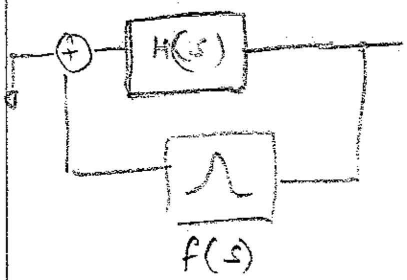
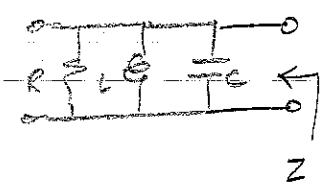
L-C Oscillators:



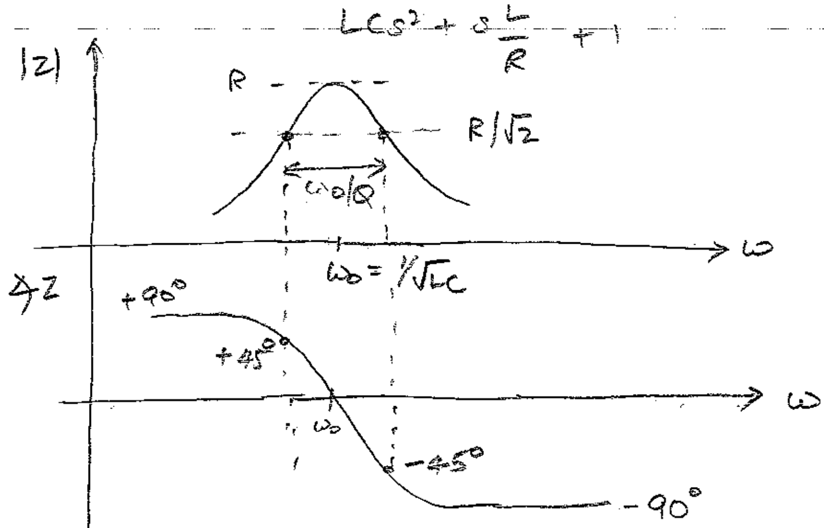
- LC network gives frequency selectivity
  - stabilizes frequency of oscillation
- Barkhausen criterion:
  - Loop gain $T(s) = H(s) f(s)$
  - For oscillation at frequency $\omega = \omega_{osc}$
    $|T(j\omega_{osc})| = 1$ and $\measuredangle T(j\omega_{osc}) = 360°$

- Although we can build oscillators without R's, these oscillators have poor performance compared to LC oscillators.

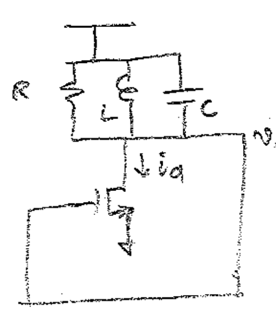
- Parallel L-C tank:



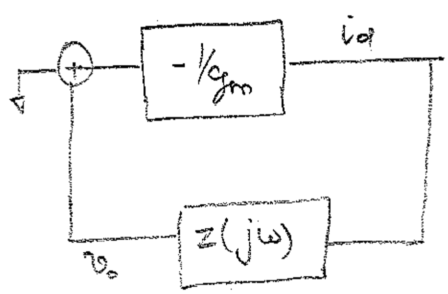
$$Z(j\omega) = \frac{sL}{LCs^2 + s\frac{L}{R} + 1}$$



- Now consider



$\Rightarrow$

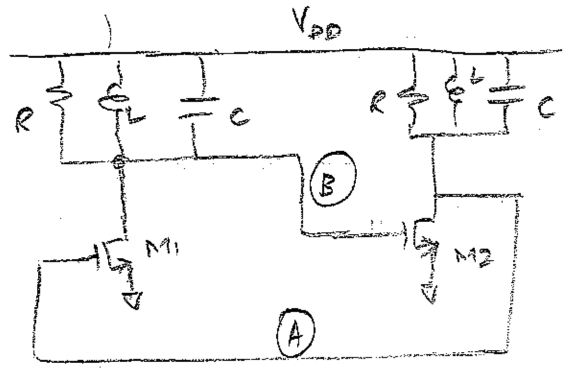


$(f_0 << f_{max})$

- At resonance, RLC tank contributes 0 phase-shift
- FET contributes 180° phase shift for frequencies $f \ll f_t$.

$\Rightarrow$ total phase shift $= 180°$ $\Rightarrow$ no oscillations.

- Next, consider



At $f_0 = \frac{1}{2\pi\sqrt{LC}}$, each tuned stage contributes 180° phase-shift

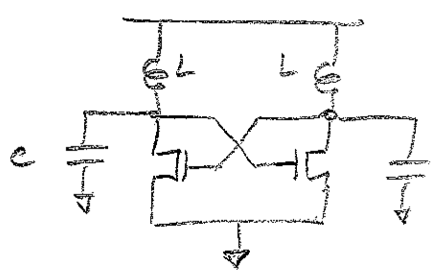
$\Rightarrow$ total $\phi = 360°$

$\Rightarrow$ oscillations possible if $|T(j\omega_0)| \geq 1$

or $\boxed{(g_m R)^2 \geq 1}$

- This is the core of most on-chip LC oscillators

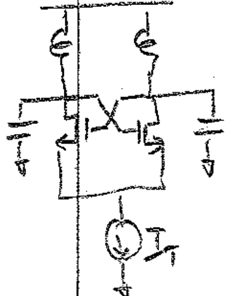
Also drawn as:

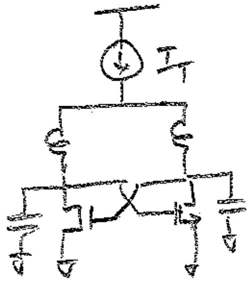


Note that at $f = f_0$, A and B must be exactly 180° out of phase

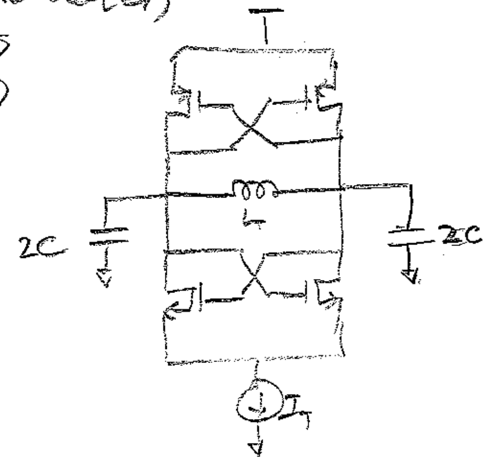
$\Rightarrow$ differential outputs for free.

Other implementations: (more later)





I_T stabilizes amplitude of oscillation

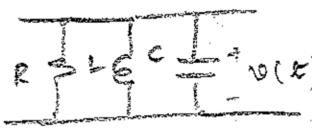


complementary cross-coupled oscillator

$\Rightarrow$ Current re-use gives more g_m

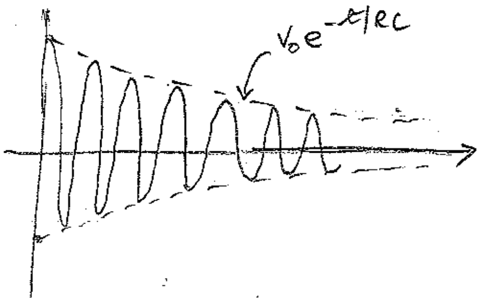
One-port view of oscillators:

- Consider a parallel R-L-C tank with an initial voltage V_0 across the capacitor.



$\rightarrow v(t=0) = V_0$

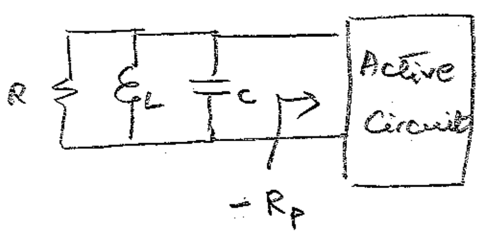
If $Q = \dfrac{R}{\omega_0 L} = \sqrt{\dfrac{L}{C}} = \dfrac{1}{\omega_0 RC} > \dfrac{1}{2}$,



the voltage will be an exponentially decaying sinusoid of freq. ω_0 and amplitude $V_0 e^{-t/RC}$.

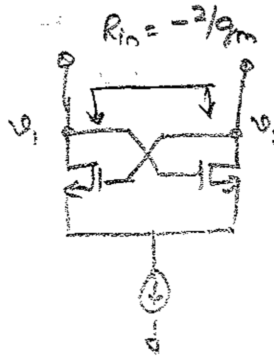
$\Rightarrow$ Amplitude decays with time constant $\tau = RC$

- As we make $R \to \infty$, τ becomes larger, and in the limit, the amplitude never decays. (i.e this is an ideal L-C tank)
- Suppose we have an active circuit that introduces a "<u>negative-R</u>" in parallel with R:

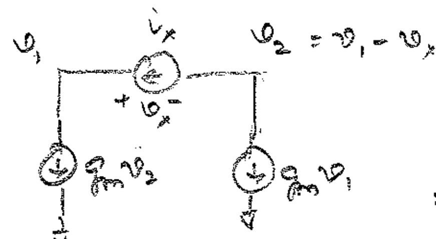


$\Rightarrow$ Amplitude envelope is now
$$V_0 \exp\left[-\frac{t}{(R-R_p)C}\right]$$

$\Rightarrow$ If $R = R_p$, amplitude never decays



$\Rightarrow$



$i_x = g_m v_2 = -g_m v_1$

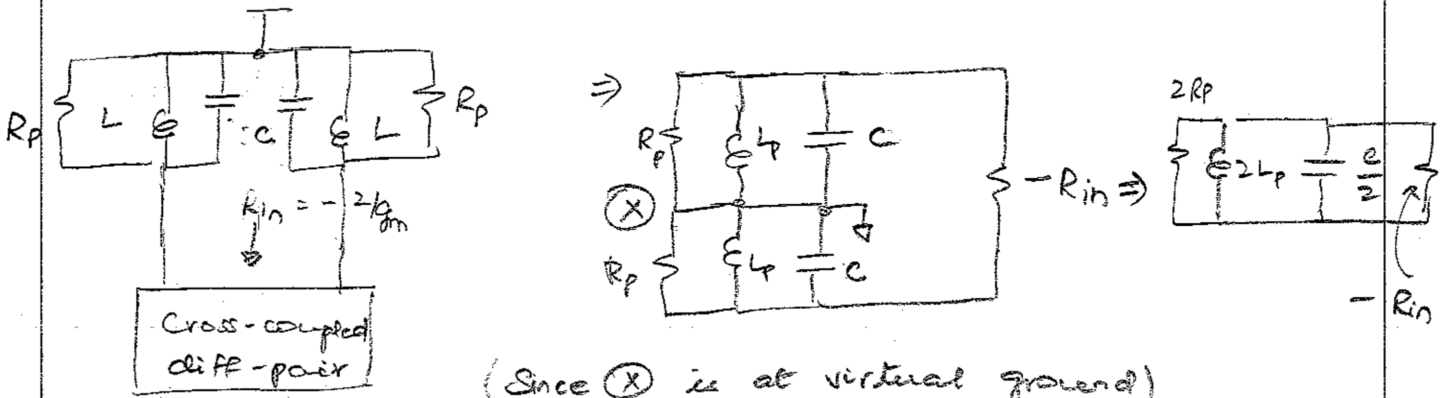
$\Rightarrow v_1 = -v_2 = -(v_1 - v_x)$

$\Rightarrow v_x = 2v_1$

$\Rightarrow i_x = -g_m \dfrac{v_x}{2}$ or $\boxed{G_{in} = \dfrac{i_x}{v_x} = -\dfrac{g_m}{2}}$

or $\boxed{R_{in} = -\dfrac{2}{g_m}}$

- In a negative-G_m cross coupled oscillator:

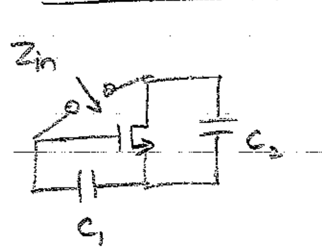


(Since X is at virtual ground)

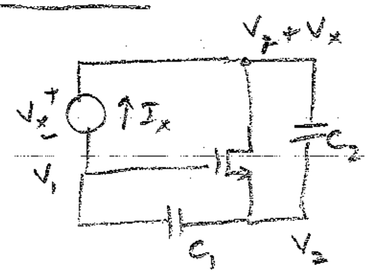
$\Rightarrow$ Set $R_{in} = 2R_p$ for oscillation

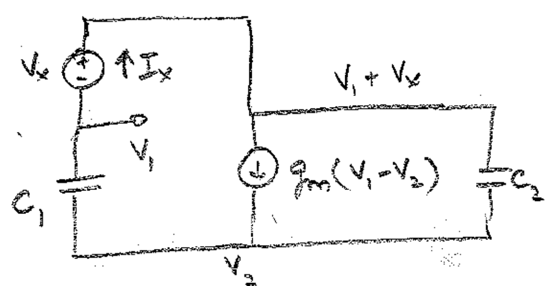
$$\Rightarrow \quad \frac{2}{g_m} = 2R_p \quad \text{or} \quad g_m R_p = 1$$

<u>Single transistor oscillators:</u>



$\Rightarrow$





- $V_1 = V_2 - I_x \cdot \frac{1}{sC_1}$

  $\Rightarrow \quad V_1 - V_2 = \frac{-I_x}{sC_1}$

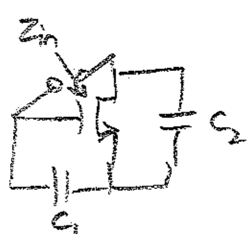
- $I_x = g_m(V_1 - V_2) + sC_2(V_1 + V_x - V_2)$

$$\Rightarrow \quad I_x = g_m\left(\frac{-I_x}{sC_1}\right) + sC_2\left(V_x - \frac{I_x}{sC_1}\right)$$

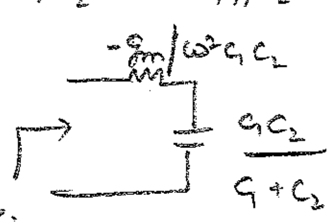
$$\Rightarrow \quad V_x\, sC_2 = 1 + \frac{g_m}{sC_1} + \frac{C_2}{C_1} \quad \text{or} \quad V_x = \frac{1}{sC_1} + \frac{1}{sC_2} + \frac{g_m}{s^2 C_1 C_2}$$

$$\Rightarrow \quad Z_{in}(j\omega) = -\frac{g_m}{\omega^2 C_1 C_2} + \frac{1}{s\, C_1 \| C_2}$$

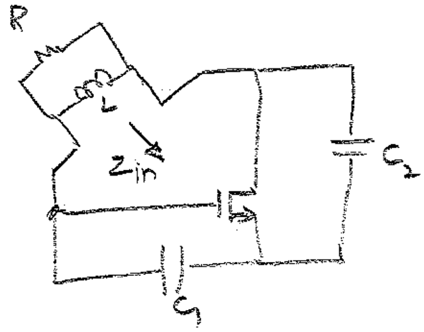
$\Rightarrow$



$\Rightarrow$



⇒

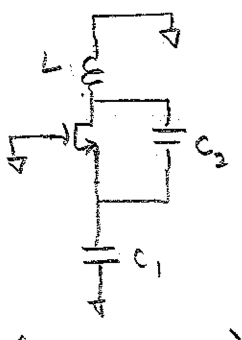


- For oscillation to start, we require

$$\frac{g_m}{\omega^2 C_1 C_2} \geq R$$

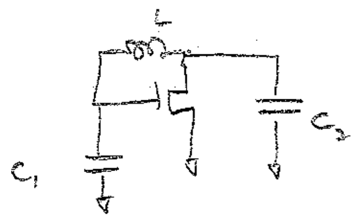
- Can get 3 oscillator topologies by simply defining an AC ground:

Ground gate:

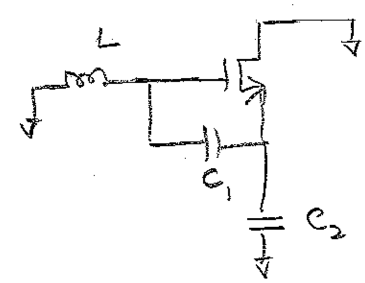


(COLPITTS)

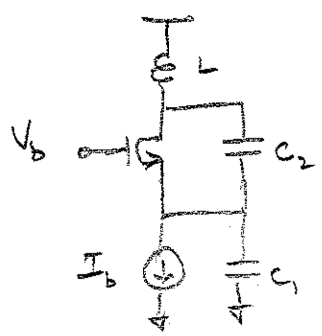
Ground source

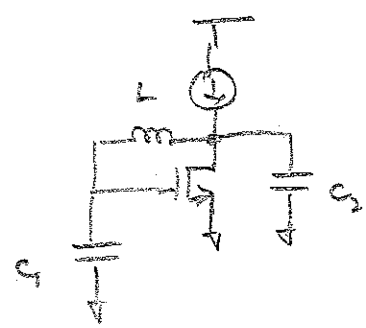


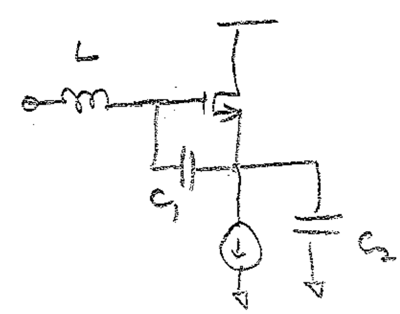
Ground drain



With biasing:

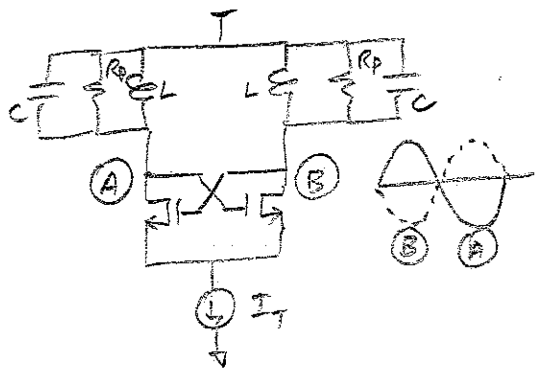




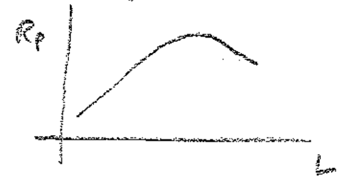


Amplitude of Oscillation & Start-up

1) Cross-coupled oscillator:



- Choose inductor for largest possible R_p
  → highest possible Q.

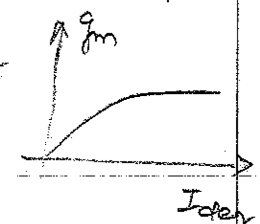


- Choose I_T for large swing, and $\left(\frac{W}{L}\right)$ of transistors to guarantee start-up:

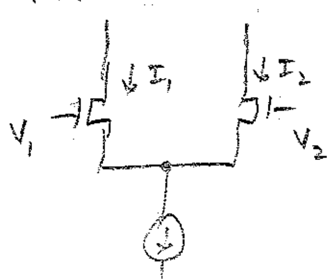
  ie <u>Small-signal</u> $g_m > \frac{1}{R_p}$ (<u>Rule of thumb</u>: $g_m \simeq \frac{2}{R_p}$)

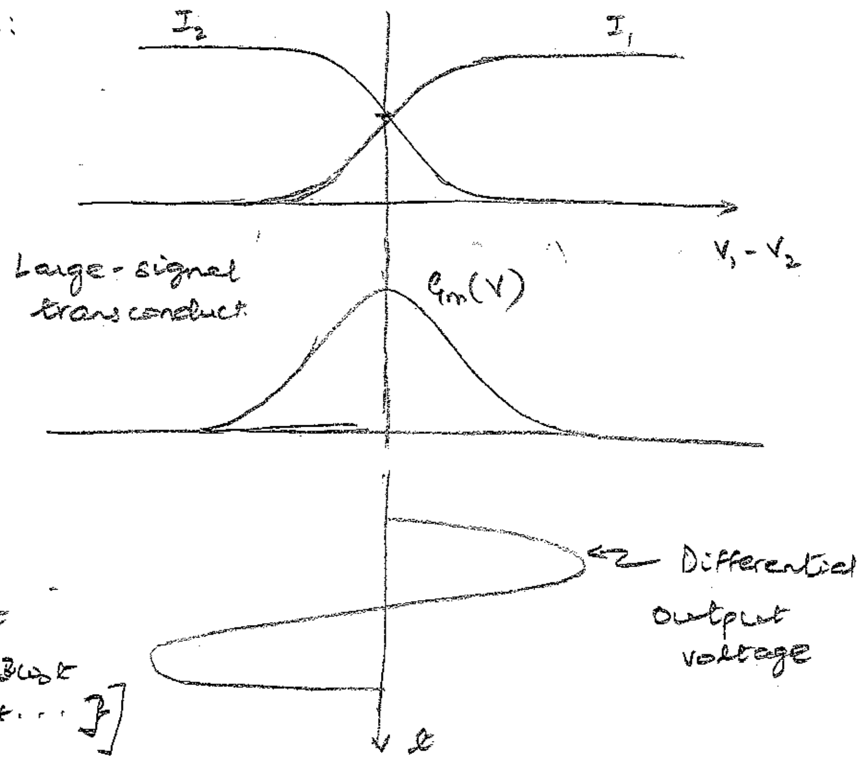
Recall

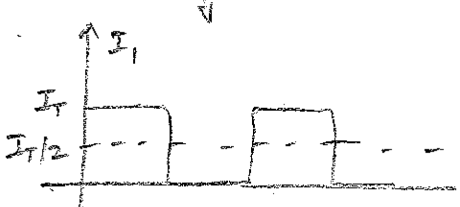
$$g_m = \begin{cases} \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T) & \text{w/o vel. sat} \\ \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(L\,\mathcal{E}_{sat}) & \text{with vel sat} \end{cases}$$



- For MOS diff pair:







$$I_1 = I_T\left[\frac{1}{2} + \frac{2}{\pi}\left\{\cos\omega_0 t - \frac{1}{3}\cos 3\omega_0 t + \cdots\right\}\right]$$

→ Only fundamental component causes voltage @ tank output. (DC & harmonics get filtered)

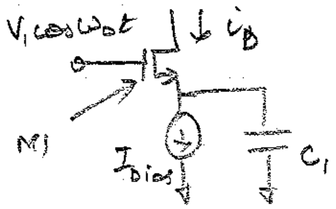
$\Rightarrow$ $\boxed{\text{Amplitude} = \frac{2}{\pi} I_T R_p}$

⇒ Large signal transconductance $\boxed{G_m = \frac{2}{\pi} I_T}$

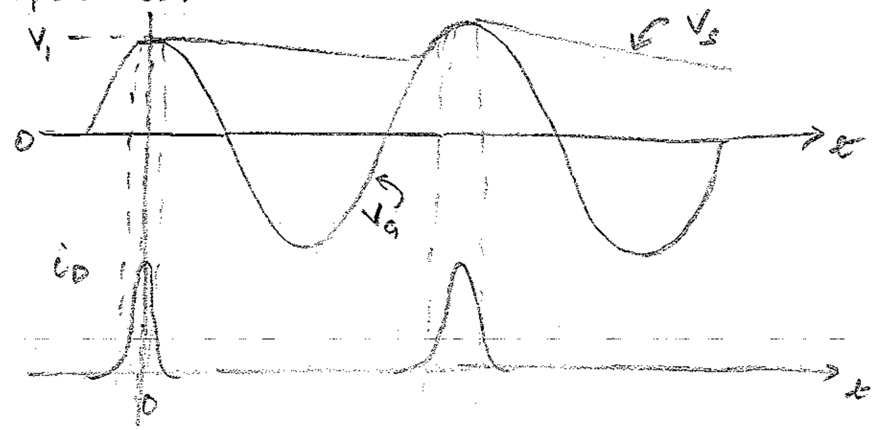
- Note that the oscillation amplitude is independent of the device size.

2) <u>Colpitts Oscillator</u>:

- First, derive large-signal transconductance of MOSFET



— Assume large-amplitude, high-frequency input sinusoid.



- M1 conducts only when V_G is large. When V_G is not close to its peak, M1 cuts off and I_{bias} discharges C_1
- Drain current waveform consists of slivers of current
- Since i_D is periodic with frequency ω_0, express as Fourier series: $i_D(t) = I_{DC} + \sum_{n=1}^{\infty} \hat{I}_n \cos n\omega_0 t$.

  (pick the time reference $t=0$ as shown above. Then, all the $\sin n\omega_0 t$ terms drop out)
- Since C_1 cannot draw a DC current, KCL at source gives $I_{DC} = I_{bias}$.
- Fundamental component $\hat{I}_1 = \frac{2}{T}\int_0^T i_D(t)\cos\omega_0 t\, dt \approx \frac{2}{T}\int_0^T i_D(t) dt$

⇒ $\boxed{\hat{I}_1 = 2 I_{bias}}$

- If the drain current flows into a tank tuned to ω_0 only the fundamental component of i_D causes an appreciable voltage across the tank (high-order harmonics get filtered)

$$\Rightarrow \boxed{\text{Large signal transconductance } G_m \approx \frac{\hat{I}_1}{V_1} = \frac{2 I_{Bias}}{V_1}}$$

- Compare G_m with g_m:

Recall short-channel model:

$$I_D = \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T)\cdot\left[(V_{GS}-V_T) \,||\, (L\,\mathcal{E}_C)\right]$$

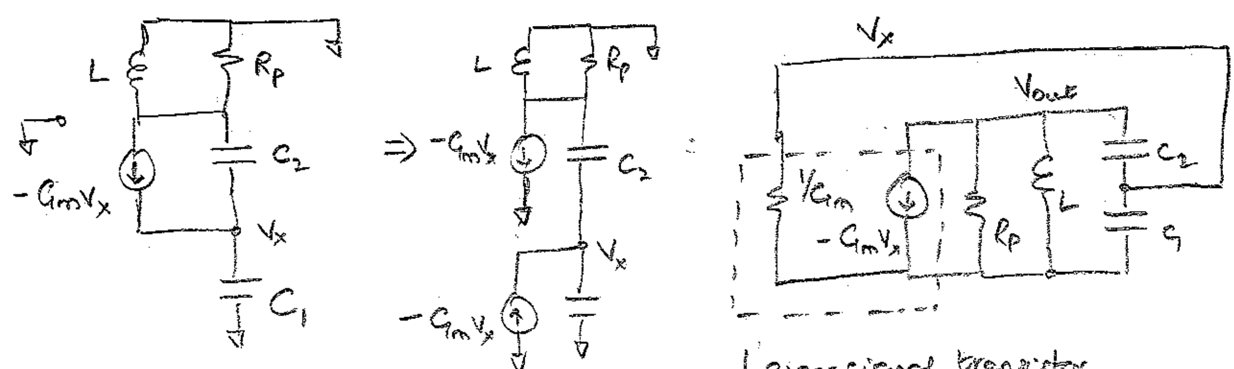
$\rightarrow \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2$ (no vel. sat)

$\rightarrow \frac{\mu_n C_{ox}}{2}\left(\frac{W}{L}\right)(V_{GS}-V_T)\,L\,\mathcal{E}_C$ (with vel. sat)

$$\Rightarrow \underbrace{\frac{2 I_{Bias}}{V_{GS}-V_T}}_{\text{(no vel. sat)}} \le g_m \quad \underbrace{\frac{I_{Bias}}{V_{GS}-V_T}}_{\text{(vel. sat)}}$$

$$\Rightarrow \boxed{\frac{V_{Dsat}}{V_1} \le \frac{G_m}{g_m} \le \frac{2V_{Dsat}}{V_1}}$$

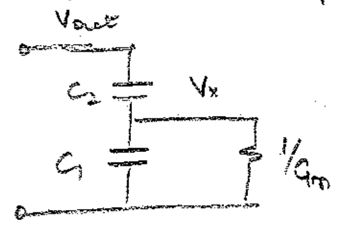
- Apply to Colpitts oscillator:



Large-signal transistor model

- Note that $G_m V_x = 2 I_{Bias}$

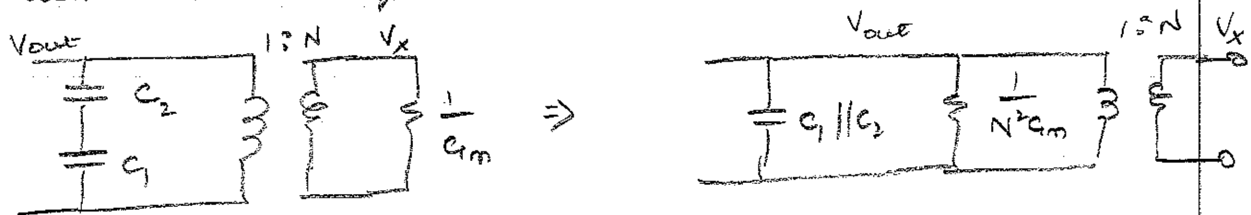
Consider the capacitive divider:



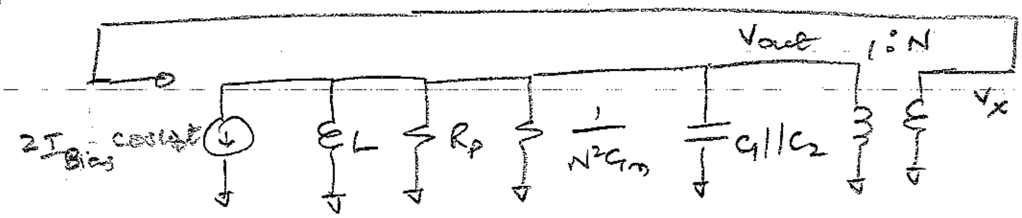
Assume that $1/G_m$ does not load the divider (ie $\frac{1}{\omega C_1} \ll \frac{1}{G_m}$, or equivalently, $Q_1 = \omega C_1 \cdot \frac{1}{G_m} \gg 1$).

$$\Rightarrow \quad V_x = \frac{C_2}{C_1 + C_2} V_{out}$$ ← transformer with turns ratio $N = \frac{C_2}{C_1 + C_2}$

This leads to an equivalent circuit



⇒ complete equivalent circuit is



For resonance, set $\omega_0 = \frac{1}{\sqrt{L\, C_1||C_2}}$

$$V_{out} = 2I_{Bias} \cdot R_p || \frac{1}{N^2 G_m} = \frac{2I_{Bias} R_p}{N^2 G_m R_p + 1} \qquad \left(G_m = \frac{2I_{Bias}}{N V_{out}}\right)$$

$$\Rightarrow \quad V_{out}\left[1 + N^2 \cdot \frac{2I_{Bias}}{N V_{out}} \cdot R_p\right] = 2I_{Bias} R_p$$

or $\boxed{V_{out} \simeq 2I_{Bias}(1-N) R_p}$

- Make R_p as large as possible (ie max Q)
- For best phase noise set $N \simeq 0.2$ or $\boxed{\frac{C_2}{C_1} = 4}$