(I) Preliminaries:

Any real modulated bandpass signal can be represented in one of three forms:

(a) Polar form: $s(t) = a(t)\cos(\omega_c t + \varphi(t))$

amplitude modulation → $a(t)$

phase-modulation (excess phase) → $\varphi(t)$

total phase → $\omega_c t + \varphi(t)$

Total phase $\theta(t) = \omega_c t + \varphi(t)$

Instantaneous frequency (in Hz) $f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}\frac{d\varphi_i(t)}{dt}$

(b) Cartesian / Rectangular form:

$$s(t) = s_I(t)\cos\omega_c t - s_Q(t)\sin\omega_c t$$

$s_I(t) = a(t)\cos\varphi(t)$ (In-phase component)

$s_Q(t) = a(t)\sin\varphi(t)$ (quadrature component)

(c) Complex envelope (or analytic signal) representation

$$s(t) = Re\left[\tilde{s}(t)e^{j\omega_c t}\right]$$

$$\tilde{s}(t) = s_I(t) + j s_Q(t) \leftarrow \text{lowpass complex envelope (or analytic signal)}$$

$$= a(t)e^{j\varphi(t)}$$

- Note that $s_I(t)$ and $s_Q(t)$ are real signals

  ⇒ Their Fourier transforms are conjugate symmetric
  (i.e magnitude is an even function
  phase is an odd function)

  $\therefore s_I(t) \rightleftharpoons S_I(f)$ and $S_I(f) = S_I^*(-f)$

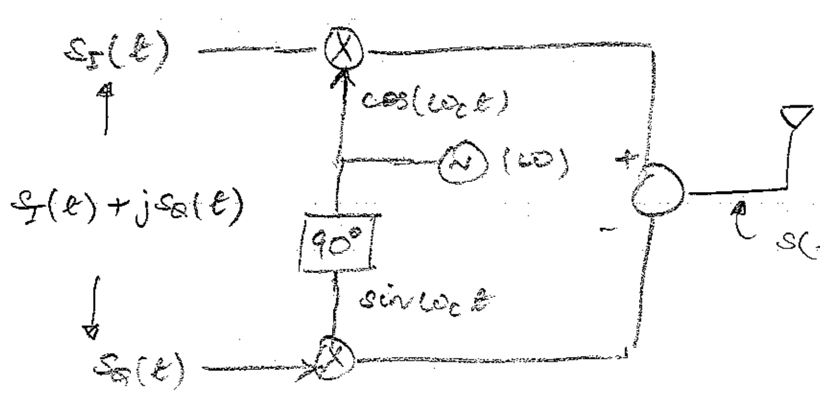
- Although signals inside a radio are real voltages or currents, pairs of signals taken together can be considered "complex" if satisfy the addition and multiplication rules of complex numbers.

ie. $c_1 = a + jb$ & $c_2 = p + jq$

$\Rightarrow c_1 + c_2 = (a+p) + j(b+q)$

and $c_1 c_2 = (ap - bq) + j(bp - aq)$

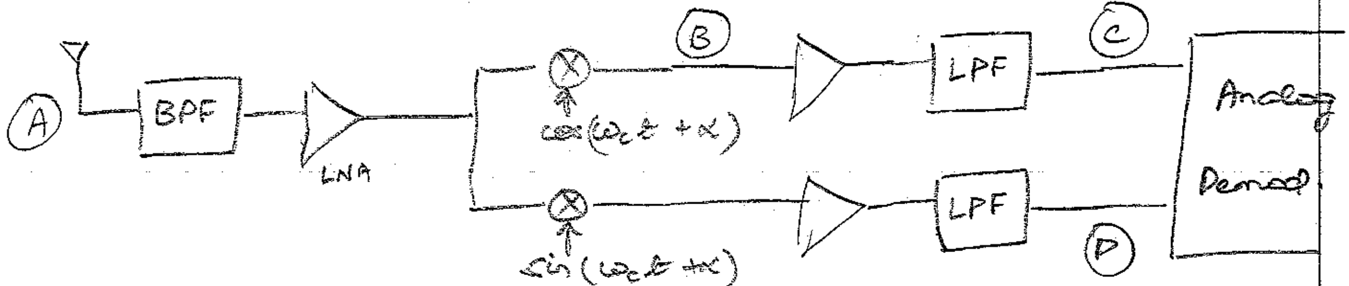
Simple direct conversion transmitter:



$s(t) = s_I(t)\cos\omega_c t - s_Q(t)\sin\omega_c t$

$= Re\left[\tilde{s}(t)e^{j\omega_c t}\right]$

II Simple Direct Conversion Receiver:



Assume $\alpha = 0$ and trace signals through the receiver:

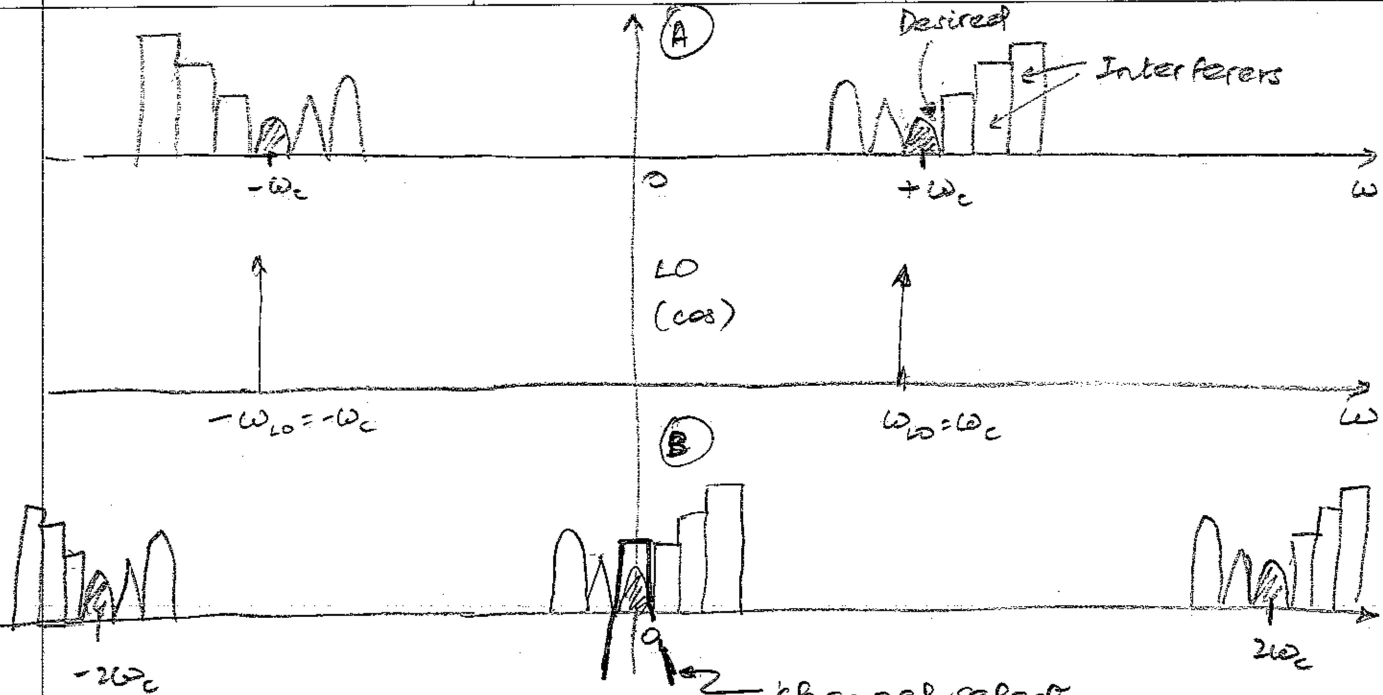
- Rx. signal (A) : $g(t) = a(t)\cos(\omega_c t + \varphi(t))$
- At (B) : $g(t)\cos(\omega_c t + \alpha) = \frac{1}{2}a(t)\left[\cos(\phi(t)+\alpha) + \cos(2\omega_c t + \phi(t) + \alpha)\right]$
- At (C) : $\frac{1}{2}a(t)\cos\{\varphi(t)\}$ $(\alpha = 0)$

Similarly, at (D) : $-\frac{1}{2}a(t)\sin\{\varphi(t)\}$

Simple demodulation algorithm:

$\Rightarrow \sqrt{C^2 + D^2} = \frac{1}{2}a(t)$

$-\text{atan2}\left(\frac{D}{C}\right) = \varphi(t)$ (4-quadrant arc-tangent)



channel select filter (highly selective)

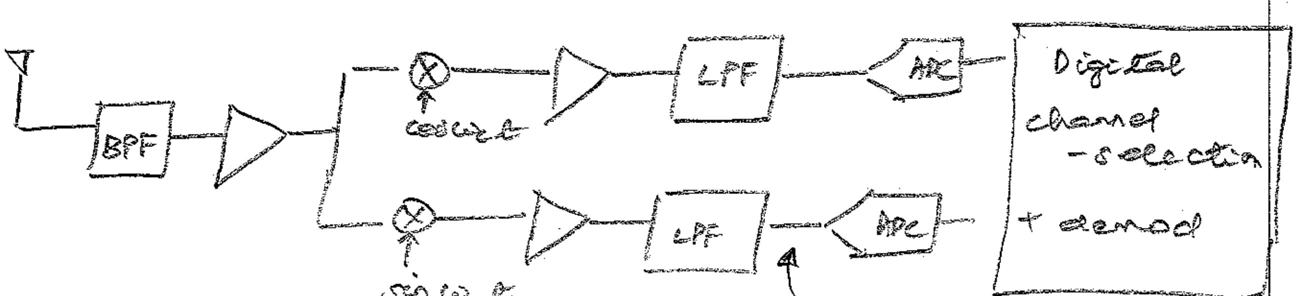
Note: The spectra are not quite accurate here because this cartoon shows (B) to have an unsymmetrical spectrum about DC although it is a real signal.

- Advantages: 1) No image problem

  2) Easy prefiltering since components exist only about DC & $2\omega_c$

Disadvantage: Highly selective analog channel-select filter required ← problematic on-chip

Alternative: Digitize signals at (C) and (D) & then use a digital channel-select filter



ADC has to digitize desired signal + large blockers ←

⇒ • High dynamic range required for ADC.

• Also, need anti-aliasing filter for ADC

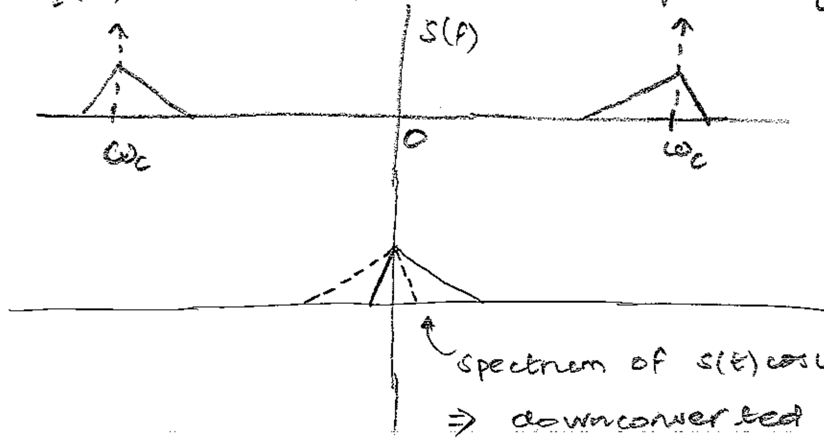
➡ There is an <u>inherent trade-off</u> here:

high-order filter ⇒ low DR ADC

low-order filter ⇒ high DR ADC (In this case, the filter must provide adequate anti-aliasing for the ADC)

<u>Why do we need I and Q in direct conversion</u> ?

(i) $s_I(t)$ and $s_Q(t)$ can be independently modulated data



(Input signal magnitude symmetric about DC, but not about $\pm\omega_c$)

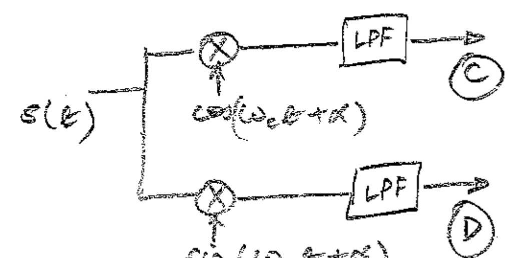
spectrum of $s(t)\cos\omega_c t$

⇒ downconverted signal folds on itself about DC

(ii) RF-LO is not generally synchronized to carrier ($\alpha \neq 0$)

Consider <u>AM only</u> i.e $\phi(t) = 0$

⇒ $s(t) = a(t)\cos\omega_c t$



C $\frac{1}{2}a(t)\cos\alpha \longrightarrow = 0$ for $\alpha = 90°$

D $-\frac{1}{2}a(t)\sin\alpha \longrightarrow = 0$ for $\alpha = 0°$

⇒ Suppose we had I-channel only, the output at C = 0 for $\alpha = 90°$ ⇒ signal lost completely

⇒ With I <u>and</u> Q, we have non-zero output at C or D or both.

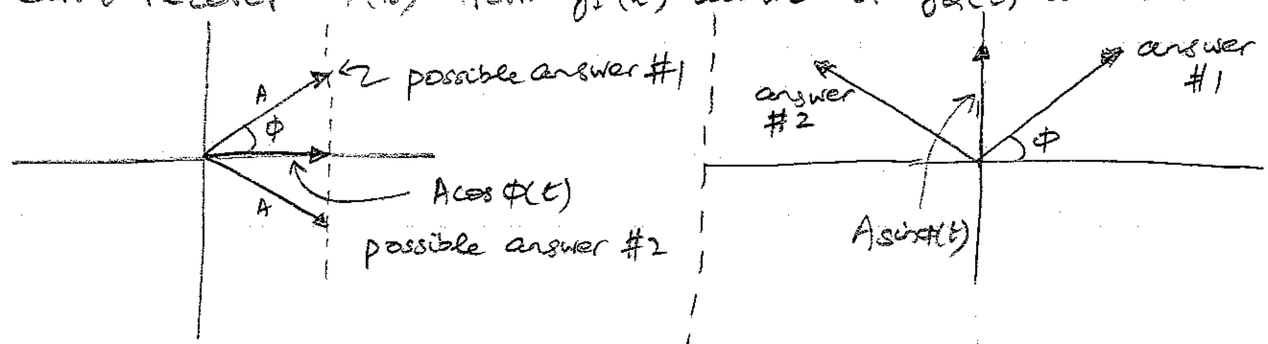
(iii) Consider PM or FM signal with perfectly synchronized LO: ($\alpha = 0$)

$s(t) = A\cos(\omega_c t + \varphi(t))$

$\Rightarrow g_I(t) = A\cos\varphi(t)$

$g_Q(t) = A\sin\varphi(t)$

Can't recover $\varphi(t)$ from $g_I(t)$ alone or $g_Q(t)$ alone:



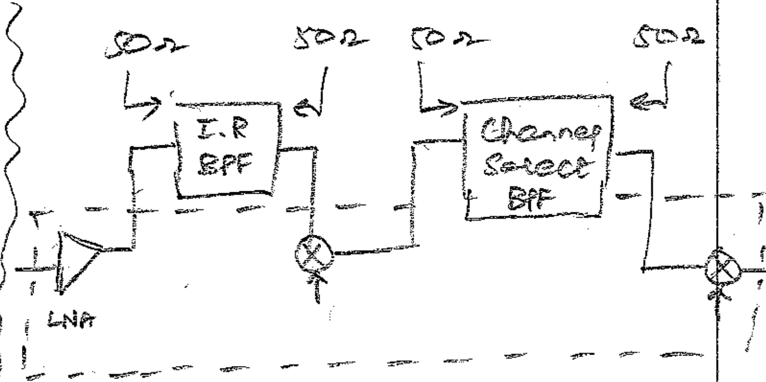
Knowlegde of $g_I(t)$ only. | Knowledge of $g_Q(t)$ only

$\Rightarrow$ If $\cos\varphi(t)$ and $\sin\varphi(t)$ are both known, we can determine $\varphi(t)$ uniquely ($\equiv$ four-quadrant arc-tangent)

DCR Advantages

- Highly integratable
  → no off-chip filters except RF BPF
- No off-chip image reject filter ⇒ don't need to drive 50Ω
- No need for on-chip channel select BPF

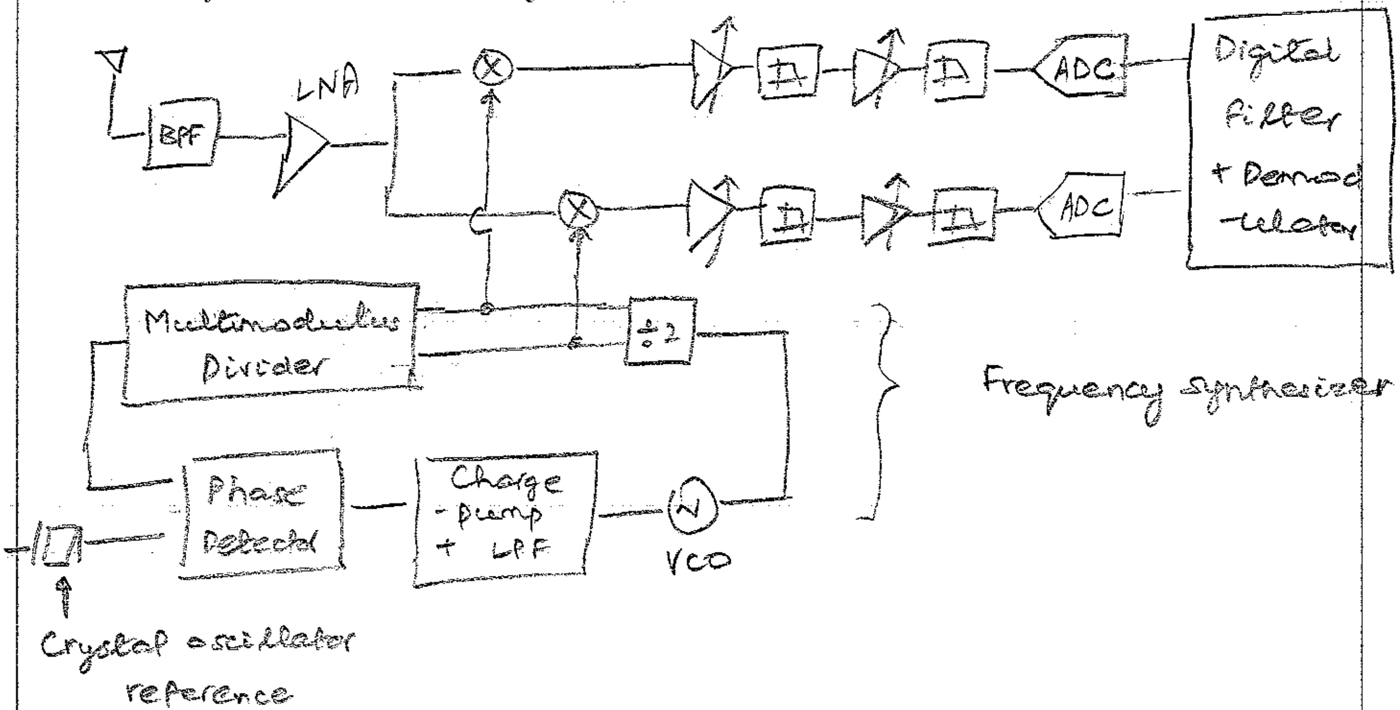
⇒ LNA and mixer need not drive 50Ω.



Super-Heterodyne architecture

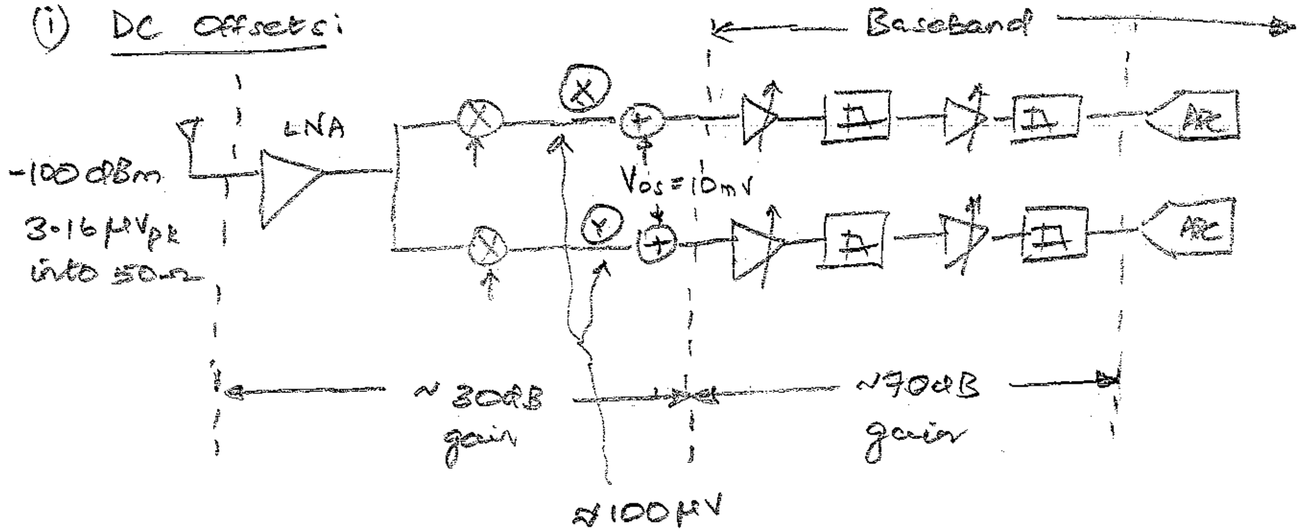
- Low-order LPF + high DR ADC
  ⇒ digital channel-select filter + demodulation
  ⇒ multi-standard radios.
- Relaxed I-Q matching requirements

- Only one frequency synthesis PLL required:

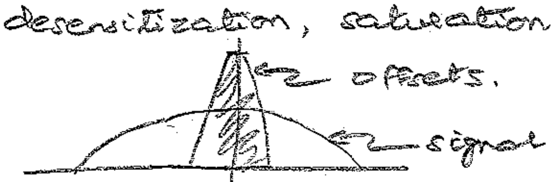


Problems with Direct Conversion receivers

(i) DC Offsets:



- At X and Y, signal of 100 μV amplitude competes with offset of 10 mV amplitude
- Baseband circuits can have large gain (~ 70 dB)
- Small signal + large undesired offset at input to large gain strip ⇒ desensitization, saturation
- DC offsets can be time varying ⇒

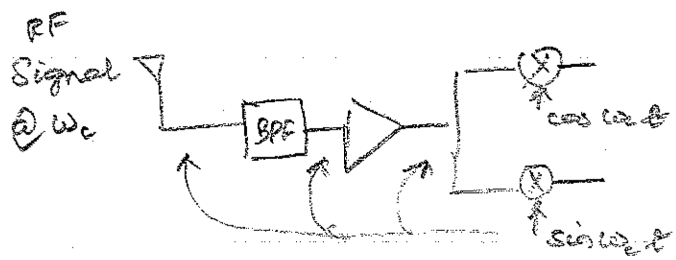


(ii) <u>1/f noise</u> from mixer & baseband circuits

→ same effect as time varying DC offsets.

→ Mixer 1/f noise is main bottleneck in CMOS DCR's.

(iii) <u>LO self-mixing</u>:



Simple model

- Suppose mixer has 60dB LO-to-RF port isolation

⇒ Leakage LO component @ RF port = $\varepsilon A\cos\omega_c t$

Self-mixing ⇒ Downconverted signal

$$= A\cos\omega_c t \cdot \varepsilon A\cos\omega_c t$$

$$= \underbrace{\frac{\varepsilon A^2}{2}} + \frac{\varepsilon A^2}{2}\cos 2\omega_c t$$

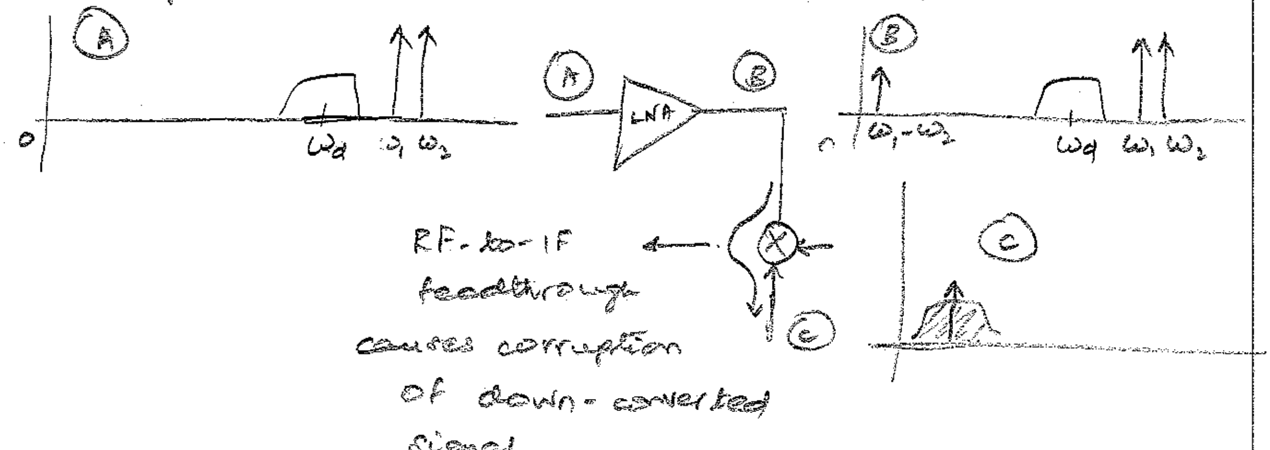
$\varepsilon = \frac{1}{1000}$, $A = 1V$ ⇒ LO-self mixing component $= 500\mu V$

→ competes with $100\mu V$ RF signal

(iv) <u>LO pulling</u>: Suppose RF signal is large, then imperfect RF-to-LO port isolation causes "LO pulling", i.e., modulation on RF signal modulates LO.

(v) <u>Even order distortion</u>:

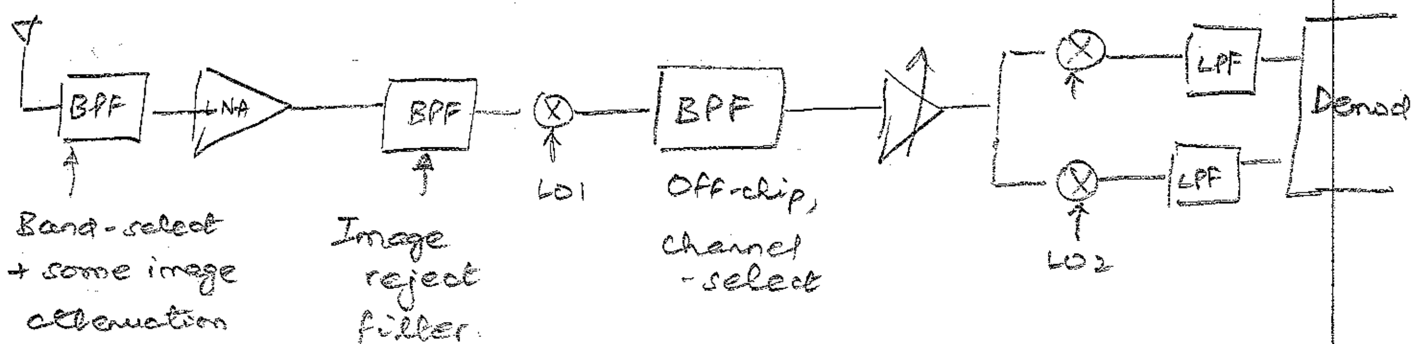
- LNA produces even order distortion components;



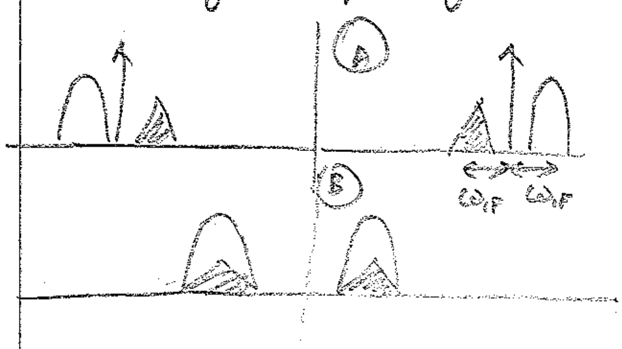
- Even order distortion in mixer's RF port has the same effect as above
- Even-order distortion in baseband circuits important.
- Even-order intermodulation specified by IIP2 (similar to IIP3).
- Differential circuits improve IIP2, but higher power dissipation

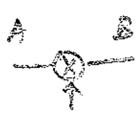
(vi) I-Q mismatch → causes higher BER.

(III) Single-IF heterodyne receiver:



- Originally developed because high gain and high selectivity were difficult to achieve at high frequencies.
- Downconvert to IF, where high gain & high selectivity are easier.
- No DC problems.
- Image frequency is a big issue:





- Image can be much larger than signal (>40dB).

Solutions to image problem:

(i) Filtering: Filter image out before mixing

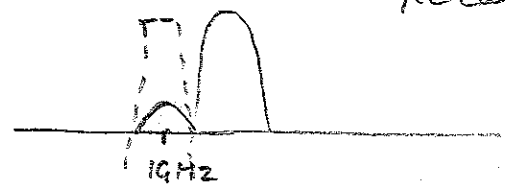
Consider an extreme example:

Recall $Q \simeq \frac{f_c}{\Delta f}$

$f_{RF} = 1GHz$

channel BW = 200kHz

$f_{IF} = 100kHz$



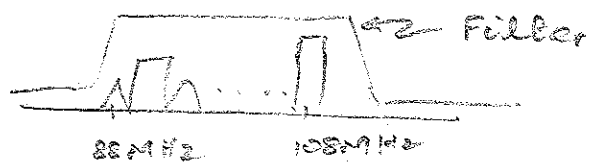
To get 40dB image attenuation, we need a $Q \simeq 40{,}000$ with a second order BPF.

- impractical on-chip or off-chip (more so because it must be tunable!)

(ii) Choose IF carefully:

Example: Broadcast FM:

Typical $f_{IF} = 10.7MHz$



Worst-case f_{image} for high-side LO injection

$= 88MHz + 2 \times 10.7MHz = 109.4MHz$

Which is outside the FM frequency band ⇒ image channel doesn't exist.

Note: The 109.4MHz falls in the "Aeronautical Radionavigation" band. Spectral emissions in this band are very tightly regulated.

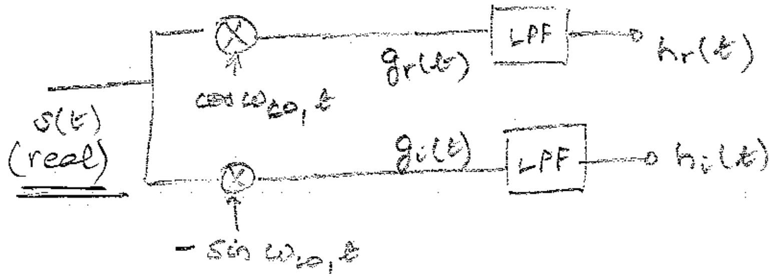
(iii) Choose a high-IF ⇒ Large separation between signal & image ⇒ highly selective filter may not be required for image rejection

- Band select filter may provide the required image rejection

(iv) Use signal cancellation (image-reject architectures)

(IV) Complex signal representation in receivers:

Consider a 'quadrature' downconverting front end:



$$s(t) = \mathrm{Re}\left[\tilde{s}(t)e^{j\omega_c t}\right] = \frac{\tilde{s}(t)e^{j\omega_d t} + \tilde{s}^*(t)e^{-j\omega_d t}}{2}$$

$g_r(t) = s(t)\cos\omega_{LO_1}t$ and $g_i(t) = -s(t)\sin\omega_{LO_1}t$

$$\Rightarrow \quad g_r(t) + j\,g_i(t) = s(t)\left[\cos\omega_{LO_1}t - j\sin\omega_{LO_1}t\right]$$

$$\Rightarrow \quad g(t) = s(t)e^{-j\omega_{LO_1}t}$$

- We can conceptually think of the pair of real signals $\{g_r(t), g_i(t)\}$ as a complex signal.

$$\therefore \quad g(t) = \frac{\tilde{s}(t)e^{j\omega_d t} + \tilde{s}^*(t)e^{-j\omega_d t}}{2} \cdot e^{-j\omega_{LO_1}t} \qquad (\omega_{IF_1} = \omega_d - \omega_{LO_1})$$

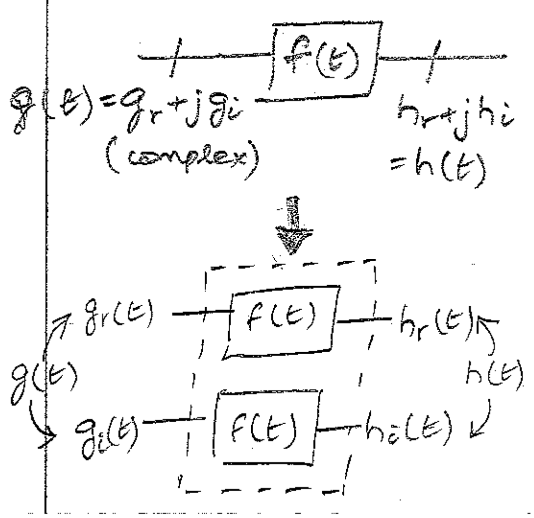
$$= \frac{1}{2}\tilde{s}(t)e^{j\omega_{IF_1}t} + \frac{1}{2}\tilde{s}^*(t)e^{-j(\omega_d+\omega_{LO_1})t}$$

subscript "d" denotes desired signal

desired complex IF term

complex high-frequency mixing terms

- Filter off the complex H.F terms using a filter with a real-valued impulse response. How do we do this?



Convolution: $h(t) = g(t) * f(t)$ ← real

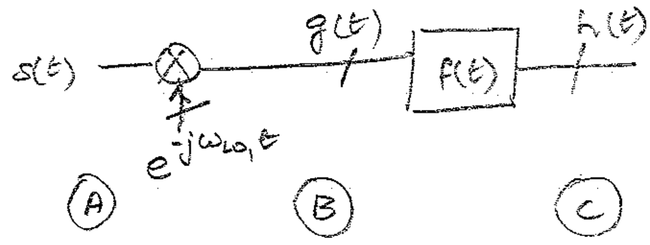
$$= \left[g_r(t) + j\,g_i(t)\right] * f(t)$$

$\Rightarrow$ by linearity,

$$h(t) = \underbrace{\{g_r(t) * f(t)\}}_{h_r(t)} + j\,\underbrace{\{g_i(t) * f(t)\}}_{h_i(t)}$$

$\Rightarrow$ filter real & imaginary parts separately.

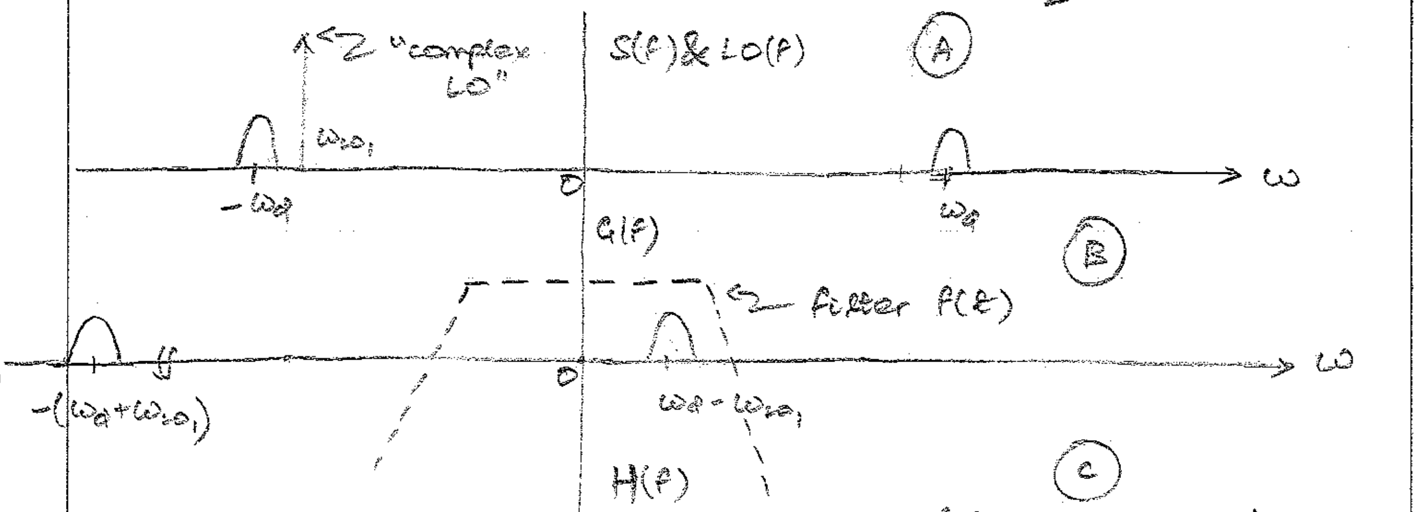
⇒ Quadrature down conversion in complex notation :



$s(t) = \mathrm{Re}\left[\tilde{s}(t)e^{j\omega_d t}\right]$

$g(t) = s(t)e^{-j\omega_{LO_1} t}$

$h(t) = \frac{1}{2}\tilde{s}(t)e^{j\omega_{IF_1} t}$



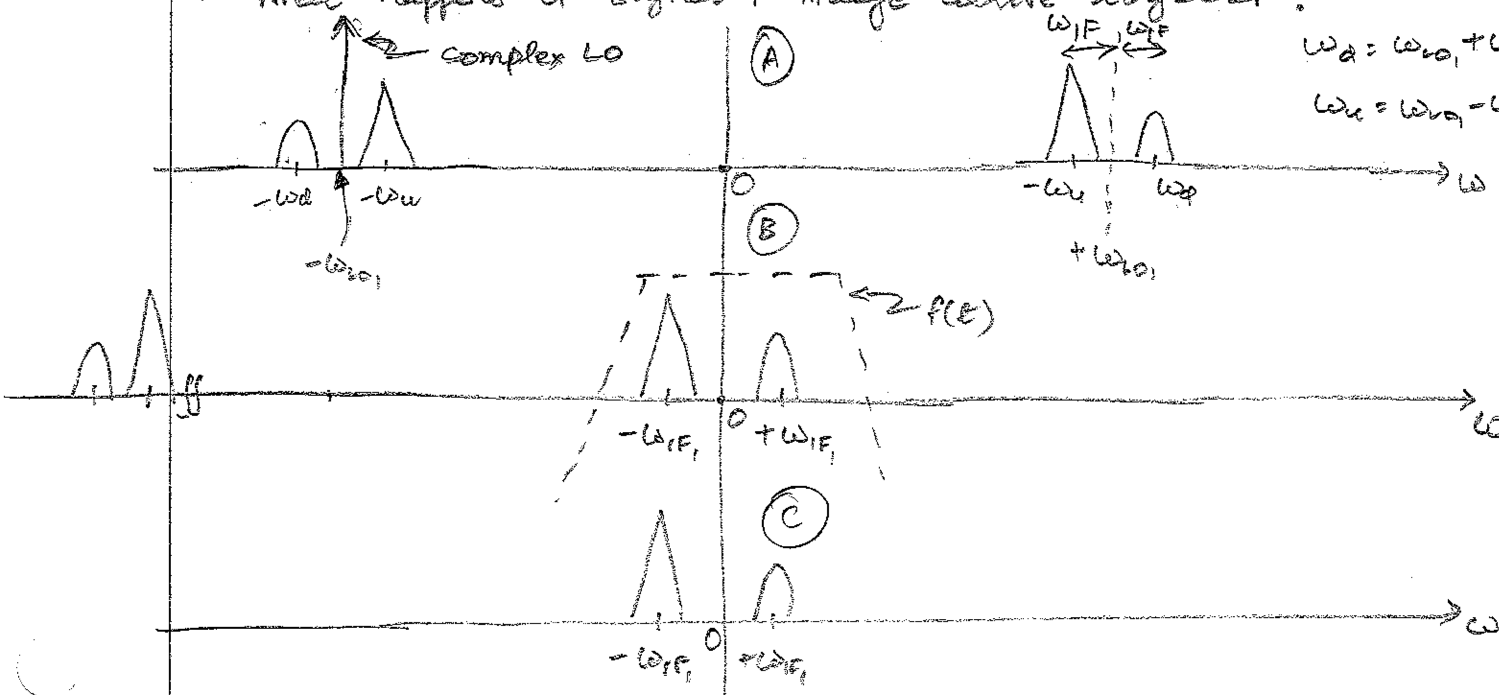
(Complex signal

⇒ spectrum unsymmetric about DC)

• What happens if signal + image arrive together?

$\omega_d = \omega_{LO_1} + \omega_{IF_1}$

$\omega_u = \omega_{LO_1} - \omega_{IF_1}$



⇒ Signal and image are down converted to <u>opposite</u> sides of DC!