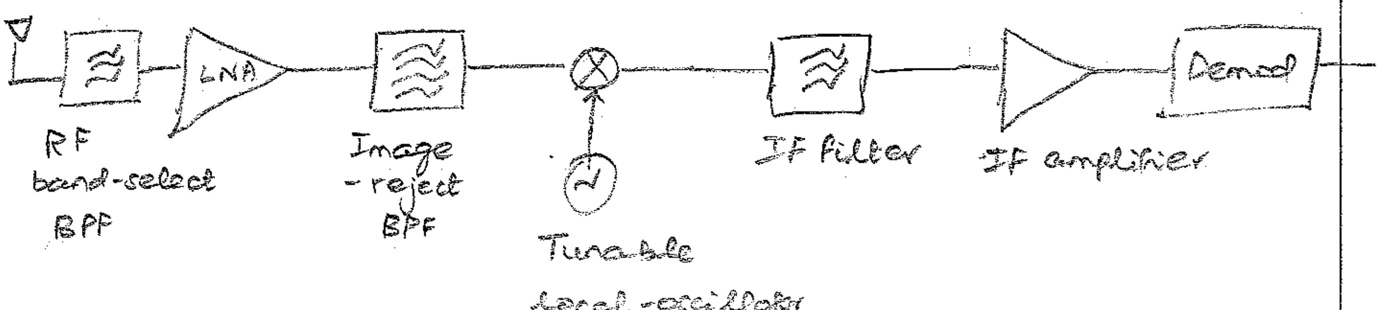
Mixers: Used for frequency translation

- upconversion of baseband signals to RF (or IF)
- downconversion of RF signals to IF or baseband

Two major types
- non-linear mixers.
- linear, periodically time-varying mixers (LPTV)

(1) Downconversion mixers (Receive mixers)

Consider a typical super-heterodyne receiver:



- Recall that the signal power at the antenna is very low ($-100\,dBm = 10^{-13}\,W$ or $3.16\,\mu V$ into $50\,\Omega$)
- Overall receiver gain must be $\sim 100\,dB$, distributed over RF, IF and baseband stages.
- Consider a simple mixer:

$$RF \longrightarrow \otimes \longrightarrow IF \quad (LO \uparrow)$$

RF: $x_{RF}(t) = A_{RF}\cos\omega_{RF}t$

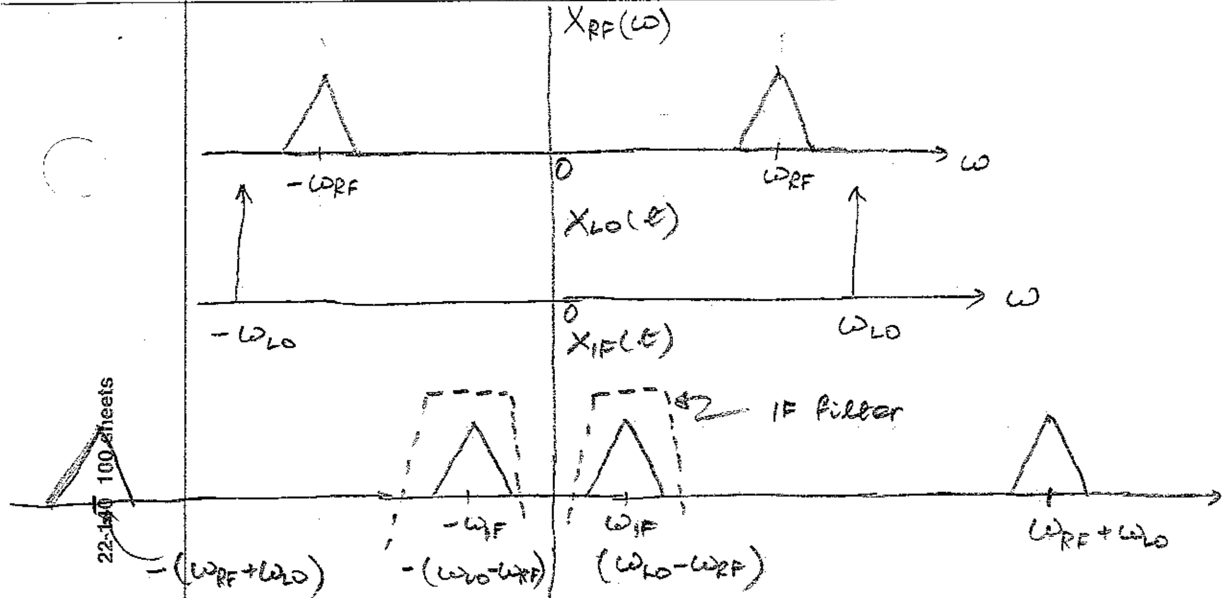
LO: $x_{LO}(t) = A_{LO}\cos\omega_{LO}t$

Mixer performs time-domain multiplication:

$$x_{IF}(t) = x_{RF}(t)\cdot x_{LO}(t) = A_{RF}A_{LO}\cos\omega_{RF}t\cos\omega_{LO}t$$

$$= \frac{1}{2}A_{RF}A_{LO}\Big[\underbrace{\cos(\omega_{RF}-\omega_{LO})t}_{\text{desired IF term}} + \underbrace{\cos(\omega_{RF}+\omega_{LO})t}_{\text{up-conversion mixing term.}}\Big]$$

- For more general modulated RF signals, recall that time-domain multiplication $\equiv$ frequency-domain convolution



(a) <u>conversion gain</u> : $G_c = \dfrac{\text{Amplitude of } \underline{\text{desired}} \text{ IF output}}{\text{Amplitude of RF input}}$

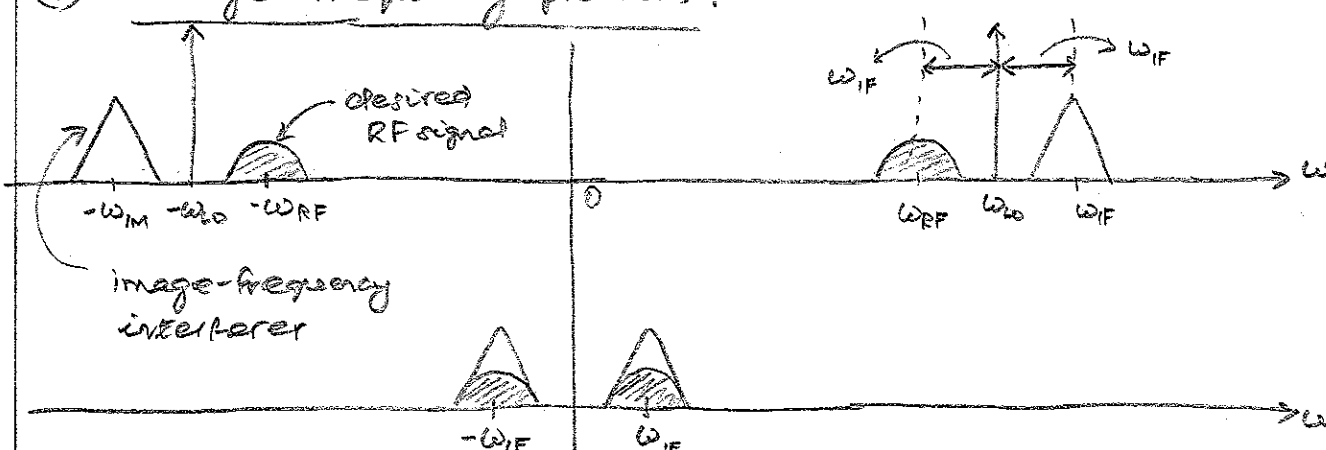
$$= \frac{\frac{1}{2} A_{RF} \cdot A_{LO}}{A_{RF}} = \frac{1}{2} A_{LO}$$

LO amplitude

Active mixers : $G_c > 1$ (generally)

Passive mixers : $G_c < 1$

(b) <u>Image-frequency problem</u> :



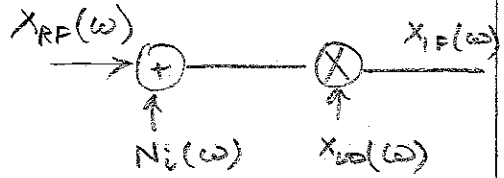
- No filtering of image prior to mixer :

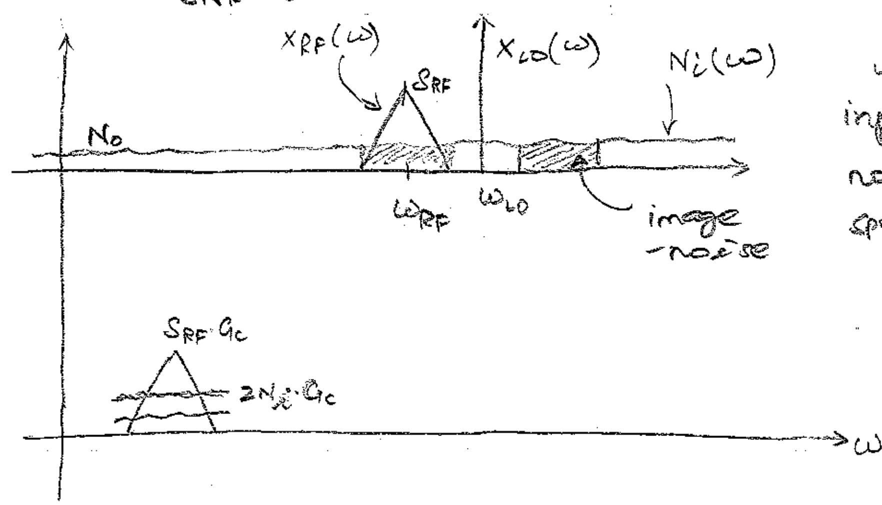
$\Rightarrow$ image folds down to the same IF $\rightarrow$ can't separate signal from image-frequency interferer after mixing.

- Solutions to the image problem:
  - image reject filter (super-heterodyne / high-IF receiver)
  - image-reject architectures (Hartley receiver, Weaver receiver)

(c) Noise figure of mixers:

$$NF = \frac{\text{SNR at RF input port}}{\text{SNR at IF output port}}$$

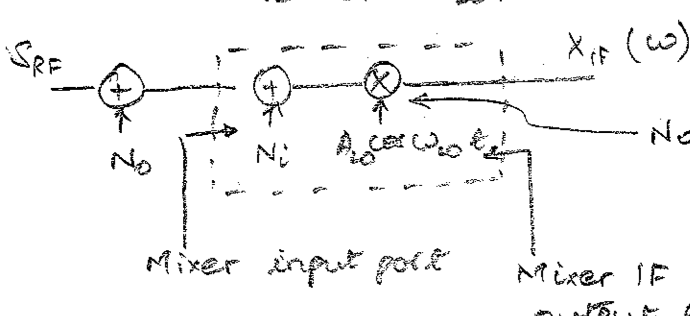




(Assume white input-referred noise spectrum)

(i) Single-sideband noise figure: (NF_{SSB})

Assume that a "useful" RF signal is present only on one side of ω_{LO}:



{ Both N_0 & N_i are broad-band white noise }

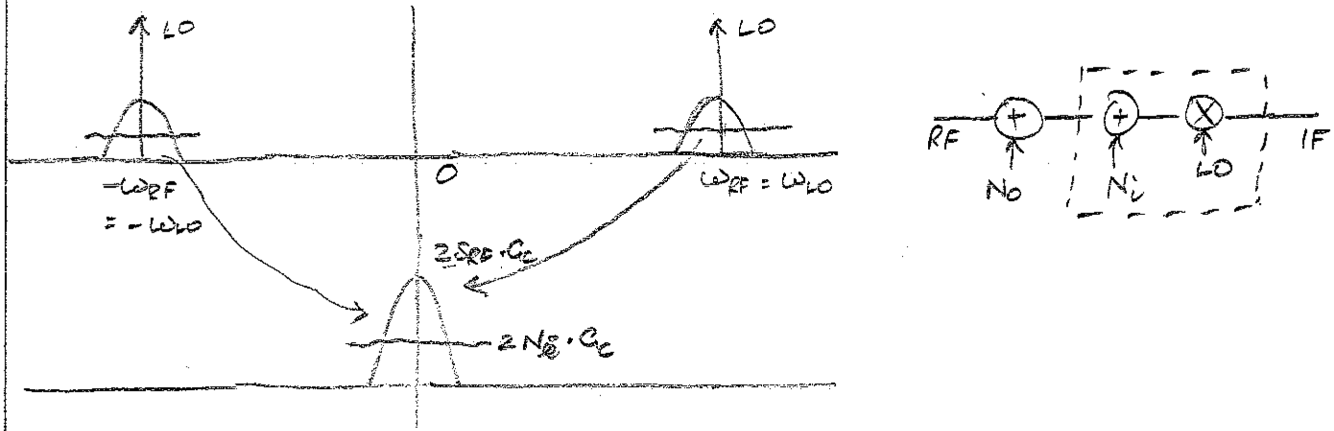
Input SNR $= \dfrac{S_{RF}}{N_0}$ & Output SNR $= \dfrac{S_{RF} \cdot G_c}{(2N_0 + 2N_i)\, G_c}$

$$\Rightarrow NF_{SSB} = \frac{2(N_0 + N_i)}{N_0} = 2 + \frac{2N_i}{N_0}$$

$\Rightarrow$ Even if mixer were <u>noiseless</u>, $NF_{SSB} = 3dB$!!

(ii) <u>Double-sideband noise figure (NF_{DSB})</u>

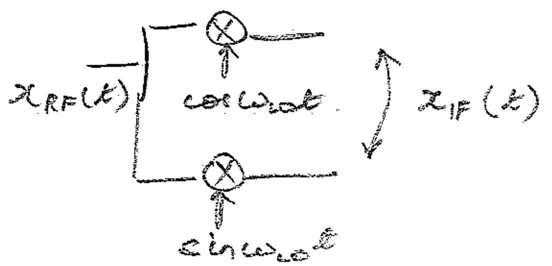
Now consider a direct-conversion receiver:



$$NF_{DSB} = \frac{SNR_{IN}}{SNR_{OUT}} = \frac{S_{RF}/N_0}{2S_{RF}\cdot G_c / (2N_0 + 2N_i)G_c} = \frac{N_0 + N_i}{N_0}$$

$\Rightarrow$ Now, if mixer were noiseless ($N_i = 0$): $NF_{DSB} = 0dB$.

Note that in general, we <u>cannot</u> use just one mixer for direct conversion. We need quadrature downconversion:

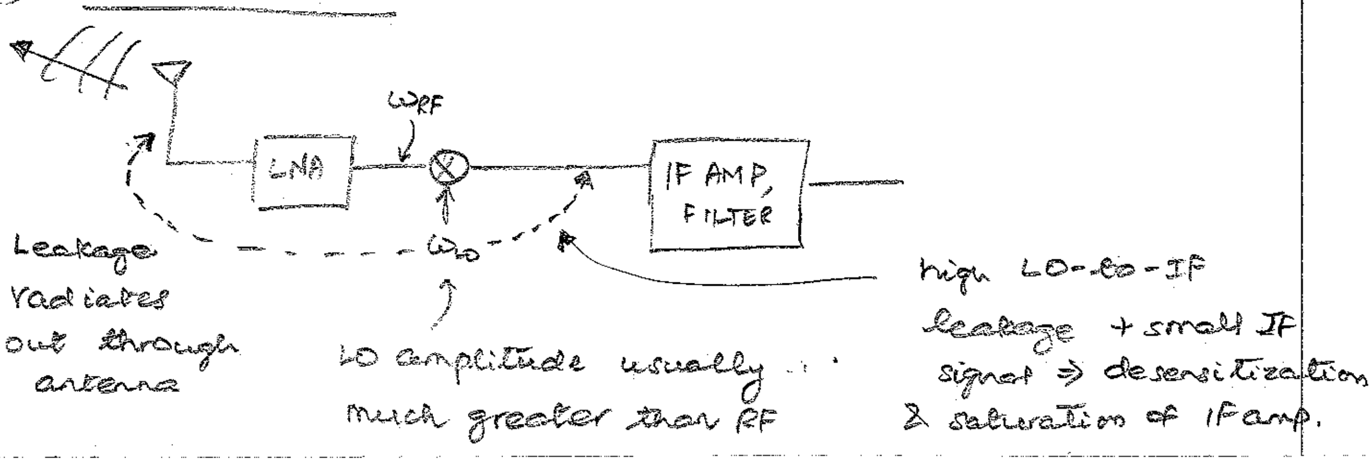


(More later!)

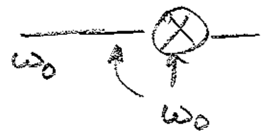
$\Rightarrow$ $NF_{DSB} = NF_{SSB} - 3dB$

Mixers are generally noisy: need to precede with LNA to keep overall receiver NF low.

(d) <u>Port Isolation:</u>



In a direct conversion receiver; $\omega_{LO} = \omega_{RF} = \omega_0$

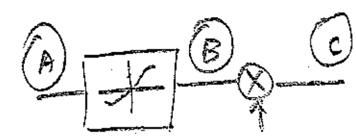


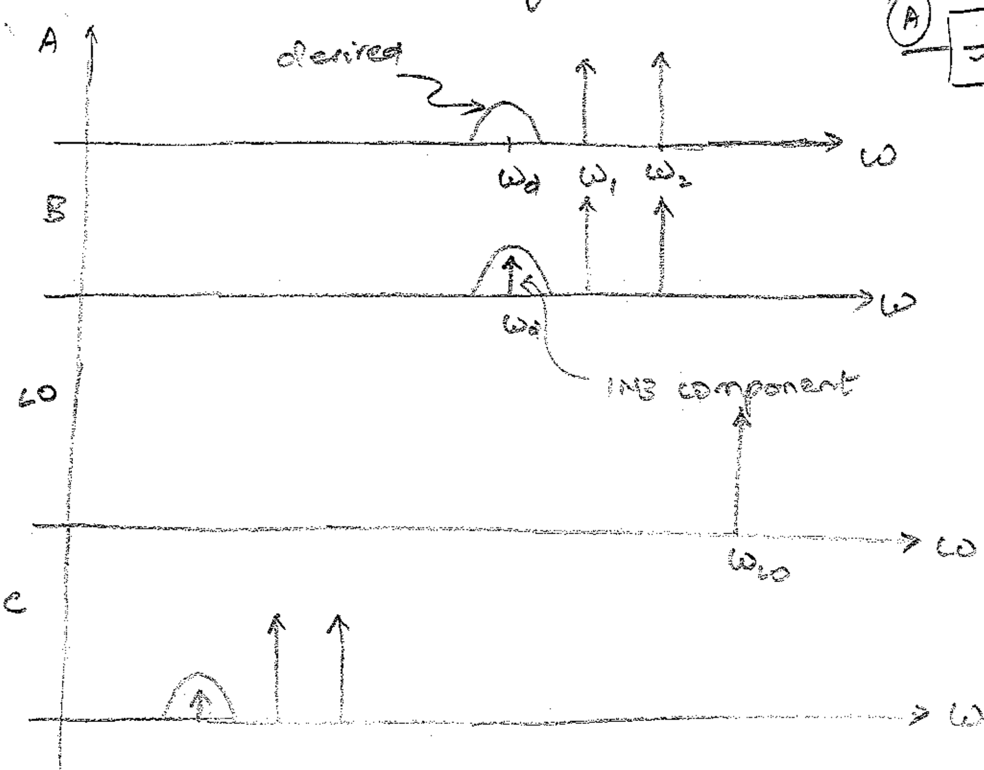
- We want RF signal <u>only</u> to be down converted to baseband
- If LO leaks to RF port and mixes with the LO, we get a low-frequency term that depends on the LO <u>only</u>. This can be much larger than the down converted RF.

→ called <u>self-mixing</u>.

(e) <u>Linearity & Spurious response</u>

(i) <u>RF path non-linearity:</u>





- Both signal & distortion component get downconverted
- Can define IIP3 for mixers similar to LNA's.

(ii) Spurs & effect of square-wave LO:

Example: Consider $\omega_{RF} = 900\,MHz$

$\omega_{IF} = 70\,MHz$

High-side injection: $\omega_{LO} = \omega_{RF} + \omega_{IF} = 970\,MHz$.

→ Suppose that the LO is a square wave; it will contain harmonics at $2\omega_{LO}, 3\omega_{LO} \ldots$ An interferer could mix with these LO harmonics.

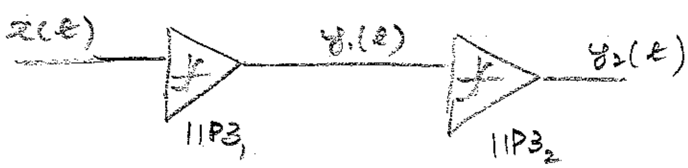
→ If an interferer were present at freq. such that

$2\omega_{LO} \pm \omega_x = \omega_{IF} = \omega_{RF} - \omega_{LO}$, then these would mix with the second LO harmonic down to ω_{IF}:

$\Rightarrow \quad \omega_x = 2\omega_{LO} \pm \omega_{IF} = 2010\,MHz$ or $1870\,MHz$.

⇒ In general we want only input frequencies at $|\omega_{LO} - \omega_{IF}|$ to be folded down to IF.

(f) Linearity of cascaded stages:



$$V_{IIP3} = \sqrt{\frac{4\alpha_1}{3\alpha_3}}$$

$y_1(t) = \alpha_1 x(t) + \alpha_3 x^3(t)$ (ignore 2nd order N.L)

$$y_2(t) = \beta_1 y_1(t) + \beta_3 y_1^3(t)$$

$$= \beta_1\left[\alpha_1 x(t) + \alpha_3 x^3(t)\right] + \beta_3\left[\alpha_1 x(t) + \alpha_3 x^3(t)\right]^3$$

$$= \alpha_1\beta_1 x(t) + \left[\alpha_3\beta_1 + \beta_3\alpha_1^3\right]x^3(t) + \ldots$$

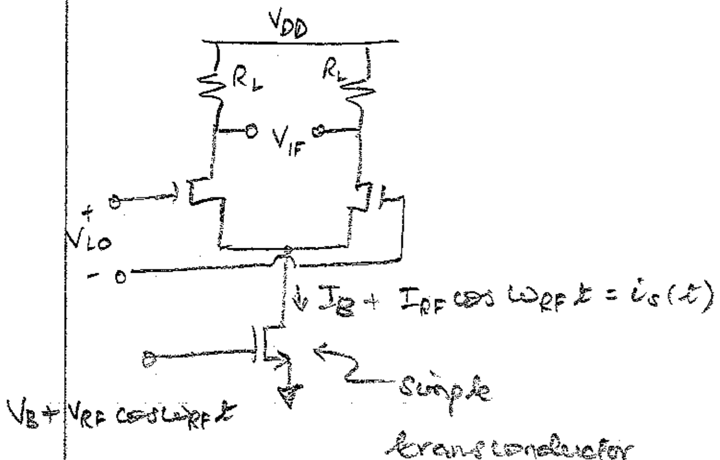
$$\Rightarrow \quad A_{IIP3} = \left[\frac{4\alpha_1\beta_1}{3(\alpha_3\beta_1 + \beta_3\alpha_1^3)}\right]^{1/2} = \left[\frac{1}{\frac{3\alpha_3}{4\alpha_1} + \left(\frac{3\beta_3}{4\beta_1}\right)\alpha_1^2}\right]^{1/2}$$

$$\Rightarrow \quad A_{IIP3} = \left[\frac{1}{\frac{1}{A_{IIP3,1}^2} + \frac{\alpha_1^2}{\underbrace{A_{IIP3,2}^2}}} \right]^{1/2}$$

If fundamental gain of preceding stage is large, we need large $A_{IIP3,2}$ for good linearity.

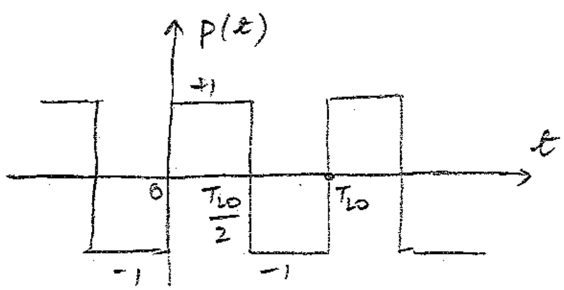
$\Rightarrow$ For good linearity, need more gain at back end
For good noise performance, need more gain in front end.

(2) Single-balanced mixer:



- Assume that V_{LO} switches the "tail" current $i_s(t)$ from one side to the other.
- The input voltage is converted to a current by a transconductor, which is a single NMOS here.

$\rightarrow$ $i_s(t) = I_B + g_m V_{RF}\cos\omega_{RF}t.$



$p(t)$ = switching function
$\rightarrow$ square wave with period T_{LO}
$\rightarrow$ expand into Fourier series:

$$p(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n\cos n\omega_{LO}t + b_n\sin n\omega_{LO}t\right] \quad \left(\omega_{LO} = \frac{2\pi}{T_{LO}}\right)$$

$$\frac{a_0}{2} = \text{DC average} = 0; \quad a_n = \frac{2}{T_{LO}}\int_0^{T_{LO}} p(t)\cos n\omega_{LO}t\,dt$$

$= 0$ for all n since $p(t)$ is an odd function

$$b_n = \frac{2}{T_{LO}}\int_0^{T_{LO}} p(t)\sin n\omega_{LO}t\,dt = 2\cdot\frac{2}{T_{LO}}\int_0^{T_{LO}/2}\sin n\omega_{LO}t\,dt$$

$$\Rightarrow b_n = \frac{4}{T_{LO}} \cdot \frac{1}{n\omega_{LO}} \Big[-\cos n\omega_{LO} t \Big]_0^{T_{LO}/2} = \frac{4}{n\pi}\Big[1 - \cos n\pi\Big]$$

$$\Rightarrow b_n = \begin{cases} 0 & \text{for } n = 2, 4, 6 \ldots \\ \frac{4}{n\pi} & \text{for } n = 1, 3, 5 \ldots \end{cases}$$

$$\Rightarrow p(t) = \frac{4}{\pi}\Big[\sin\omega_{LO} t + \frac{1}{3}\sin 3\omega_{LO} t + \frac{1}{5}\sin 5\omega_{LO} t + \cdots\Big]$$

$$\therefore V_{IF}(t) = \left(I_B + I_{RF}\cos\omega_{RF} t\right) \cdot \frac{4}{\pi}\Big[\sin\omega_{LO} t + \frac{1}{3}\sin 3\omega_{LO} t + \cdots\Big]$$

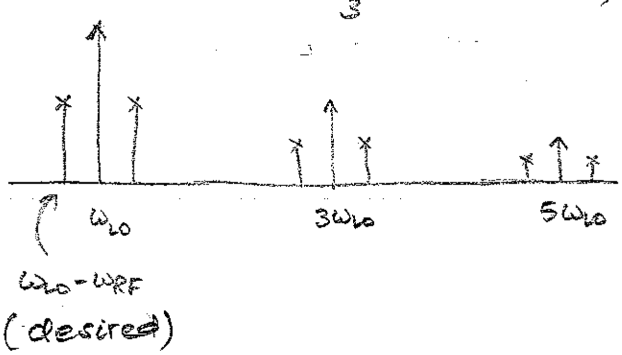
$$= I_B \cdot \frac{4}{\pi}\Big[\sin\omega_{LO} t + \frac{1}{3}\sin 3\omega_{LO} t + \cdots\Big] \leftarrow \text{LO + LO harmonics feedthrough}$$

$$+ \frac{4}{\pi} I_{RF}\cos\omega_{RF} t \cdot \Big[\sin\omega_{LO} t + \frac{1}{3}\sin 3\omega_{LO} t + \frac{1}{5}\sin 5\omega_{LO} t + \cdots\Big]$$

$$= (\text{LO feedthrough terms})$$

$$+ \frac{2 I_{RF}}{\pi}\Big[\sin(\omega_{LO} - \omega_{RF})t + \sin(\omega_{LO} + \omega_{RF})t + \frac{1}{3}\sin(3\omega_{LO} - \omega_{RF})t + \frac{1}{3}\sin(3\omega_{LO} - \omega_{RF})t + \cdots\Big]$$

(i)

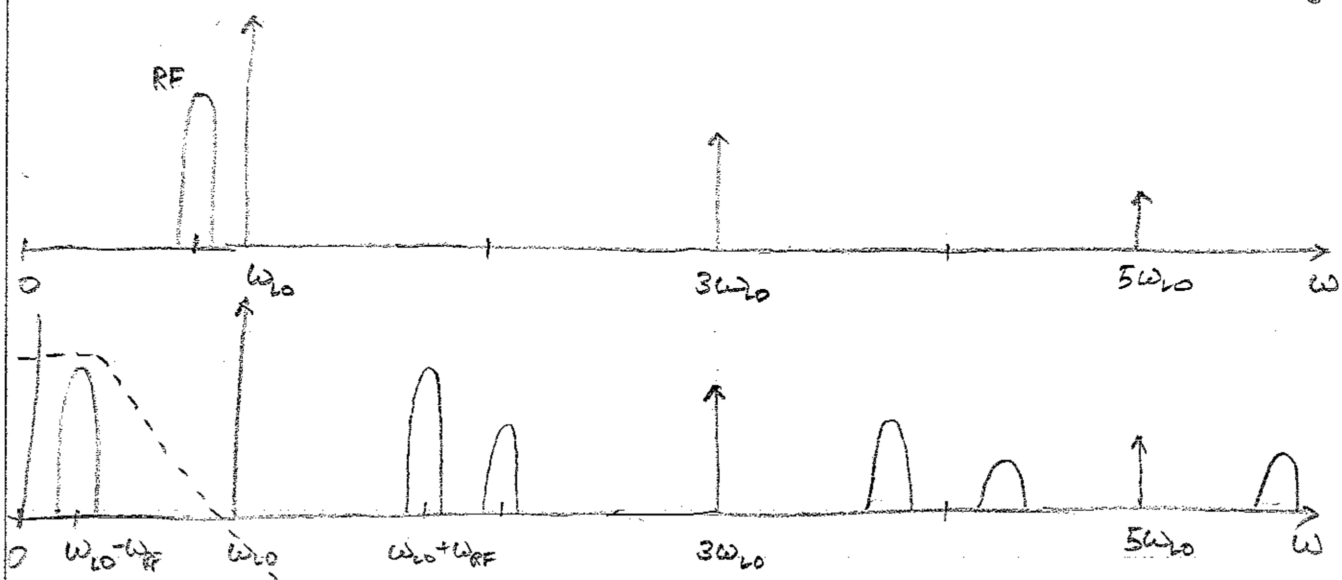


Note: This spectrum is for an upconversion mixer whose input $\omega_{RF} << \omega_{LO}$

→ poor isolation between LO & output ports can be problematic here.

In upconversion, because the desired output, mixing terms and LO feedthrough terms are spaced closely in frequency, filtering to isolate the desired term is difficult.

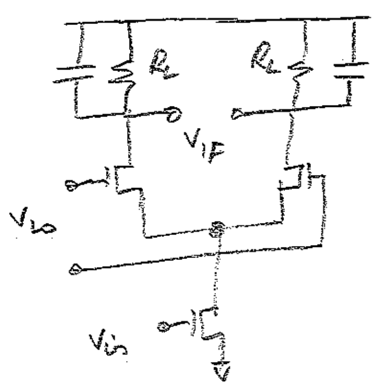
(ii) Downconversion : ω_{RF} and ω_{LO} close together in frequency.



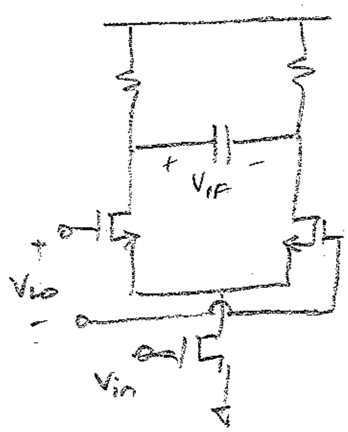
mixing terms + LO feedthrough far away from IF ⇒ easy filtering.

LO-to-IF port isolation not very problematic

⇒

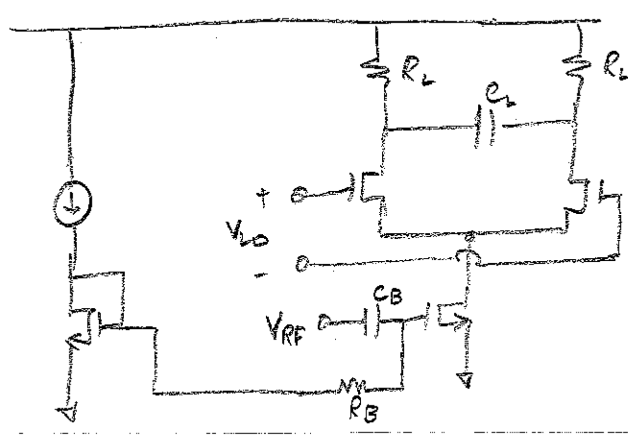


or



Conversion gain $G_C = \dfrac{\text{IF amplitude}}{\text{RF amplitude}} = \dfrac{2 g_m V_{RF}/\pi \cdot R_L}{V_{RF}}$

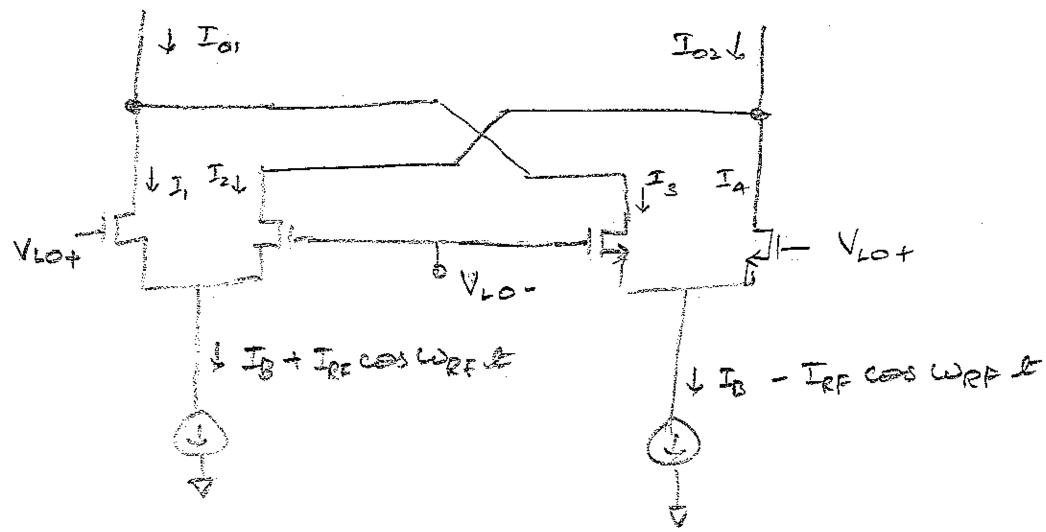
$$\Rightarrow \boxed{G_c = \frac{2}{\pi} g_m R_L}$$



Complete single-balanced mixer with biasing details

Double-balanced mixer:

- Use a differential (or balanced) RF input, along with two cross-connected single-balanced mixers to cancel LO feedthrough terms:



$$I_1 - I_2 = (I_B + I_{RF}\cos\omega_{RF}t)\,p(t)$$

$$I_3 - I_4 = -(I_B - I_{RF}\cos\omega_{RF}t)\,p(t)$$

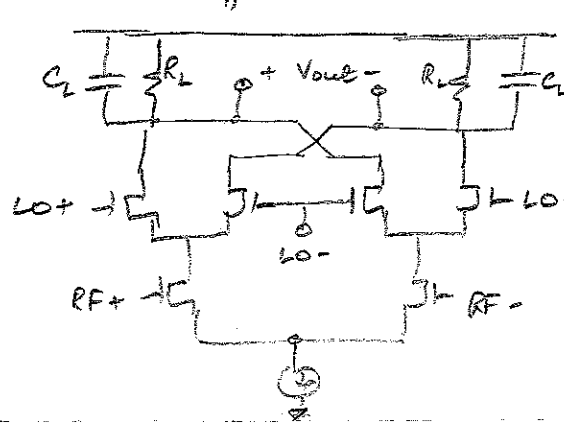
Differential output current $I_d = (I_1 + I_3) - (I_2 + I_4)$

$$= (I_1 - I_2) + (I_3 - I_4)$$

$$I_d = 2I_{RF}\cos\omega_{RF}t \cdot p(t)$$

$$= \frac{4I_{RF}}{\pi}\Big[\sin(\omega_{LO}-\omega_{RF})t + \sin(\omega_{LO}+\omega_{RF})t + \frac{1}{3}\sin(3\omega_{LO}-\omega_{RF})t + \frac{1}{3}\sin(3\omega_{LO}-\omega_{RF})t + \cdots\Big]$$

$$\Rightarrow\quad G_C = \frac{4g_mR_L}{\pi}$$



Gilbert cell mixer