{1}{2\pi \cdot 5GHz \cdot 50 \cdot \sqrt{3}} = 367.55fF}$$

$$C_s = C_p\left(\frac{Q^2+1}{Q^2}\right) \Rightarrow \quad C = C_s\left(\frac{Q^2}{Q^2+1}\right) = \frac{(\sqrt{3})^2}{(\sqrt{3})^2+1}\cdot 367.5fF$$

or $\boxed{C = 275.5fF}$

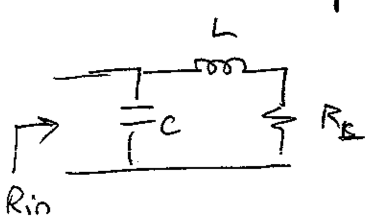
Since L and C_s resonate @ 5GHz,

$$\omega_0^2 = \frac{1}{LC_s} \Rightarrow L = \frac{1}{(2\pi\times 5GHz)^2(367.55fF)} = 2.75nH$$

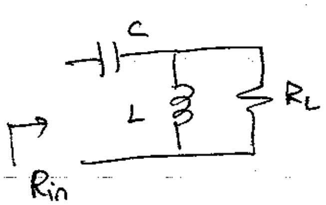
or $\boxed{L = 2.75nH}$

(Not surprisingly, the values are the same as the upward transformation case looking backwards)

<u>Mnemonic</u> : How to find if an L-match network gives an upward or downward transformation?



← since we add L in series, we expect $R_{in} > R_L \Rightarrow$ <u>upward tx.</u>



← add "something" (i.e, L here) in parallel $\Rightarrow R_{in} < R_L \Rightarrow$ downward transformation