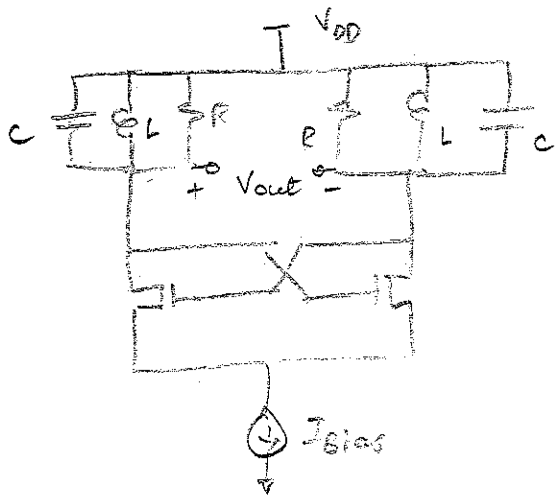
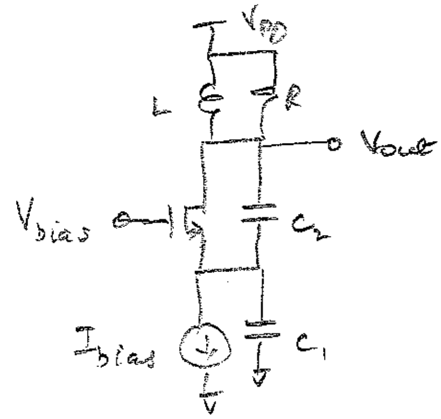
Negative-g_m oscillator



Colpitts oscillator



- $f_{osc} = \dfrac{1}{2\pi\sqrt{LC}}$
- Amplitude (peak, single-ended) $\doteq \dfrac{2}{\pi} I_{bias} R$
- Start-up condition $g_m R > 1$

- $f_{osc} = \dfrac{1}{2\pi\sqrt{L\, C_1 \| C_2}}$
- Amplitude $= 2 I_{bias} (1-N) R$ where $N = \dfrac{C_2}{C_1 + C_2}$ ($N \simeq 0.2$ for best phase noise)
- Start-up condition $g_m R \cdot \dfrac{C_1 \| C_2}{C_1 + C_2} = g_m R (N - N^2) > 1$ (see derivation @ end of lecture)

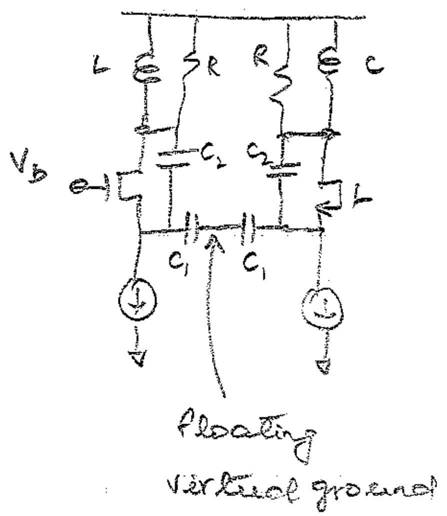
Comparison:

- Easier start-up conditions for negative-g_m
  $g_m > 1/R$ vs. $g_m \geq \dfrac{1}{R(N-N^2)} \simeq \dfrac{5}{R}$
- Negative-g_m oscillator offers differential outputs, but colpitts can be extended to a differential form.
- Phase-noise : colpitts better (allegedly).

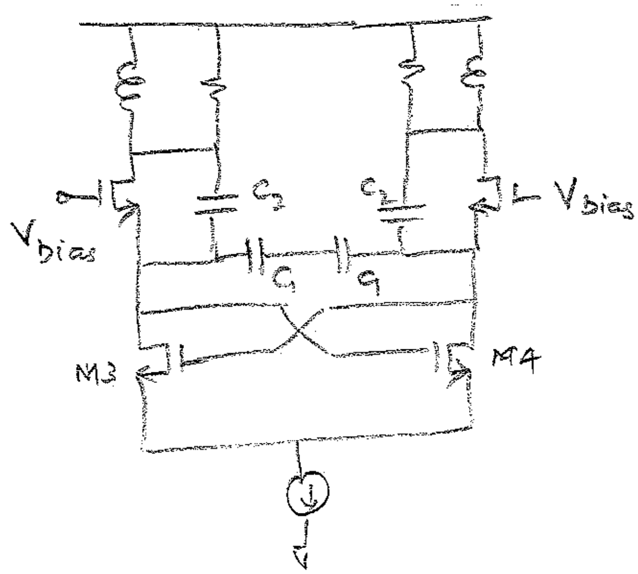
Intuitively, the noisy active device must conduct for extremely short durations in every cycle, preferably at the peak of the voltage swing. This is satisfied in the

Colpitts; but in the -ve-g_m oscillator, one of the two devices is always on, and noise from the ON device always disturbs the tank.

<u>Differential Colpitts oscillators:</u>



- Good phase noise
- start-up condition now requires 2X power for same oscillation amplitude



"Noise-shifting" Colpitts

Ref: Aparacio & Hajimiri, JSSC, Dec 2002

- Retains good phase noise property of Colpitts
- Cross-coupled diff pair (M3, M4) improves start-up condition

<u>Phase-Noise</u>

Ideal oscillator: $V(t) = V_0 \cos \underbrace{\omega_0 t}$

$\theta(t)$ = total phase

→ $\theta(t)$ sweeps an angle <u>exactly</u> equal to 2π over every period of duration T

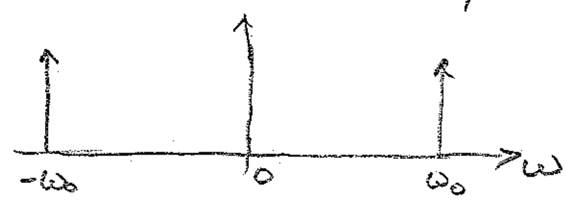
Real Oscillator: The angle swept over every interval of duration T will randomly fluctuate about 2π.

⇒ called phase noise → modulates zero crossings.

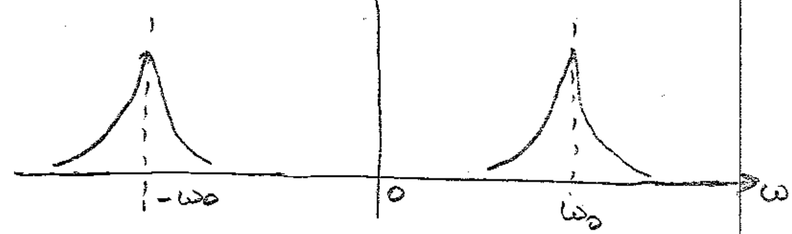
Additionally, the amplitude of a real oscillator will not be constant in a real oscillator.

Real oscillator $\Rightarrow$ $v(t) = \underbrace{a(t)}_{\text{amplitude Noise}} \cos\left[\omega_0 t + \underbrace{\varphi(t)}_{\text{phase-noise}}\right]$

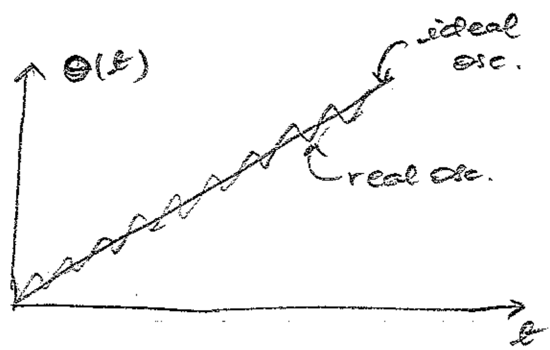
Instantaneous frequency $\omega(t) = \dfrac{d\theta(t)}{dt} = \omega_0 + \dfrac{d\varphi(t)}{dt}$



(Ideal Oscillator)

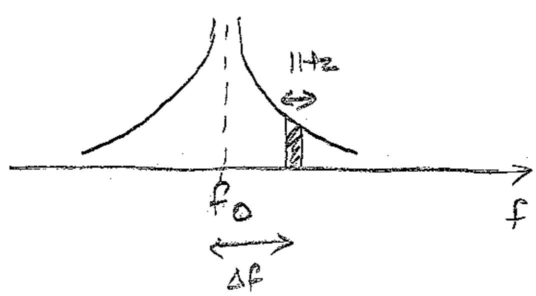


(Real oscillator)



- Autonomous oscillator has no time reference
  $\rightarrow$ phase fluctuations (i.e. jitter) accumulate for ever.

<u>SSB phase-noise:</u>



$$L(\Delta f) = 10\log_{10}\left\{\frac{P_{1Hz}(f_0+\Delta f)}{P_s}\right\}$$

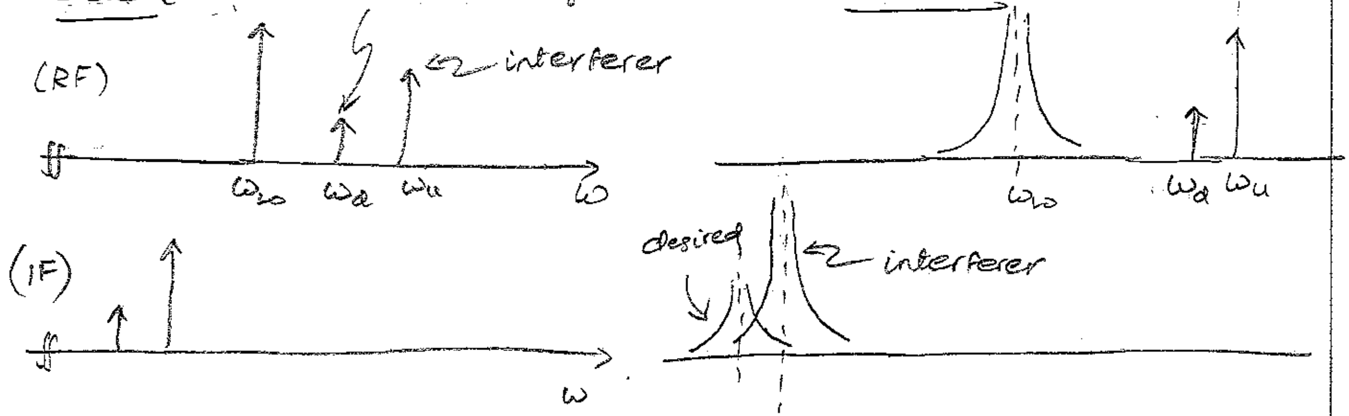
dBc/Hz

$\Rightarrow$ $\dfrac{\text{power in a 1Hz BW at offset } \Delta f}{\text{mean-square carrier power}}$ expressed in dB's

Units = dB's below carrier per Hz, at offset Δf from carrier frequency.

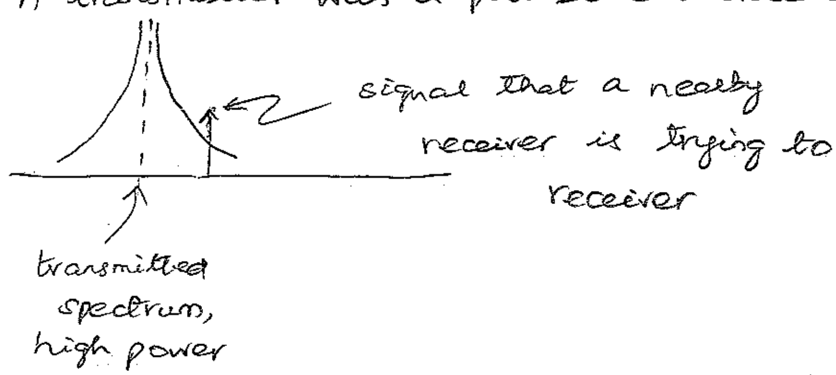
Effects of phase-noise ⇒ Reciprocal mixing

(a) Ideal   Actual



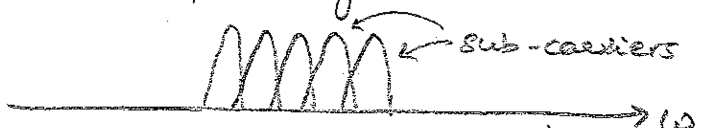
Desensitizes receiver.

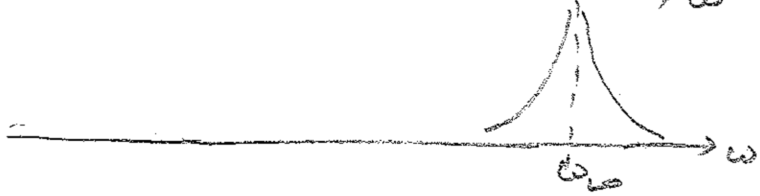
(b) A transmitter with a poor LO can desensitize a nearby receiver.

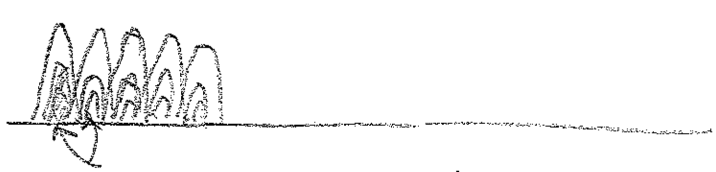


(c) Orthogonal frequency division multiplexing (OFDM)

→ each channel consists of a parallel stream of independently modulated subcarriers

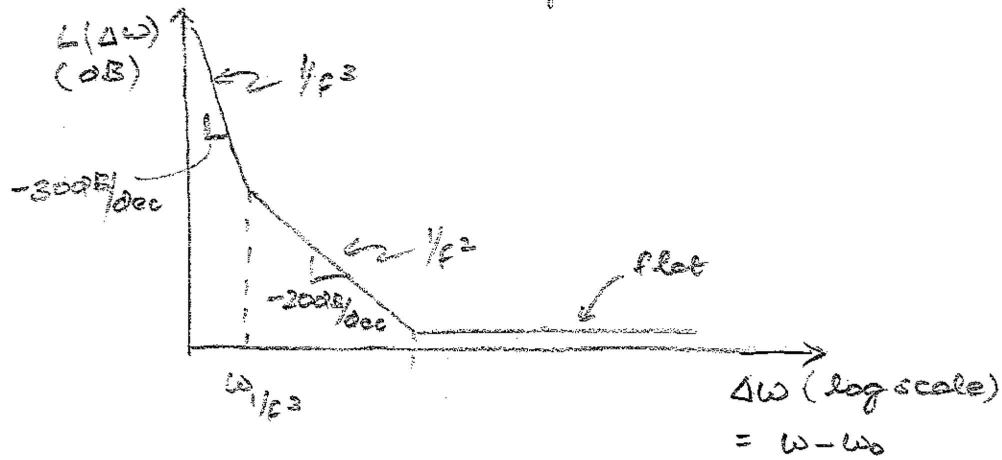






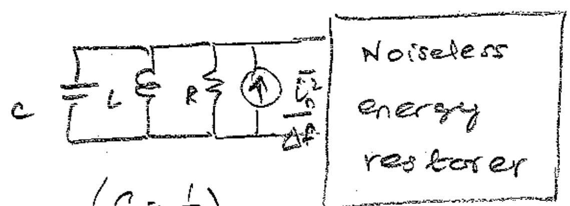
inter-carrier interference (ICI) ⇒ SNR seriously degraded even without other interferers if phase-noise is high.

Close-in view of LO spectrum:



Leeson's phase-noise model:

- Consider an idealized oscillator where the resistor is the only source of noise.



$(G = \frac{1}{R})$

$$\overline{i_n^2} = 4kTG\,\Delta f \quad A^2/Hz$$

- For small offsets $\Delta\omega$ from the center frequency,

$$Z(\omega_0 + \Delta\omega) = j\omega_0 L \,\|\, \frac{1}{j\omega_0 C}$$ (since the admittance of the noiseless energy restorer exactly cancels G)

(see Lecture #2) $$\simeq \frac{j\omega_0 L}{2(\Delta\omega/\omega_0)}$$

$$\Rightarrow |Z(\omega_0 + \Delta\omega)| \simeq \frac{\omega_0 L}{2(\Delta\omega/\omega_0)} = \frac{1}{G}\cdot\frac{\omega_0}{2Q\,\Delta\omega}$$

Mean-square noise voltage density

$$\frac{\overline{v_n^2}}{\Delta f} = \frac{\overline{i_n^2}}{\Delta f}|Z|^2 = 4kTG\left(\frac{1}{G}\cdot\frac{\omega_0}{2Q\Delta\omega}\right)^2 = 4kTR\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2$$

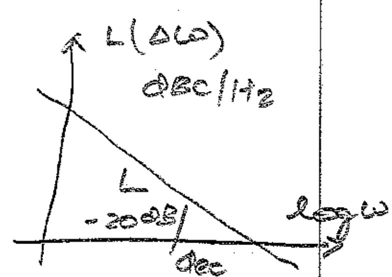
→ Noise increases as frequency increases
→ higher Q ⇒ lower noise.
→ The above expression includes amplitude noise and phase noise. However, we always use an amplitude

limiter in all VCO's, and are mostly interested in only the phase noise. According to the equipartition theorem of thermodynamics, the total energy is split equally between AM noise and PM noise. Amplitude limiting ideally eliminates all AM noise, so that the total phase-noise is ½ the noise calculated above.

- Finally, to get phase-noise $L(\Delta\omega)$ in dBc/Hz, divide $\frac{v_n^2}{\Delta f}$ by ½, and normalize to P_{sig}

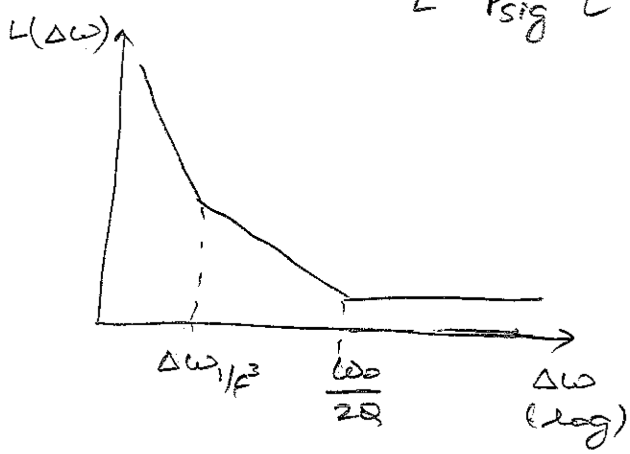
$$\Rightarrow \quad L(\Delta\omega) = 10\log_{10}\left[\frac{\overline{v_n^2}/\Delta f \cdot \frac{1}{2}}{\overline{v_{sig}^2}}\right]$$

$$\boxed{L(\Delta\omega) = 10\log_{10}\left[\frac{2kT}{P_{sig}}\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right]} \rightarrow$$



- Actual oscillators have regions with different slopes not accounted for in the simple model above. Specifically, real oscillators have a $1/f^3$ region with a -30dB/dec slope and a flat noise floor.
- Leeson's model extends the above to empirically include these effects:

$$L(\Delta\omega) = 10\log_{10}\left[\frac{2kTF}{P_{sig}}\left\{1+\left(\frac{\omega_0}{2Q\Delta\omega}\right)^2\right\}\cdot\left\{1+\frac{\Delta\omega_{1/f^3}}{|\Delta\omega|}\right\}\right]$$



F = empirical factor

$\Delta\omega_{1/f^3}$ = $1/f$ noise corner (not always equal to the $1/f$ noise corner of the device)

→ Leeson's model accounts for trends in all regions of $L(\Delta\omega)$

→ provides some insight (e.g. $\omega_0 \uparrow \Rightarrow PN\uparrow$, and $Q\uparrow \Rightarrow PN\downarrow$)

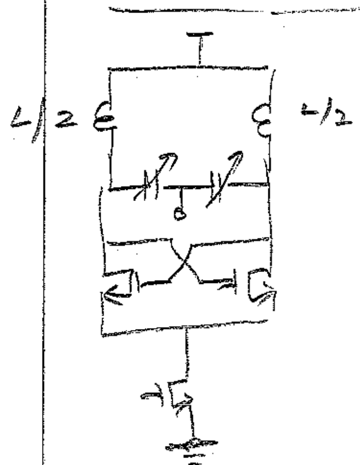
→ provides no guidelines on how to reduce F or $\Delta\omega_{1/f^3}$.

→ Finally, oscillators are not LTI

More sophisticated models:

- A. Hajimiri & T. H. Lee, " A General Theory of phase-noise in Oscillators," JSSC, Feb 1998
  → oscillator modeled as linear, time varying circuit
- J. Rael & A. Abidi, " Physical processes of phase noise in differential LC oscillators," IEEE Custom Integrated Circuits Conf. 2000
  → similar to mixer noise analyses

Guidelines for differential LC-oscillators: (Rael/Abidi)



- Rael/Abidi provide an expression for the factor F in Leeson's model:

$$\boxed{F = 2 + \frac{8\gamma R I_T}{\pi V_0} + \gamma \frac{8}{9} g_m R}$$

where $V_0 = \frac{4}{\pi} I_T R$ = peak differential amplitude (= when swing is not limited by supply rails.)

$$P_{sig} = \frac{V_0^2}{2R}$$

- If we increase I_T while keeping g_m constant, F remains constant from equation above.
  But since $L(\Delta\omega) \propto \frac{4FKT}{V_0^2}(\dots)$, V_0 increases and $L(\Delta\omega)$ falls.
- If we increase I_T beyond the point where the swing is limited by the rails, V_0 does not increase while F increases and $L(\Delta\omega)$ increases ← optimum bias current!

I/Q generation:

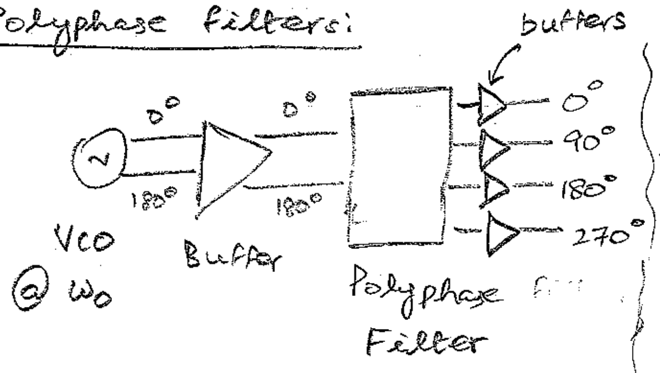
Three main methods:

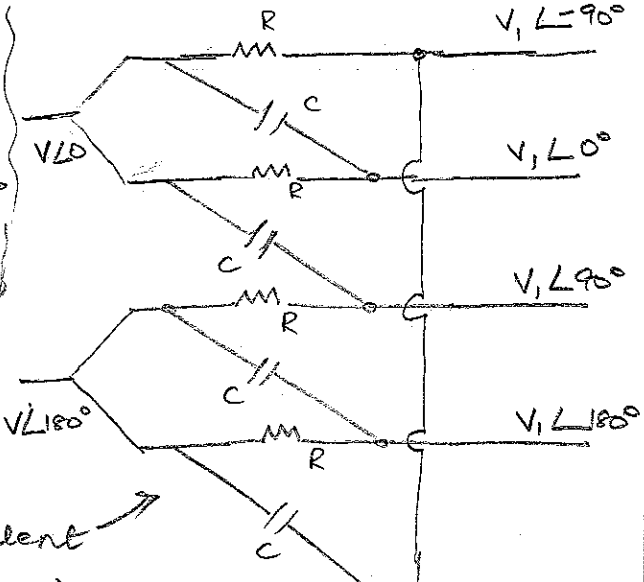
(a) Oscillator at frequency ω_0, driving a <u>polyphase filter</u>

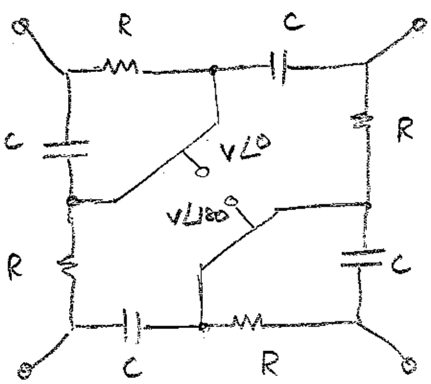
(b) Oscillator at frequency $2\omega_0$ driving a 2x frequency divider

(c) Quadrature oscillator at frequency ω_0

(a) <u>Polyphase filters:</u>



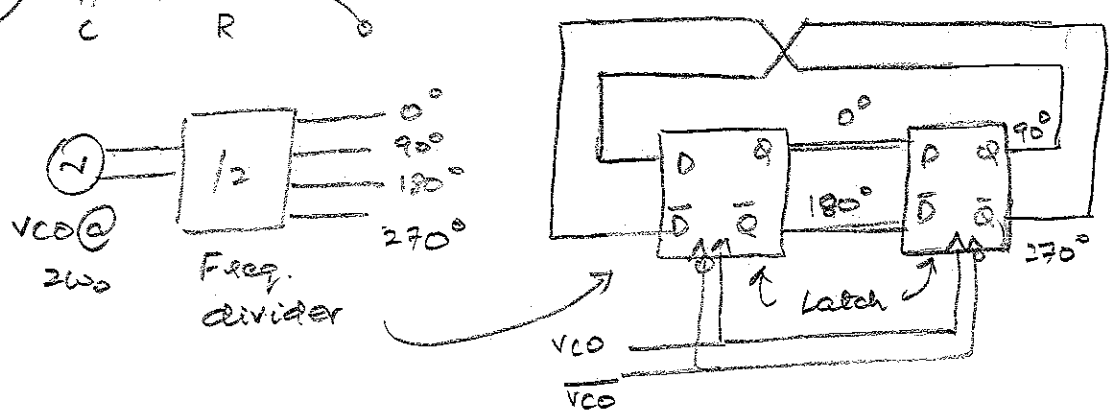




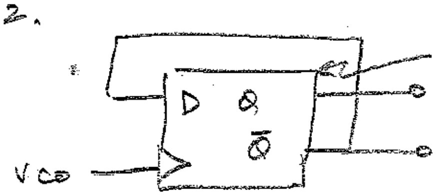
(equivalent drawing)

- Buffers consume extra power

(b)



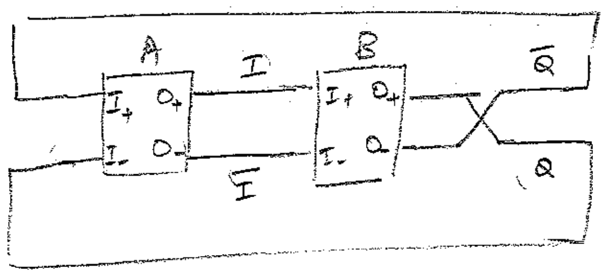
→ This is basically a synchronous counter that counts to 2.



Master-slave D-flip flop.

(e) Quadrature Oscillators:

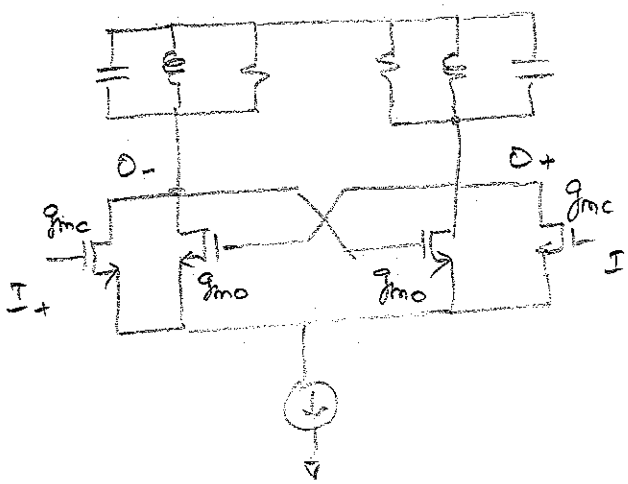
- Couple two identical oscillators:



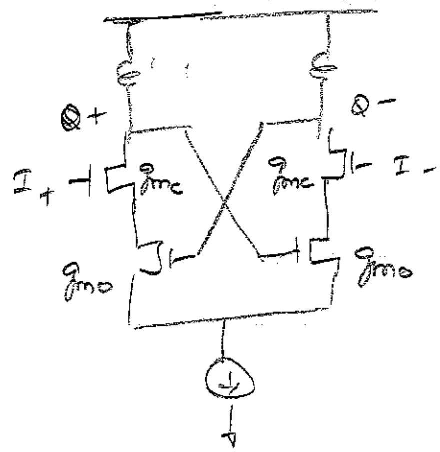
$I, \bar{I} \rightarrow 0, 180^\circ$

$Q, \bar{Q} \rightarrow \pm 90^\circ, \mp 90^\circ$

A, B:



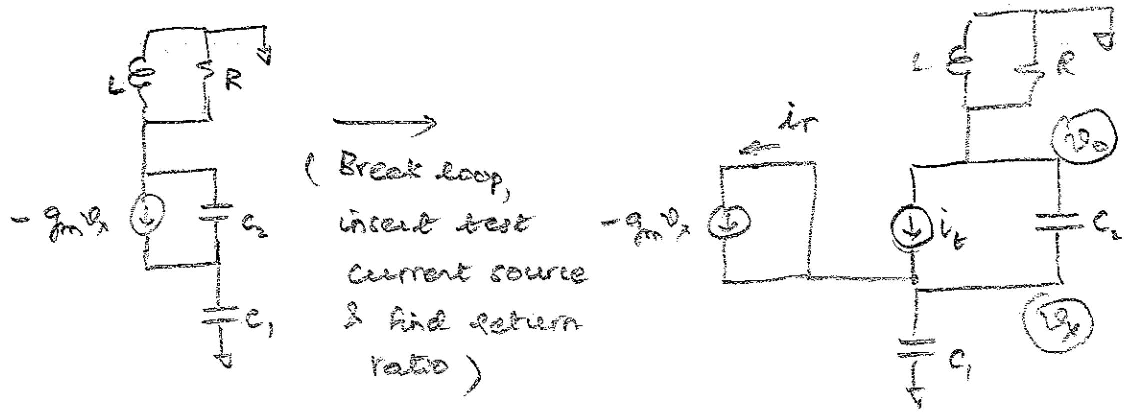
or



Parallel coupling

Series coupling

Loop-gain Analysis of the Colpitts oscillator:



Since current in C_1 = current in parallel L-R,

$$-v_o / sL\|R = sC_1 v_x \Rightarrow v_o = -sC_1 \cdot sL\|R\, v_x \quad (1)$$

KCL at v_x: $i_t = s(C_1+C_2)v_x - sC_2 v_o$

$$= \left[s(C_1+C_2) + sC_1 \cdot sC_2 \cdot sL\|R\right] v_x \quad (2)$$

$$i_r = -g_m v_x = \frac{-g_m}{s(C_1+C_2) + sC_1 \cdot sC_2 \cdot sL\|R}$$

$\Rightarrow$ Loop gain
$$T(s) = -\frac{i_r}{i_t} = \frac{g_m(sL/R + 1)}{s(C_1+C_2)\left[s^2L(C_1\|C_2) + \frac{sL}{R} + 1\right]} \quad (3)$$

Barkhausen criteria for oscillation:

(i) $\angle T(j\omega_0) = 0 \Rightarrow -90° + \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\left\{\frac{\omega L/R}{(1-\omega^2 L C_1\|C_2)}\right\} = 0$

$$\Rightarrow \tan^{-1}\frac{\omega L}{R} - \tan^{-1}\frac{\omega L/R}{1-\omega^2 L C_1\|C_2} = 90°$$

Taking tan on both sides & expanding,

$$\frac{\frac{\omega L}{R} - \frac{\omega L/R}{1-\omega^2 L C_1\|C_2}}{1 + \frac{(\omega L/R)^2}{1-\omega^2 L C_1\|C_2}} = \tan 90° = \infty$$

$$\Rightarrow 1 - \omega^2 L C_1\|C_2 + \left(\frac{\omega L}{R}\right)^2 = 0 \quad \text{or} \quad \omega_0 = \frac{1}{\left[L C_1\|C_2 - \left(\frac{L}{R}\right)^2\right]^{1/2}}$$

$$\Rightarrow \quad \boxed{\omega_0 \simeq \frac{1}{\sqrt{L C_1 \| C_2}}} \qquad \text{since } R \to \infty \text{ for a good inductor.}$$

(ii) $$|T(j\omega_0)| = 1 \Rightarrow \frac{g_m \cdot \left[1 + \left(\frac{\omega_0 L}{R}\right)^2\right]^{1/2}}{\omega_0 (C_1 + C_2)\left[\left\{1 - \omega_0^2 L C_1 \| C_2\right\} + \left(\frac{\omega_0 L}{R}\right)^2\right]^{1/2}} > 1$$

Since $Q_{inductor} = \frac{R}{\omega_0 L} \gg 1$, $\frac{\omega_0 L}{R} \ll 1$

$$\therefore \quad \frac{g_m}{\omega_0 (C_1 + C_2) \cdot \frac{\omega_0 L}{R}} > 1 \Rightarrow \frac{g_m R}{L (C_1 + C_2) \cdot \frac{1}{L C_1 \| C_2}} > 1$$

$$\Rightarrow \quad g_m R \cdot \frac{C_1 C_2}{(C_1 + C_2)^2} > 1 \quad \text{or} \quad \boxed{g_m R (N - N^2) > 1 \text{ where } N = \frac{C_2}{C_1 + C_2}}$$