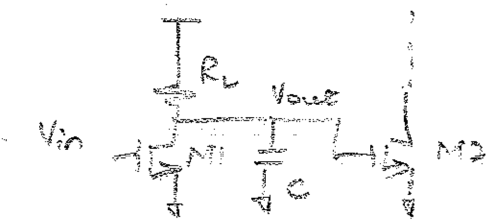
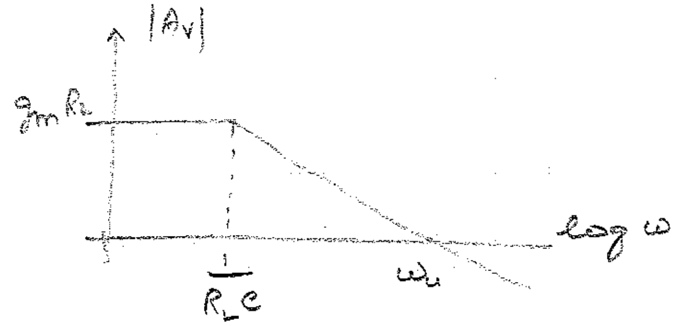
Lecture #10

Bandwidth Enhancement using zeros:

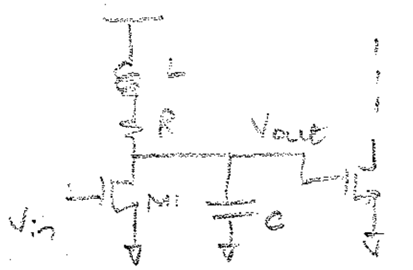
- Basic C-S stage:



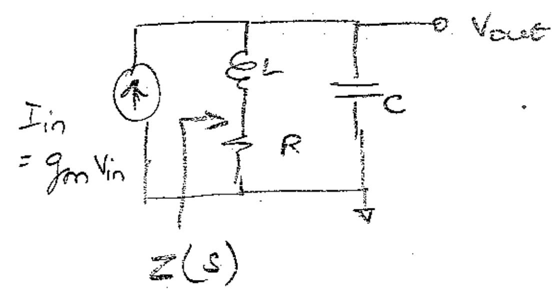


Basic idea: Use an inductor to introduce "peaking" into the frequency response to increase 3dB frequency.

(1) Shunt peaking:



⇒



- At low frequencies, inductor is a short and gain is a constant $g_m R$
- At very high frequencies, C presents a diminishing impedance, and gain falls off.
- At mid-band frequencies, L introduces a zero i.e presents an impedance component that increases with frequency & offsets the decreasing component presented by C.

More intuition: Recall that in L, current lags voltage. Here, L delays the flow of current to the L-R branch, which leaves more current to charge the C ⇒ faster rise time ⇒ more BW

(A) <u>Analysis</u>: $z(s) = (sL+R) \,\|\, \frac{1}{sC} = R\,\frac{sL/R + 1}{s^2LC + sRC + 1}$

- Since voltage gain $|A_v| = g_m |z(j\omega)|$, it is sufficient to study the frequency response of $z(s)$.
- Here: one zero $\omega_z = -R/L$

  & two poles, possibly complex conjugate.
- Note that when $L \to 0$, this boils down to the "un-peaked" C-S amp: $z_0(s) = R\,\frac{1}{1+s\underbrace{RC}} = R\,\frac{1}{1+s/\omega_1}$

- Define the following variables:

(i) uncompensated 3dB BW : $\omega_1 = \frac{1}{RC}$

(ii) "New" time constant $\tau = L/R$

(iii) $m = \frac{\text{Uncompensated 3dB time constant}}{\text{New time constant}} = \frac{RC}{L/R}$

$$\Rightarrow \quad \frac{z(s)}{R} = \frac{s\tau + 1}{s^2\tau^2 m + s\tau m + 1}$$

- Find the <u>compensated</u> 3dB BW:

At $\omega = \omega_{3dB}$, $\left|\frac{Z(j\omega)}{R}\right| = \frac{1}{\sqrt{2}}$

$$\Rightarrow \left|\frac{1 + j\omega\tau}{(1-\omega^2\tau^2 m) + j\omega\tau m}\right| = \frac{1}{\sqrt{2}}$$

or
$$\frac{1+\omega^2\tau^2}{(1-\omega^2\tau^2 m)^2 + (\omega^2\tau^2 m^2)} = \frac{1}{2}$$

Define $x = \omega^2\tau^2$:

$$\Rightarrow \frac{x+1}{(1-mx)^2 + m^2x} = \frac{1}{2}$$

$$\Rightarrow \quad m^2x^2 + (m^2 - 2m - 2)x - 1 = 0$$

$$\Rightarrow \quad x = \frac{(2m+2-m^2) + \left[(2m+2-m^2)^2 + 4m^2\right]^{1/2}}{2m^2}$$

or $$x = \frac{1}{m^2}\left\{\left(m+1-\frac{m^2}{2}\right) + \left[\left(m+1-\frac{m^2}{2}\right)^2 + m^2\right]^{1/2}\right\}$$

But $$x = (\omega_{3dB}\cdot\tau)^2 = \left(\frac{\omega_{3dB}}{\omega_1}\cdot\omega_1\tau\right)^2 = \left(\frac{\omega_{3dB}}{\omega_1}\right)^2\cdot\frac{1}{m^2}$$

$$\therefore \quad \boxed{\frac{\omega_{3dB}}{\omega_1} = \left[\left(m+1-\frac{m^2}{2}\right) + \left\{\left(m+1-\frac{m^2}{2}\right)^2 + m^2\right\}^{1/2}\right]^{1/2}} \quad (BWXF)$$

- This gives us the ratio of the new 3dB BW to the uncompensated BW in terms of "m".

(a) Optimization of "m" for max. ω_{3dB}/ω_1 (see plot of BWXF vs m)

$$\Rightarrow \boxed{\begin{array}{l}\text{Max. BW extension occurs when } m \simeq 1.4. \\ \text{and } \dfrac{\omega_{3dB}}{\omega_1} = 1.85 \leftarrow \text{BW extension factor (BWXF)}\end{array}}$$

But, is this a good value for m? To investigate normalize the frequency axis and plot $|Z(j\omega)|$

→ Define $s_1 = \dfrac{s}{\omega_1} \Rightarrow s = s_1\omega_1$

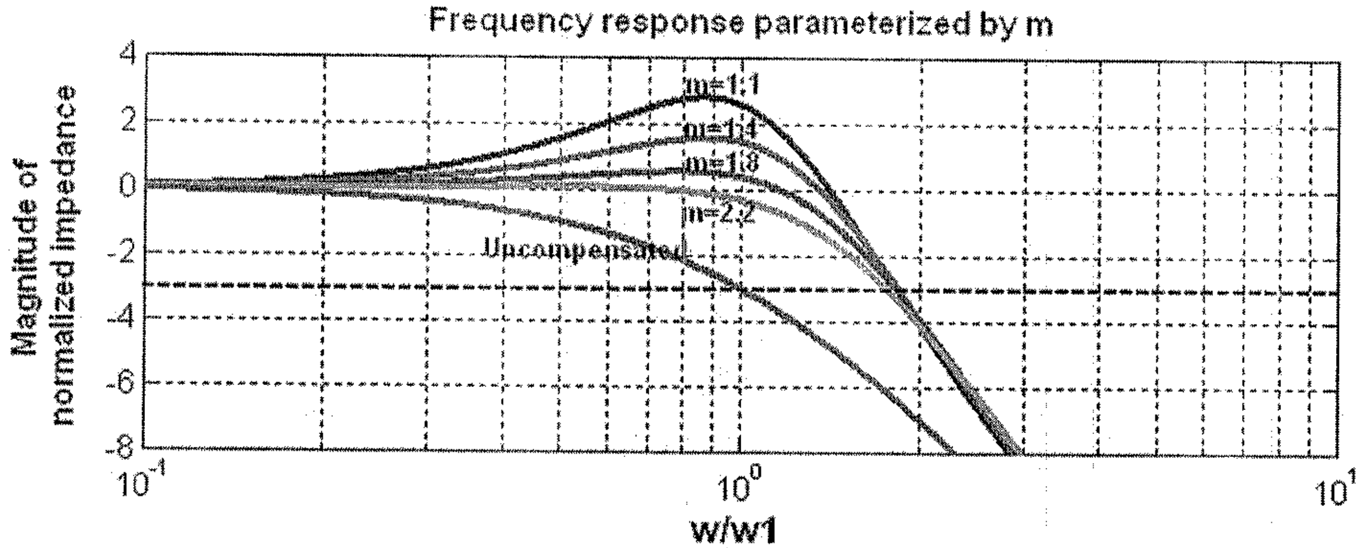
$$\therefore \quad \frac{Z(s)}{R} = \frac{s_1\omega_1\tau + 1}{s_1^2(\omega_1\tau)^2 m + s_1(\omega_1\tau)m + 1}$$

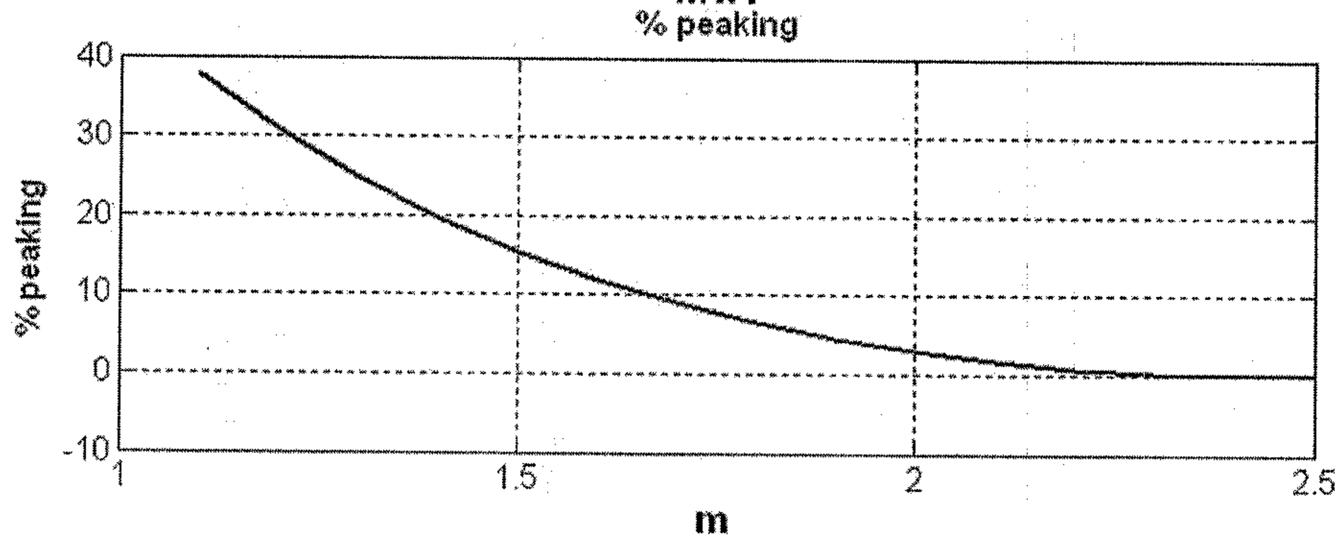
But $m = \dfrac{RC}{L/R} = \dfrac{1}{\omega_1\tau}$ $\Rightarrow \omega_1\tau = 1/m$

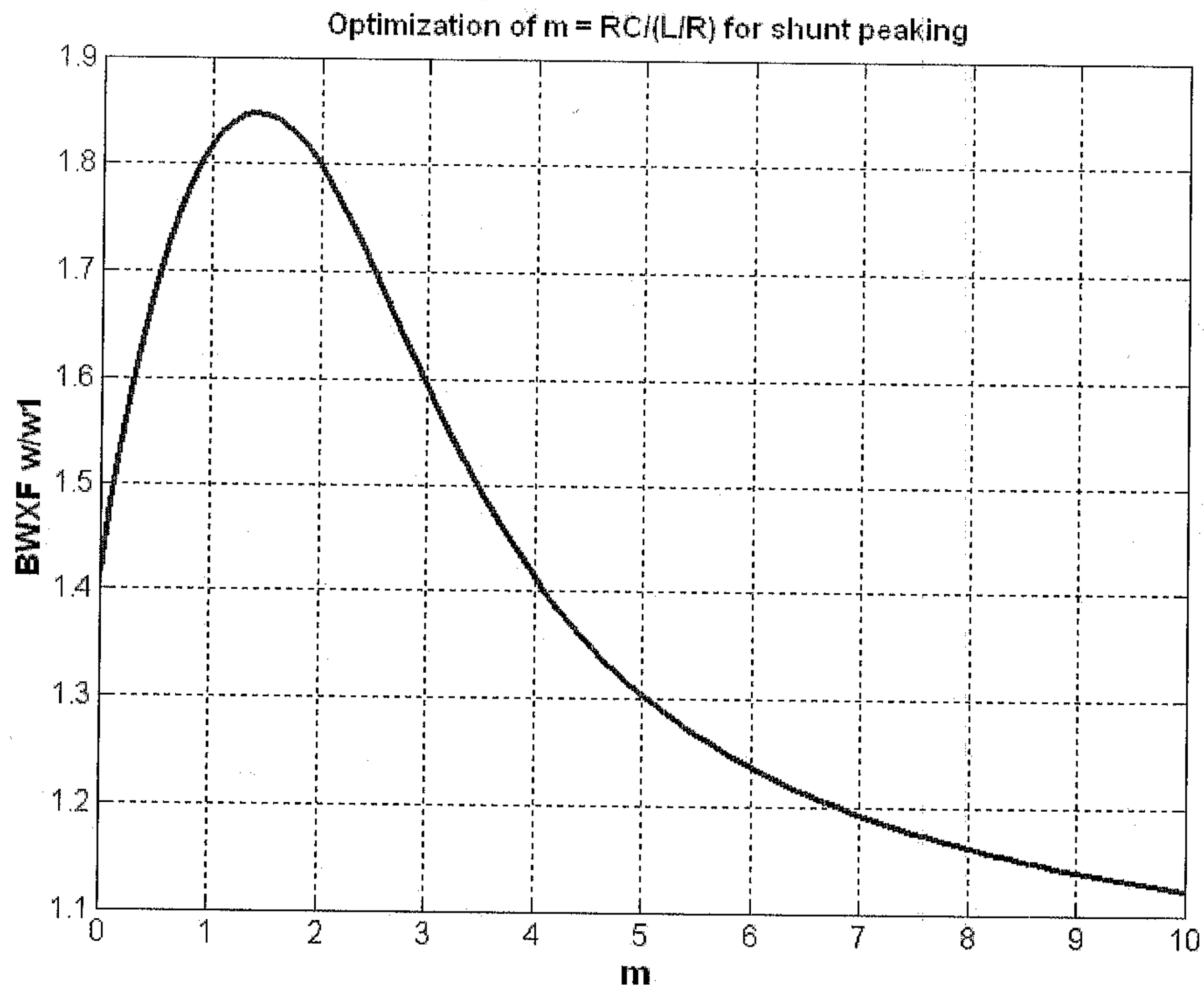
$$\Rightarrow \quad \frac{Z(s_1)}{R} = \frac{s_1/m + 1}{s_1^2/m + s_1 + 1}$$

Now define $s = j\omega$ so that $s_1 = j\dfrac{\omega}{\omega_1} = j\omega_{norm}$

& plot $|Z(j\omega)|/R$ vs ω_{norm}. (see graph)


Frequency response parameterized by m
Magnitude of normalized impedance
4
2
0
-2
-4
-6
-8
m=1.1
m=1.4
m=1.8
m=2.2
Uncompensated
10^{-1}
10^{0}
10^{1}
w/w1


% peaking
% peaking
40
30
20
10
0
-10
1
1.5
2
2.5
m


Optimization of m = RC/(L/R) for shunt peaking
BWXF w/w1
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
0
1
2
3
4
5
6
7
8
9
10
m

$\dfrac{|Z(j\omega)|}{R}$ is shown as a function of the normalized frequency for various values of m. Also plotted is the % peaking as a function of m.

- Note that for the optimum m (= 1·4), the peaking is about 20%, with a BWXF = 1·85. This is often unacceptable
- If we set m = 2, we still get BWXF ≃ 1·8, but the peaking is only 3%. ⟸ <u>frequently used "optimum"</u>

(b) <u>Optimization for maximally flat response</u> (Butterworth)

⟶ used when peaking is absolutely unacceptable

⇒ Form expression for $\dfrac{|Z(j\omega)|}{R}$ and maximize the number of derivatives w.r.t ω that are zero at DC

⇒ $m = 1 + \sqrt{2} \simeq 2 \cdot 41$

From the graph of BWXF vs m, when $\boxed{m = 2\cdot41,\ BWXF = 1\cdot72}$

(c) <u>Optimization for minimum group-delay distortion</u> (Bessel)

- Important in optical communication application
- Ideally phase response of wideband amp. must increase <u>linearly</u> with frequency. In this case, we get equal delay on all frequency components of the signal. ⇒ $d\phi(\omega)/d\omega$ = constant at all frequencies.
- Non-linear phase response causes unequal delay of the frequency components and causes "<u>group-delay distortion</u>"
- Impossible to obtain group delay flatness over infinite BW with a finite order network

⇒ compromise → optimize for <u>maximally</u> <u>flat group</u> <u>delay</u>:

Define $T_D(\omega) = -\dfrac{d\phi(\omega)}{d\omega}$

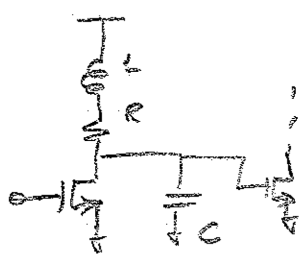
→ Maximize number of derivatives of $T_D(\omega)$ whose value is zero at DC

→ $\boxed{m = 3.10 \quad \& \quad BW \times F = 1.6}$

(different result from optimization for maximally flat magnitude response)!

- Group delay distortion causes inter-symbol interference and hence <u>closes</u> the eye opening at the sampling instant. This is often the limiting factor in high data-rate digital communication systems (eg. optic fibre communication, wireless UWB)

(B) <u>Design</u>



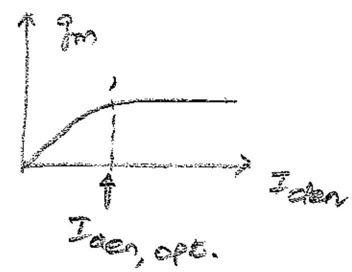
— Given the DC gain, load cap C, ω_{3dB} and an optimization constraint (max. BW, maximally flat magnitude or minimum group-delay distortion)

Find $\omega_1 = \dfrac{\omega_{3dB}}{BW \times F} = \dfrac{1}{RC}$ → Find R

→ From $|A_V(0)| = g_m R$, find g_m

→ Find L using $m = \dfrac{RC}{L/R} \Rightarrow L = \dfrac{R^2 C}{m}$

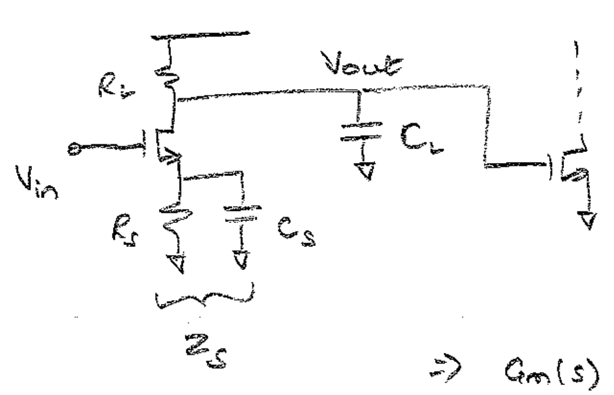
- For power-efficient design: recall:



- Usually, C is not known at the beginning of the design, and is dictated by other constraints
- Can use crummy, low-Q inductors:

Since R is in series with it anyway, we can absorb the series resistance of L into the total resistance.

② Zero-peaked C-S amplifier:



Effective transconductance:

$$G_m(s) = \frac{g_m}{1 + g_m Z_s}$$

$$Z_s = R_s \,||\, \frac{1}{sC_s} = \frac{R_s}{1 + sC_sR_s}$$

$$\Rightarrow G_m(s) = \frac{g_m}{1+g_mR_s} \cdot \frac{1+sC_sR_s}{1 + sC_s\left(\frac{R_s}{1+g_mR_s}\right)}$$

$$\simeq G_{mo} \frac{1+sC_sR_s}{1+sC_s/g_m}$$

$$A_V(s) = G_m(s)\cdot Z_L(s)$$

$$= G_{mo} \frac{1+sC_sR_s}{1+sC_s/g_m} \cdot \frac{R_L}{1+sC_LR_L}$$

$$= (G_{mo}R_L)\left(\frac{1+sC_sR_s}{1+sC_LR_L}\right)\left(\frac{1}{1+sC_s/g_m}\right)$$

D.C gain (I)

Pole-zero cancellation (II)

high-frequency pole (III)