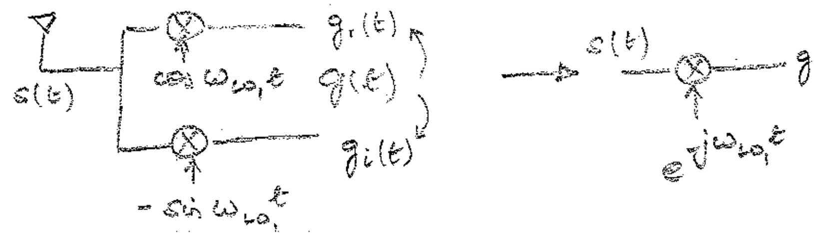
Image-reject receivers:

Recall the complex-signal notation in receivers:

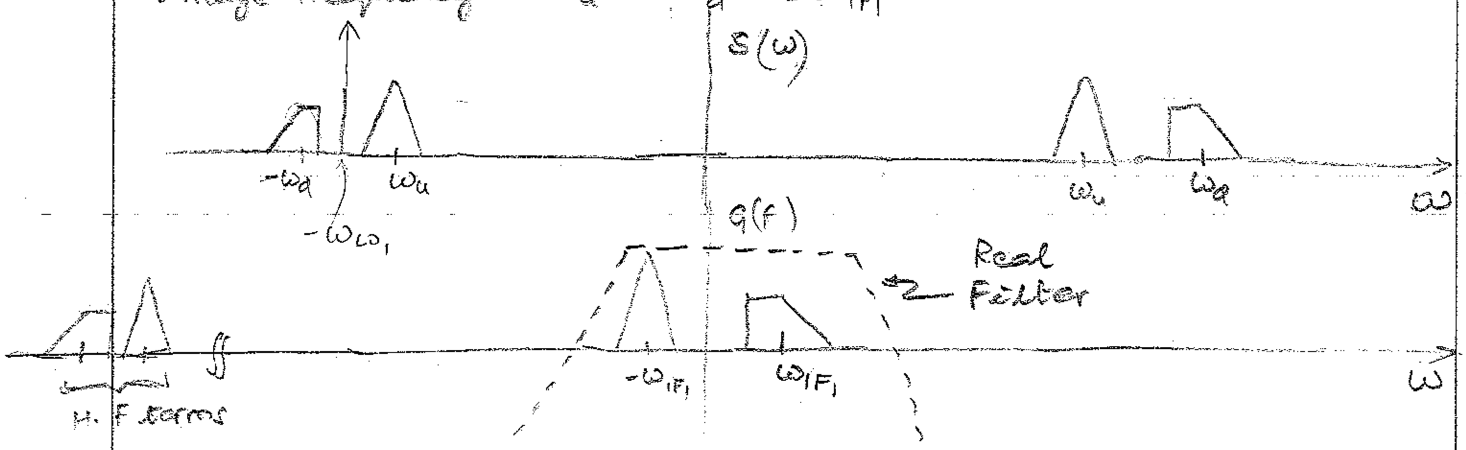


(Complex signal equivalent)

$$s(t) = \underbrace{Re\left[\tilde{s}_d(t)e^{j\omega_d t}\right]}_{\text{Desired Signal}} + \underbrace{Re\left[\tilde{s}_u(t)e^{j\omega_u t}\right]}_{\text{Undesired image}}$$

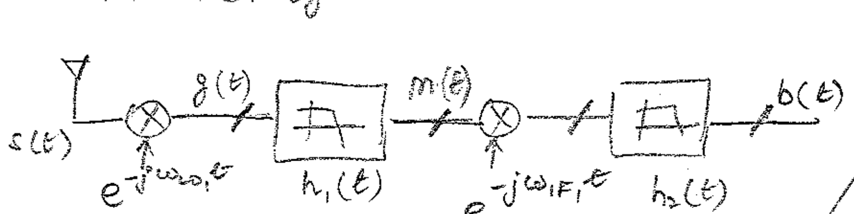
IF frequency $\omega_{IF_1} = \omega_d - \omega_{LO_1}$

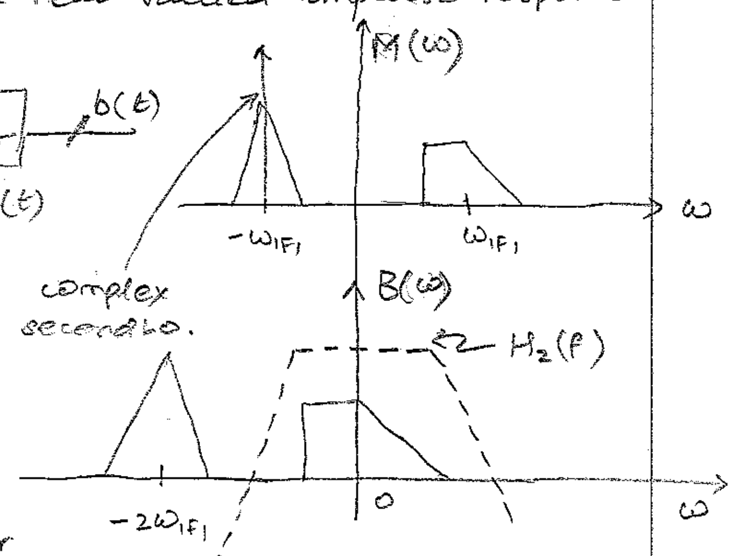
Image frequency $\omega_u = \omega_d - 2\omega_{IF_1}$



How to isolate desired channel?

(a) Weaver Receiver: Use a second complex downconversion followed by a filter with a real-valued impulse response.





- Actually, $h_1(t)$ is optional
- We can digitize the signal at IF, before the second down conversion, or at baseband either before or after the filter $h_2(t)$.

- But $m(t)$ is a complex-valued signal. How do we do complex downconversion?

→ Recall that $m(t) = m_r(t) + j m_i(t)$ where $m_r(t)$ and $m_i(t)$ are <u>real-valued</u>.

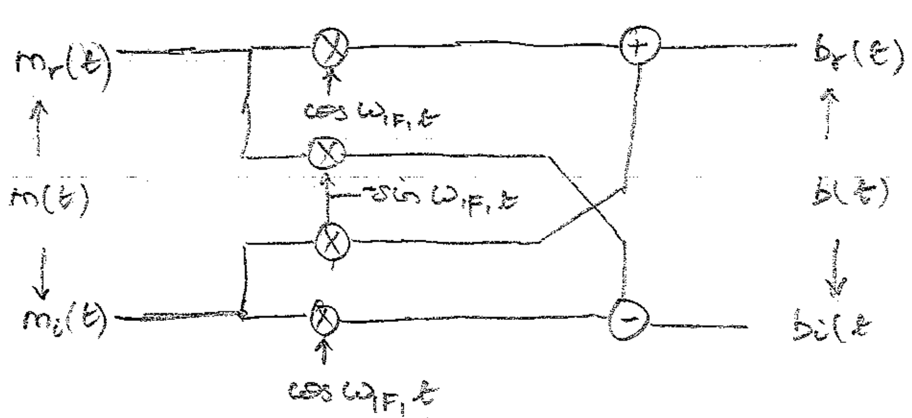
$$\therefore \quad b(t) = m(t)\, e^{-j\omega_{IF_1} t}$$

$$= \left[m_r(t) + j m_i(t)\right]\left[\cos\omega_{IF_1} t - j \sin\omega_{IF_1} t\right]$$

$$= \underbrace{\left[m_r(t)\cos\omega_{IF_1} t + m_i(t)\sin\omega_{IF_1} t\right]}_{\leftarrow\, b_r(t)}$$

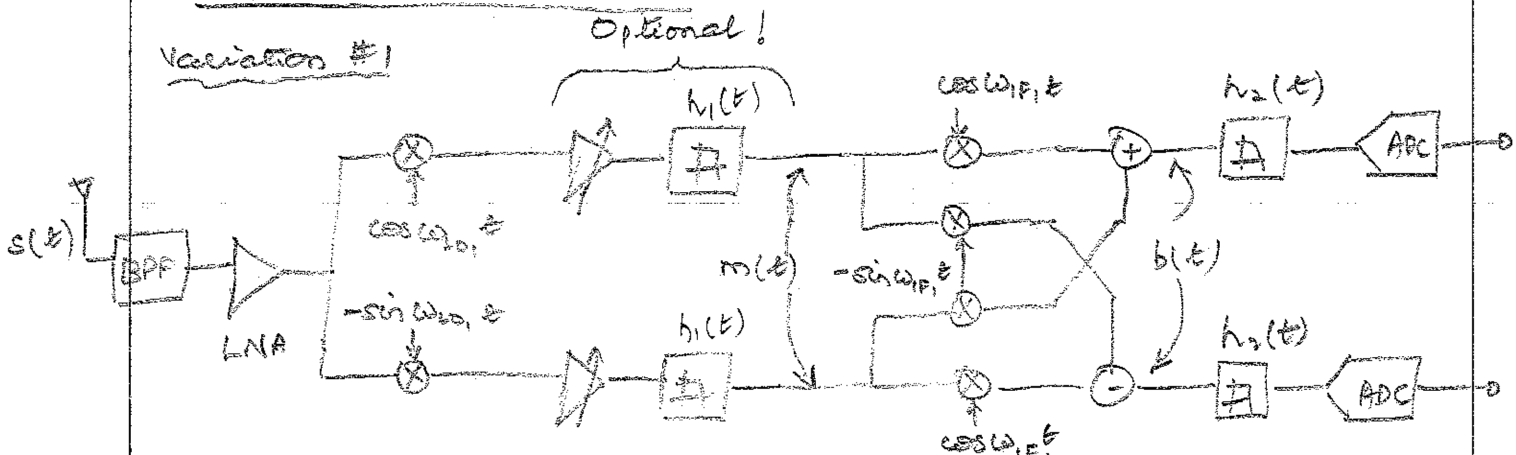
$$+ j \underbrace{\left[-m_r(t)\sin\omega_{IF_1} t - m_i(t)\cos\omega_{IF_1} t\right]}_{\leftarrow\, b_i(t)}$$

⇒ Implement complex mixing with 4 <u>real</u> multiplications and 2 real additions:



<u>Complete Weaver Receiver:</u>

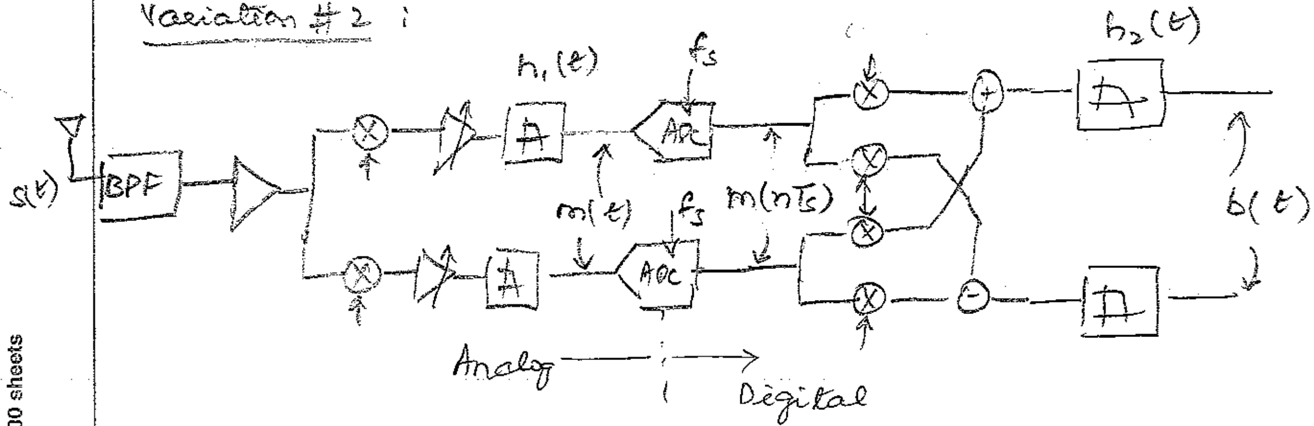
<u>Variation #1</u>



- Fully analog Weaver ← $h_2(t)$ performs channel selection before ADC
- Can use high-IF

<u>Example</u> "A 1.9GHz Wideband Double Conversion CMOS Receiver..."
J. C. Rudell et al. JSSC Dec 1997, pp 2071 – 2088

Variation # 2 :



- $h_1(t)$ required to perform anti-aliasing for ADC.
- $h_2(t) \rightarrow$ digital channel-select filter.
- IF is usually low, otherwise ADC requires very high sampling rate

  (Advanced topics here: bandpass ADC's, sub-sampling)
- If sampling rate of ADC is chosen such that

  $f_{IF_1} = \frac{f_s}{4}$, then the 4 mixers in the second downconversion become ANDgates since

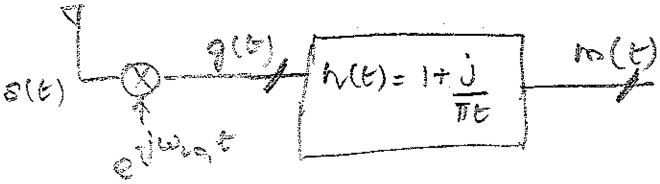
  $$\cos \omega_{IF_1} t \rightarrow \cos \omega_{IF_1} \cdot nT_s = \cos \frac{2\pi f_{IF_1} \cdot n}{f_s} = \cos \frac{n\pi}{2} = \{ \ldots +1, 0, 1, 0 \ldots \}$$

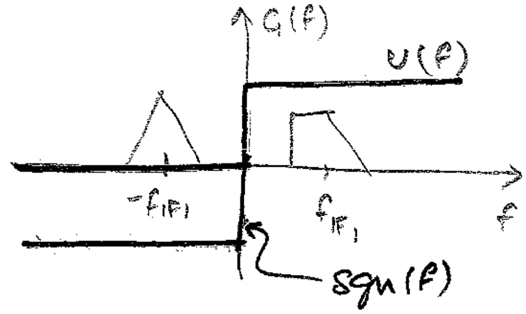
and $$\sin \omega_{IF} t \rightarrow \sin \omega_{IF_1} nT_s = \sin \frac{2\pi f_{IF_1}}{f_s} \cdot n = \sin \frac{n\pi}{2} = \{ \ldots 0, 1, 0, -1 \ldots \}$$

> Example: S. Dow et. al., "A dual-band direct-conversion / VLIF tranceiver for GSM/DCS/PCS", IEEE ISSCC, Feb 2002, pp 230 - 231

$\rightarrow$ This is the topology of choice for modern GSM cellphones

OPTIONAL: Hilbert Transforms & the Generalized Hartley Receiver:





- Suppose we have a filter $h_1(t)$ whose Fourier transform is the unit-step function $U(f)$, ie., $h_1(t) \xrightleftharpoons{F.T} H_1(f) = U(f)$
- We can isolate the desired signal by filtering the complex signal $G(f)$ with the complex filter $H_1(f) = U(f)$

ie: $G(f)$ — $\boxed{U(f)}$ — $M(f)$

$$M(f) = U(f) \cdot G(f) = \frac{1 + \mathrm{sgn}(f)}{2} G(f)$$

$$= \frac{1}{2} G(f) + \frac{j}{2} \{-j\,\mathrm{sgn}(f)\} G(f)$$

- Now, recall that $-j\,\mathrm{sgn}(f) G(f)$ is just the F.T of the Hilbert transform of $g(t)$.

Ref: Haykin, "Communication Systems", John Wiley & Sons

ie: $G(f)$ — $\boxed{\text{Hilbert Transformer}}$ — $\hat{G}(f) = -j\,\mathrm{sgn}(f) G(f)$

Impulse response $= \frac{1}{\pi t}$ ← real-valued.

- $\therefore \; M(f) = \frac{1}{2}\left[G(f) + j\hat{G}(f)\right]$

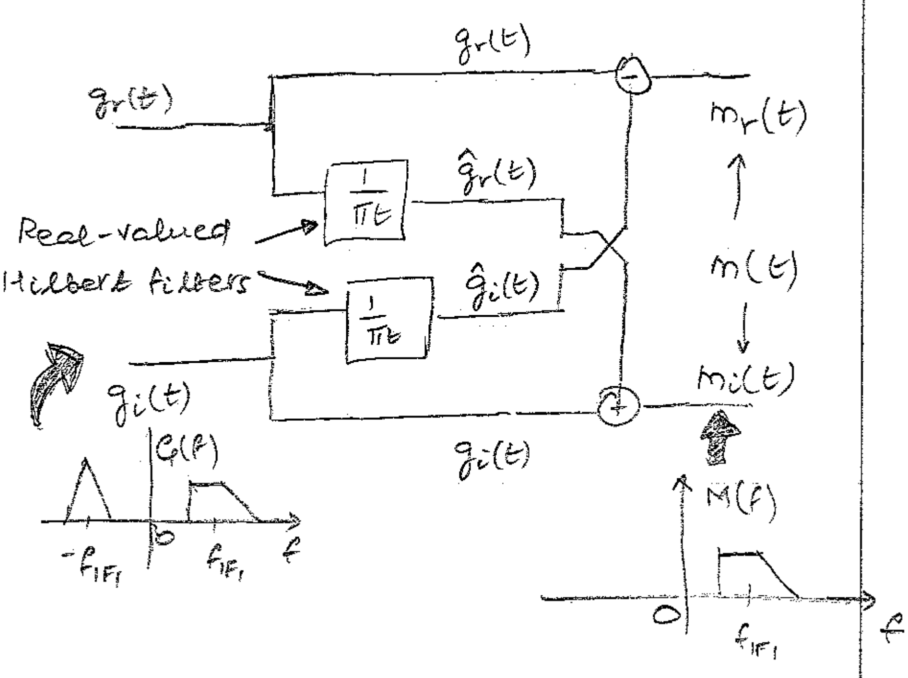
The carat symbol denotes Hilbert transform.

But $G(f) = G_r(f) + jG_i(f)$

$$\Rightarrow M(f) = \frac{1}{2}\left[\{G_r(f) + jG_i(f)\} + j\{\hat{G}_r(f) + j\hat{G}_i(f)\}\right]$$

$$= \frac{1}{2}\left[\underbrace{\{G_r(f) - \hat{G}_i(f)\}}_{M_r(f)} + j\underbrace{\{\hat{G}_r(f) + G_i(f)\}}_{M_i(f)}\right]$$

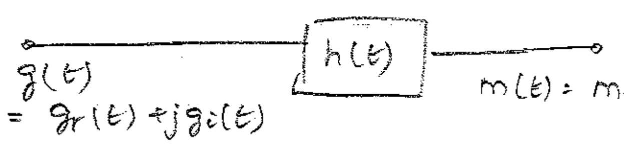
- Note that $G_r(f)$, $G_i(f)$, $\hat{G}_r(f)$ and $\hat{G}_i(f)$ are each Fourier transforms of real-valued signals, and are, therefore, physically implementable.



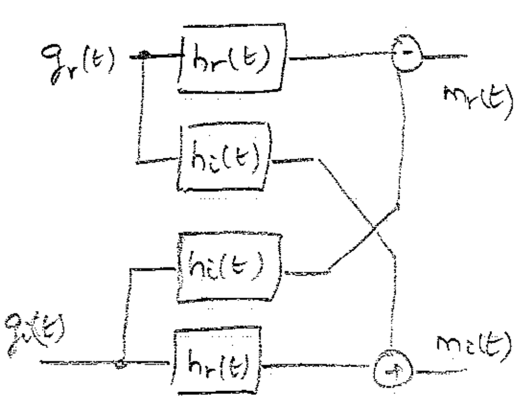
- We can express this in a compact form using the notion of a complex-valued filter. Specifically, define

$$h(t) = h_r(t) + j h_i(t).$$

Here $h_r(t) = 1$ and $h_i(t) = \frac{1}{\pi t}$ (impulse-response of Hilbert filter)



- The input-output convolution follows the rules of complex addition, multiplication & superposition:



$$m(t) = g(t) * h(t) = (g_r + j g_i) * (h_r + j h_i)$$
$$= \underbrace{(g_r * h_r - g_i * h_i)}_{m_r(t)} + j \underbrace{(g_i * h_r + g_r * h_i)}_{m_i(t)}$$

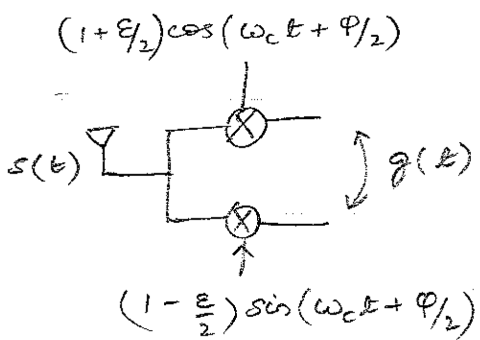
Image rejection ratio (IRR)

Until now, we have assumed perfect quadrature mixing

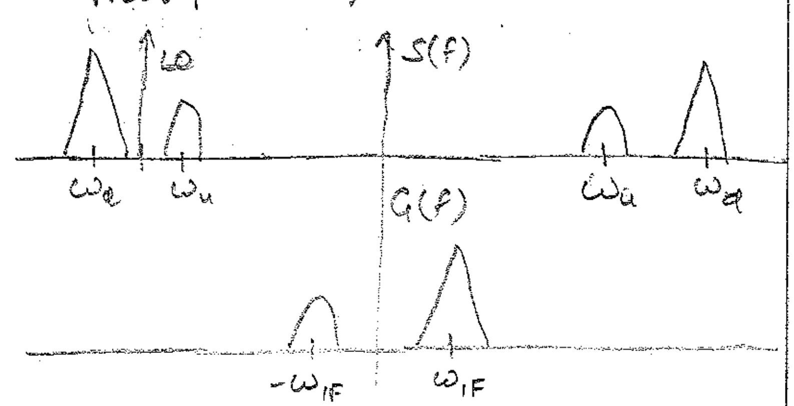
- equal gain and phase-shifts in each I & Q paths
- I and Q LO's are exactly 90° apart.

Under these assumptions, the image signal is cancelled perfectly. In practice, the image is attenuated imperfectly due to gain & phase mismatches:

Consider a simple model:



With perfect quadrature



With I/Q mismatch:

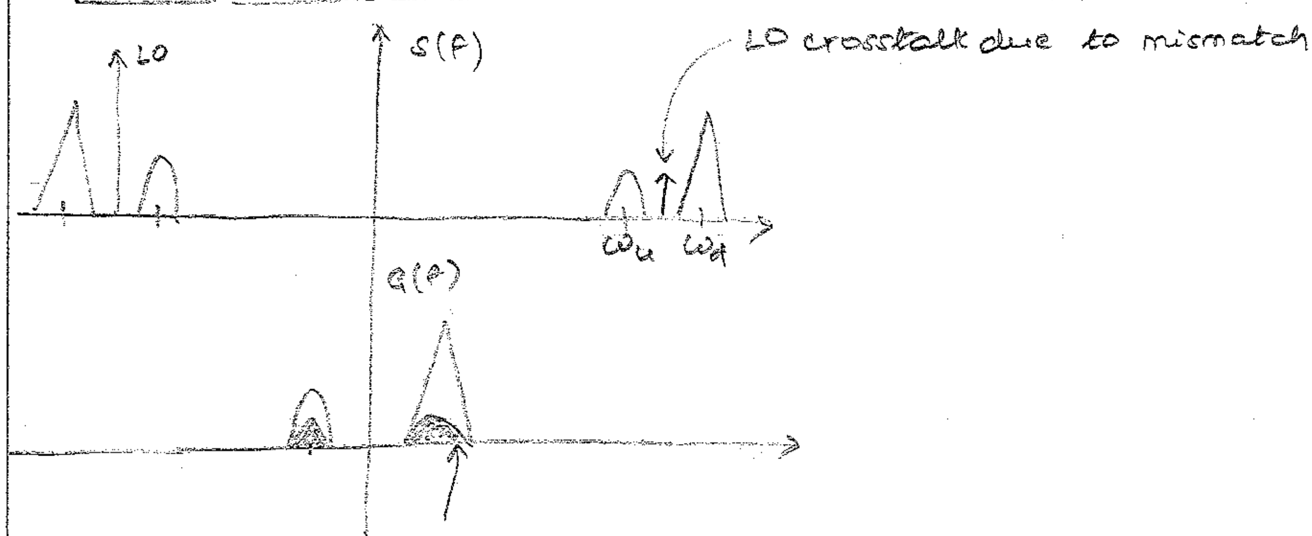


Image imperfectly attenuated

$\rightarrow$ degrades SNR.

- Instead of down conversion as $g(t) = s(t)e^{-j\omega_{LO}t}$,

We get down conversion as $g(t) = s(t)\left[A_1 e^{-j\omega_{LO}t} + A_2 e^{+j\omega_{LO}t}\right]$

where A_1 & A_2 are complex constants dependent on the gain mismatch ε & phase mismatch φ.

With simple algebra,

$$A_1 = \cos\varphi/_2 + j\,\varepsilon/_2 \sin\varphi/_2$$

$$A_2 = \frac{\varepsilon}{2}\cos\frac{\varphi}{2} + j\sin\varphi/_2$$

$$\text{Image rejection ratio IRR} = \frac{\text{Signal/Image @ input}}{\text{Signal/Image @ Output}}$$

$$= \frac{1 + 2(1+\varepsilon)\cos\varphi + (1+\varepsilon)^2}{1 - 2(1-\varepsilon)\cos\varphi + (1+\varepsilon)^2} \simeq \frac{\varepsilon^2 + \varphi^2}{4}$$

($\varepsilon = \frac{\Delta G}{G}$ = relative gain mismatch & φ is phase mismatch in radians)

→ For $\varepsilon = 1\%$ and $\varphi = 1\%$ → IRR = 40dB

- These numbers are typical in IC implementations.
- With digital calibration, IRR can be improved to 60-80dB

<u>OPTIONAL</u>: Derivation of A_1 & A_2:

$$\text{LO} = \left(1+\frac{\varepsilon}{2}\right)\cos\left(\omega_0 t + \frac{\varphi}{2}\right) - j\left(1-\frac{\varepsilon}{2}\right)\sin\left(\omega_0 t - \frac{\varphi}{2}\right)$$

$$\left(1+\frac{\varepsilon}{2}\right)\left(\cos\omega_0 t\cos\frac{\varphi}{2} - \sin\omega_0 t\sin\frac{\varphi}{2}\right) + j\left(1-\frac{\varepsilon}{2}\right)\left(\sin\omega_0 t\cos\frac{\varphi}{2} - \cos\omega_0 t\sin\frac{\varphi}{2}\right)$$

$$= \cos\frac{\varphi}{2}\left[\left(1+\frac{\varepsilon}{2}\right)\cos\omega_0 t - j\left(1-\frac{\varepsilon}{2}\right)\sin\omega_0 t\right]$$

$$- \sin\frac{\varphi}{2}\left[\left(1+\frac{\varepsilon}{2}\right)\sin\omega_0 t + j\left(1-\frac{\varepsilon}{2}\right)\cos\omega_0 t\right]$$

$$= \cos\frac{\varphi}{2}\left[(\cos - j\sin) + \frac{\varepsilon}{2}(\cos + j\sin)\right] - \sin\frac{\varphi}{2}\left[(\sin + j\cos) + \frac{\varepsilon}{2}(\sin - j\cos)\right]$$

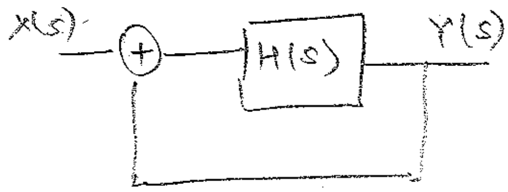
$$= \cos\frac{\varphi}{2}\left(e^{-j\omega_0 t} + \frac{\varepsilon}{2}e^{j\omega_0 t}\right) - \sin\frac{\varphi}{2}\left(je^{j\omega_0 t} - j\frac{\varepsilon}{2}e^{-j\omega_0 t}\right)$$

$$= \underbrace{\left(\cos\frac{\varphi}{2} + j\frac{\varepsilon}{2}\sin\frac{\varphi}{2}\right)}_{A_1}e^{-j\omega_0 t} + \underbrace{\left(\frac{\varepsilon}{2}\cos\frac{\varphi}{2} + j\sin\frac{\varphi}{2}\right)}_{A_2}e^{j\omega_0 t}$$

Voltage-Controlled Oscillators (VCO's)

- Autonomous circuit, always used in RF transceivers for up/down conversion: Used in phase-locked loop frequency synthesizer for RF channel selection

Feedback model:



$$Y(s) = \frac{H(s)}{1 - H(s)}$$

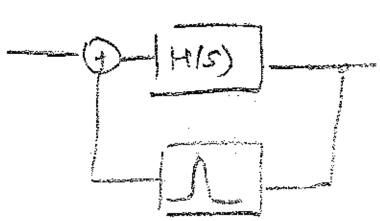
$\Rightarrow$ If at $s_0 = j\omega_0$, $H(s_0) = 1$, then $Y(s) = \infty \rightarrow$ possible oscillations with constant amplitude.

Barkhausen criteria for oscillations:

Two necessary conditions are required for steady-state oscillation. At $s = s_0$, $H(s_0) = 1$

$\Rightarrow$
- $|H(j\omega_0)| = 1$
- $\angle H(j\omega_0) = 360°$.

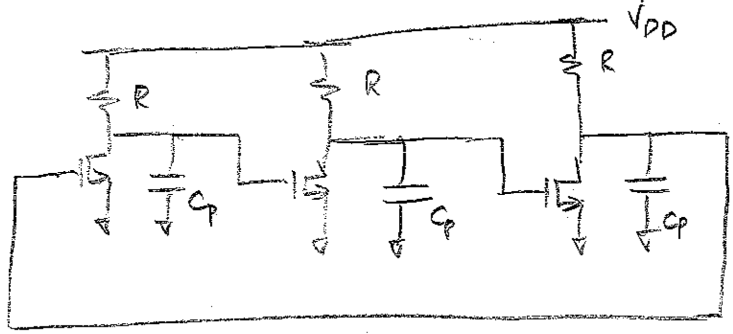
In many RF oscillators, an LC tank is used inside the loop to stabilize frequency:



Note: If we have DC negative feedback, we would have a 180° phase shift around the loop at DC. H(s) would need to contribute only 180° of excess phase with $|H(j\omega_0)| > 1$ for oscillations

- Barkhausen criterion not sufficient for oscillations $\Rightarrow$ even if met, oscillations may not occur (Ex. 4 CMOS Inverters in a loop)

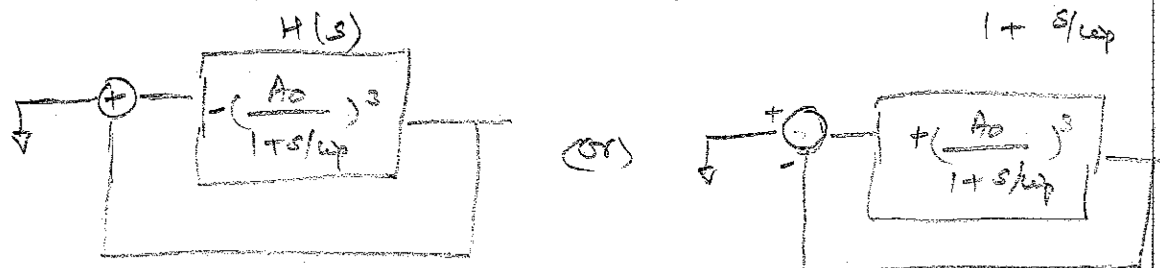
Example: Consider 3-stage C-S ring oscillator:



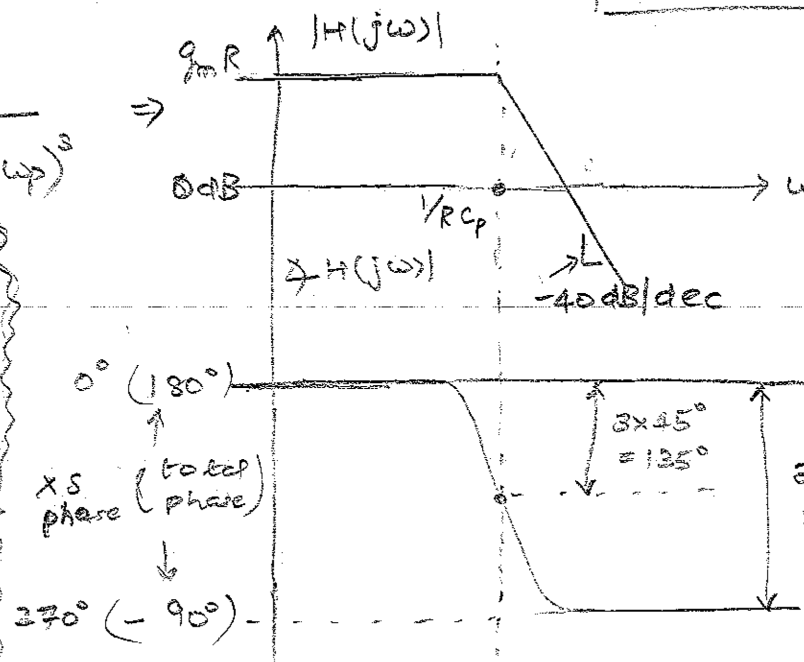
At DC, 3 inversions
$\Rightarrow$ 180° phase-shift around loop

Small signal response of each stage: $A_0(s) = \dfrac{A_0}{1 + s/\omega_p}$

$\Rightarrow$



$$H(s) = -\frac{A_0^3}{(1 + s/\omega_p)^3} \Rightarrow$$



For oscillation, total phase shift around loop = 360° at ω_x:

i.e., $\pi - 3\tan^{-1}\dfrac{\omega_x}{\omega_p} = 2\pi$ or $\tan^{-1}\dfrac{\omega_x}{\omega_p} = \dfrac{\pi}{3} \Rightarrow \boxed{\omega_x = \sqrt{3}\omega_p}$

At $\omega = \sqrt{3}\omega_p$, $|H(j\omega_x)| = \dfrac{A_0^3}{\left[1 + \left(\frac{\omega_x}{\omega_p}\right)^2\right]^{3/2}} = \dfrac{A_0^3}{8}$

For oscillation, we require $|H(j\omega_x)| = 1 \Rightarrow \boxed{A_0 = 2}$