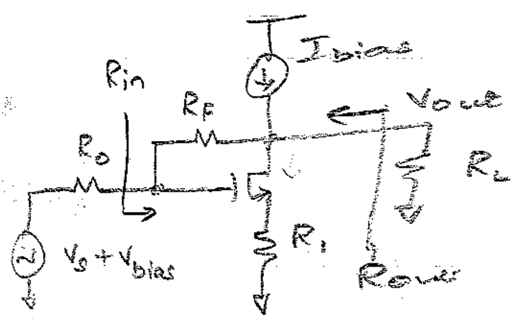
EE 538A ~~Lecture~~ #9 Wideband Amplifiers

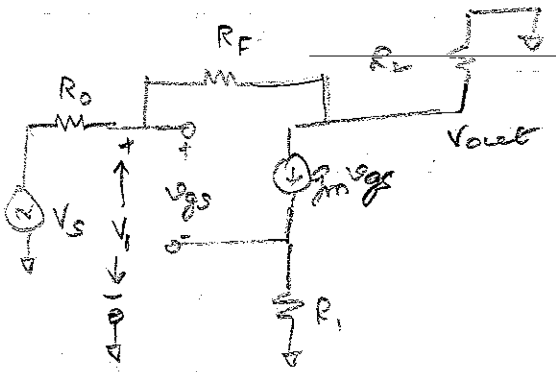
General wideband amplifier techniques:

1) Cascade of single-pole amplifiers (with or w/o feedback)
2) Bandwidth enhancement techniques (shunt peaking, series peaking, T-coil etc.)
3) Regeneration (positive feedback)
4) Distributed amplifiers

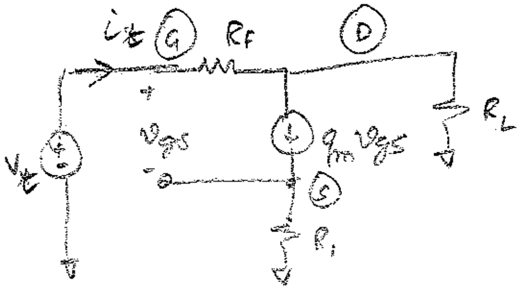
Shunt-series Amplifier



- Use feedback to obtain accurate input & output impedances & gain
- Gain accuracy over PVT variations
- R_F gives shunt feedback (takes a fraction of the output current which feeds back and subtracts from input current)
- R_1 gives series feedback (senses output current, converts it to voltage and subtracts this fedback voltage from the input voltage)



(a) Input resistance:

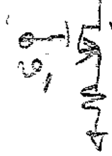


(1) $V_t = V_{gs} + g_m V_{gs} R_1 = (1 + g_m R_1) V_{gs}$

(2) KCL at D

$\Rightarrow \; i_t = g_m v_{gs} + \dfrac{v_D}{R_L}$

$$= \frac{g_m V_t}{1 + g_m R_1} + \frac{V_t - i_t R_F}{R_L}$$

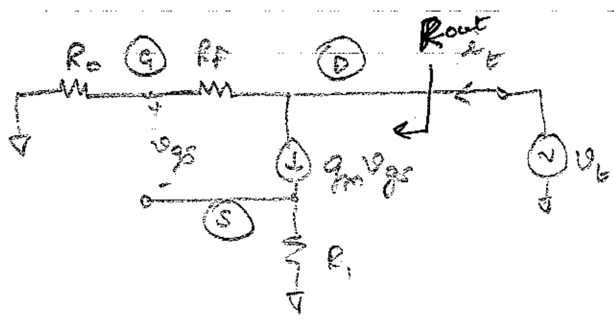
Define $G_m = \dfrac{g_m}{1+g_m R_1}$ (effective transconductance of: )

Note: When $g_m R_1 \gg 1$, $G_m \simeq \dfrac{1}{R_1}$

$$\therefore \left(1+\frac{R_F}{R_L}\right) i_t = v_t\left[G_m + \frac{1}{R_L}\right]$$

$$\Rightarrow \boxed{R_{in} = \frac{R_L + R_F}{1 + G_m R_L} \simeq \frac{R_L + R_F}{1 + R_L/R_1}}$$

(b) Output resistance:



$$v_{gs} = v_g - v_s$$

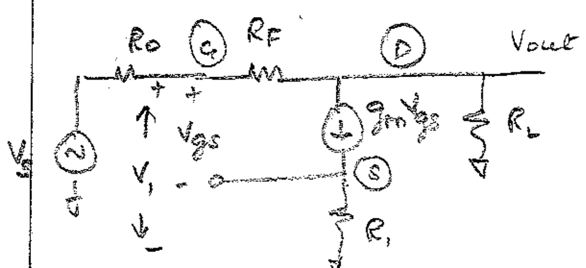
$$= \frac{R_o}{R_o + R_F} v_t - g_m v_{gs} R_1$$

$$\therefore v_{gs} = \frac{R_o}{R_o + R_F} \cdot \frac{1}{1+g_m R_1} v_t$$

$$i_t = g_m v_{gs} + \frac{v_t}{R_o + R_F} = \left[\frac{g_m}{1+g_m R_1} \cdot \frac{R_o}{R_o + R_F} + \frac{1}{R_o + R_F}\right] v_t$$

$$\Rightarrow \boxed{R_{out} = \frac{V_t}{I_t} = \frac{R_o + R_F}{1 + G_m R_o} \simeq \frac{R_o + R_F}{1 + R_o/R_1}}$$

(c) Voltage gain:



First calculate $A_{V_1} = \dfrac{V_{out}}{V_1}$ and then use the voltage divider rule to calculate $A_V = \dfrac{V_{out}}{V_s}$;

$$V_1 = V_{gs} + g_m V_{gs} R_1 \Rightarrow V_{gs} = \frac{V_1}{1+g_m R_1}$$

KCL at (D) : $g_m V_{gs} + \frac{V_{out}}{R_L} + \frac{V_{out} - V_1}{R_F} = 0$

$$\therefore \quad \frac{g_m V_1}{1+g_m R_1} - \frac{V_1}{R_F} = -\left(\frac{V_{out}}{R_L} + \frac{V_{out}}{R_F}\right)$$

$$\therefore \quad (G_m - 1/R_F) V_1 = -\frac{V_{out}}{R_L \| R_F}$$

or $\frac{V_{out}}{V_1} = A_{V_1} = -\frac{R_L \| R_F}{R_F}(G_m R_F - 1)$

$$\Rightarrow \boxed{A_{V_1} = \frac{R_L}{R_L + R_F}(1 - G_m R_F)}$$

usually choose $R_F \gg R_L$ and $G_m R_F \gg 1$

$\Rightarrow A_{V_1} \simeq -G_m R_L$

Also, $V_{out} = V_s \cdot \frac{R_{in}}{R_{in} + R_o} A_{V_1}$ and $A_V = \frac{V_{out}}{V_s}$

$$\Rightarrow \boxed{A_V = \frac{R_{in}}{R_{in} + R_o} \frac{R_L}{R_L + R_F}(1 - G_m R_F)} \simeq -\frac{R_{in}}{R_{in} + R_o} G_m R_L$$

(d) Bandwidth: Until now, we have neglected all capacitances. To calculate the BW, we will use the method of open-circuit time constants (OC τ). Consider a network with only R's, C's, independent I or V sources and dependent sources (VCCS, VCVS etc.) The OCτ technique consists of the following steps:

(a) Calculate the effective resistance R_{jo} facing the j^{th} cap C_j with all other capacitors removed.

(b) Find the time constant $\tau_{jo} = R_{jo} C_j$ for every capacitor

(c) 3dB BW of the network $= \frac{1}{(\Sigma \text{ all such "open-circuit" time constants})}$

$$\Rightarrow \quad \omega_{3dB,OC\tau} = \frac{1}{\sum_{j=1}^{m} R_{oj} C_j} \quad \text{for a total of } m \text{ capacitors}$$

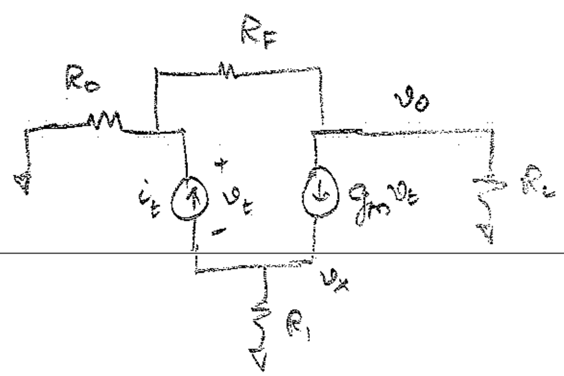
<u>Beware</u>: There is a long list of caveats associated with this method. In particular, it assumes the existence of a dominant pole. For details, see Section 7.3.2 of Gray & Meyer or Chapter 8 of Tom Lee's "Design of CMOS RFICs".

However, OCTs can give a pessimistic estimate of BW with much less computational complexity than an exact analysis.

For our circuit we will consider only C_{gs} and C_{gd}:

(i) $C_{gs} \rightarrow$ calculate resistance facing C_{gs} ($R_{C_{gs}}$)

$$R_{C_{gs}} = \frac{v_t}{i_t}$$



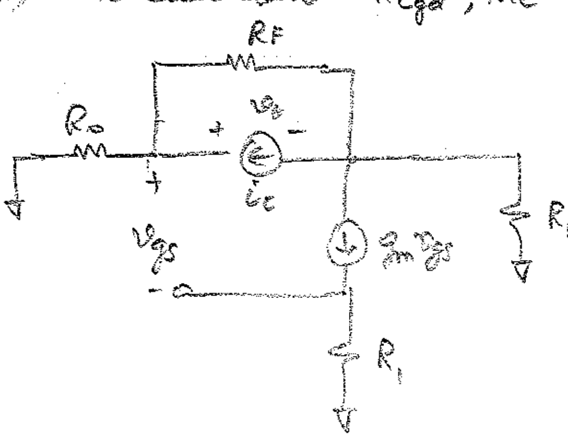
The algebra involved is tedious, but to simplify things, assume that

- A_v is large
- $R_o = R_L = R$

Then, $\boxed{R_{C_{gs}} = \frac{R}{R_1} \cdot \frac{1}{g_m}}$

(ii) To calculate $R_{C_{gd}}$, the resistance facing C_{gd}:

$$R_{C_{gd}} = \frac{v_t}{i_t}.$$



Again, after painful algebra and same assumptions as before,

$$R_{C_{gd}} \simeq |A_v| \frac{R}{2}$$

$$\Rightarrow \boxed{\omega_{3dB} \simeq \frac{1}{C_{gd} R_{C_{gd}} + C_{gs} R_{C_{gs}}} = \frac{1}{|A_v| \left(\frac{C_{gs}}{g_m} + \frac{R C_{gd}}{2} \right)}}$$

Design: Very easy to design for equal source and load resistances, i.e., $R_0 = R_L = R$ (which are usually known)

$$\Rightarrow R_{in} = \frac{R_L + R_F}{1 + G_m R_0} = \frac{R + R_F}{1 + G_m R}$$

$$R_{out} = \frac{R_0 + R_F}{1 + G_m R_L} = \frac{R + R_F}{1 + G_m R}$$

$R_{in} = R_{out}$

For power match at both ports, set $R_{in} = R_{out} = R$

$$\Rightarrow R = \frac{R + R_F}{1 + G_m R} \Rightarrow 1 + G_m R = 1 + \frac{R_F}{R} \text{ or } \boxed{G_m = \frac{R_F}{R^2}} \quad \text{— (1)}$$

With a given gain specification

$$A_V \simeq \frac{1}{2} G_m R_L \Rightarrow \boxed{G_m = \frac{2|A_V|}{R_L}} \quad \text{— (2)}$$

$$\Rightarrow \boxed{R_F = \frac{2|A_V| R^2}{R_L}} \quad \text{— (3)}$$

Now, we only need to set g_m and R_1 to get

$$G_m = \frac{g_m}{1 + g_m R_1}$$

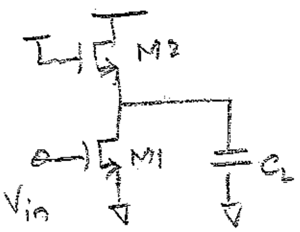
Advantages of shunt-series amp:

- broadband
- stable, accurate gain, R_{in} & R_{out}
- easy to design

Disadvantages:

- performance so-so
- Noisy
- With $R_{in} \neq R_{out}$, cannot get simultaneous power match at input & output. (the equations become inconsistent!)

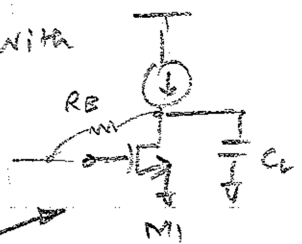
Other wideband approaches:



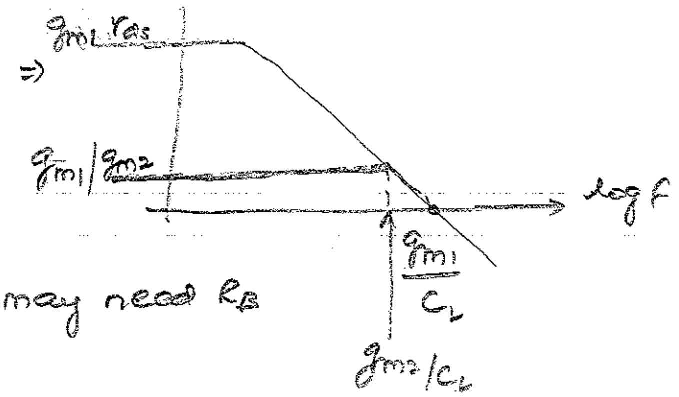
$$A_v = -\frac{g_{m1}}{g_{m2}} = \sqrt{\frac{2K_n'(W/L)_1 I_{D1}}{2K_n'(W/L)_2 I_{D2}}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}}$$

$$\omega_p \cong \frac{g_{m2}}{C_L}$$

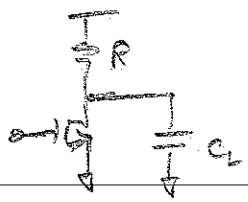
Compare with



⇒



output bias not accurately set ⇒ may need R_B

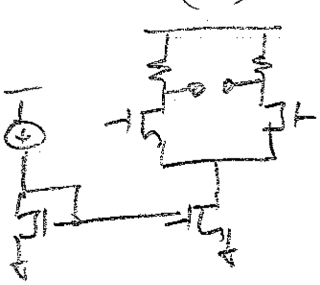


$A_v = -g_m R$

$\omega_p \cong \frac{1}{RC_L}$

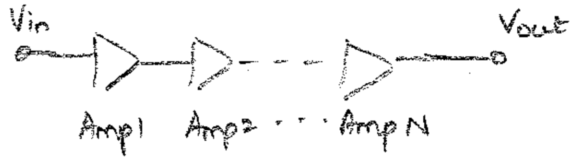
usually the fastest because resistor has lower parasitic cap. compared to MOS current source

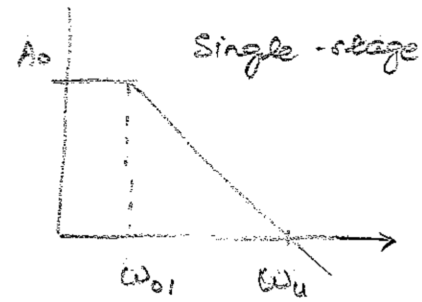
(or)



self-biased

Cascading amplifiers:





Consider a cascade of N identical amplifiers, each with a single-pole response

$$A(s) = \frac{A_0}{1 + s/\omega_p}$$

Recall gain-BW tradeoff for single stage! If we wanted $2A_0$ gain, we would necessarily have to give up BW

Overall transfer function is $H(s) = \left(\frac{A_0}{1+s/\omega_p}\right)^N$

→ Calculate the 3dB frequency of the cascade, (ω_{0N})

At $\omega = \omega_{0N}$, $|H(j\omega)| = \frac{1}{\sqrt{2}} |H(0)|$

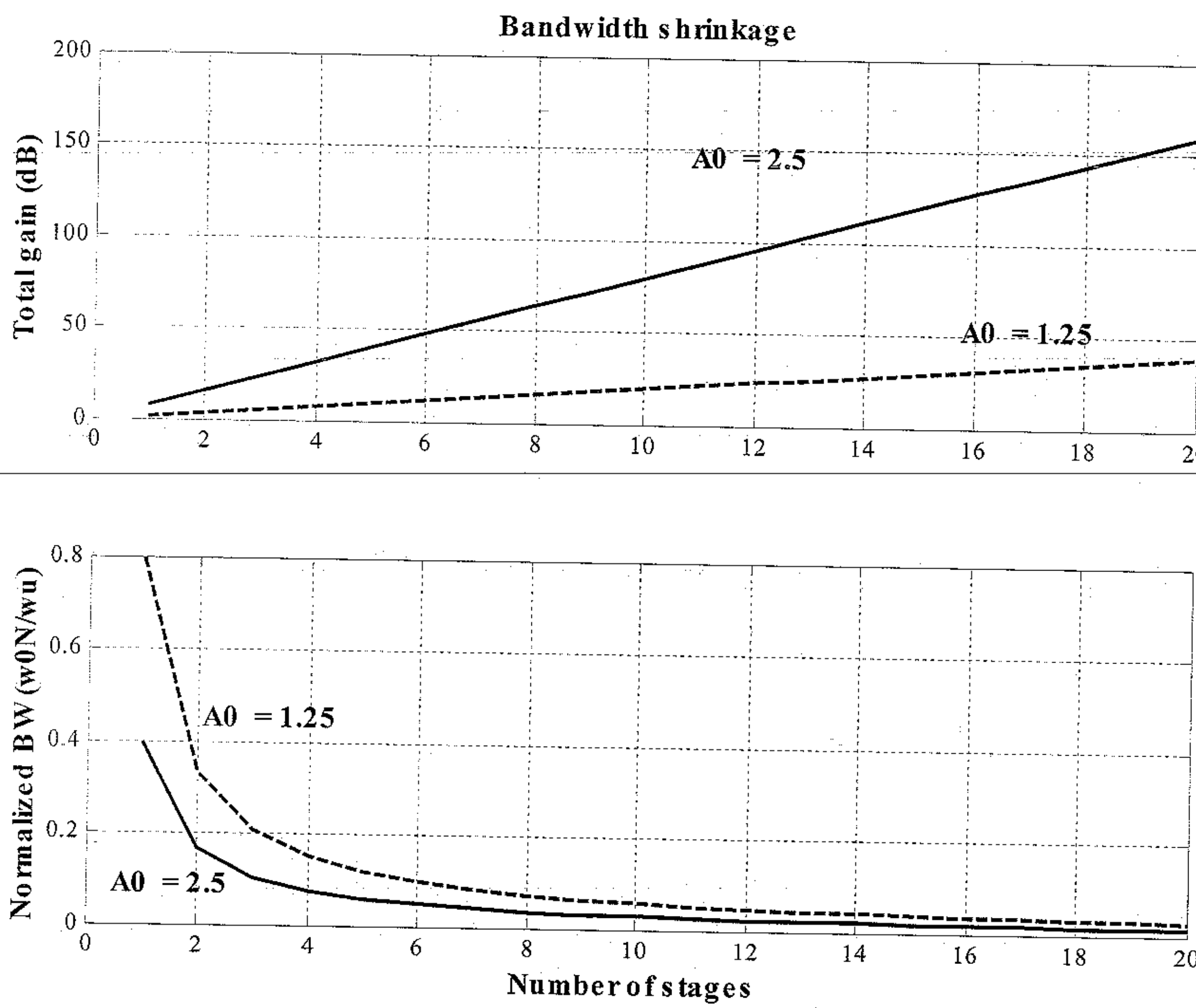
$$\Rightarrow \left(\frac{A_0}{\sqrt{1 + \frac{\omega_{0N}^2}{\omega_p^2}}}\right)^N = \frac{1}{\sqrt{2}} A_0^N \Rightarrow \frac{\omega_{0N}}{\omega_p} = \sqrt{2^{1/N} - 1}$$

$$\text{Recall } A_0 \omega_p = \omega_u \Rightarrow \omega_{0N} = \frac{\omega_u}{A_0} \sqrt{2^{1/N} - 1}$$

(a) Bandwidth shrinkage: Suppose we fix the gain of each stage to be A_0 and cascade more and more stages. Assuming $(A_0 > 1)$:

→ Total DC gain $A_{tot} = A_0^N \to \infty$ as $N \to \infty$

→ Cascaded 3dB frequency $\omega_{0N} \to 0$ as $N \to \infty$


Bandwidth shrinkage
Total gain (dB)
A0 = 2.5
A0 = 1.25
Normalized BW (w0N/wu)
A0 = 1.25
A0 = 2.5
Number of stages

(b) Now, fix total gain A_{tot} and find an optimal N to <u>maximize</u> BW:

→ The DC gain A_0 and pole frequency of each stage ω_p are <u>variable</u>, but the unity gain frequency of each stage is constant: $\omega_u = A_0 \omega_p$

$$\Rightarrow \quad A_0^N = A_{tot} \quad \text{or} \quad A_0 = A_{tot}^{1/N}$$

$$\Rightarrow \quad \omega_{0N} = \frac{\omega_u}{A_{tot}^{1/N}} \sqrt{2^{1/N} - 1}$$

⇒ Maximize BW: $\dfrac{d\omega_{0N}}{dN} = 0$

After some algebra: $N_{opt} = \dfrac{\ln 2}{\ln\left(1 + \dfrac{\ln 2}{2 \ln A_{tot}}\right)}$

For large A_{tot},

$$\ln\left(1 + \frac{\ln 2}{2 \ln A_{tot}}\right) \simeq \frac{\ln 2}{2 \ln(A_{tot})} \Rightarrow \boxed{N_{opt} \simeq 2 \ln A_{tot}}$$

⇒ Find optimum gain of each stage:

$$A_{0,opt} = A_{tot}^{1/N} = \exp\left(\frac{1}{N} \ln A_{tot}\right) \simeq e^{1/2}$$

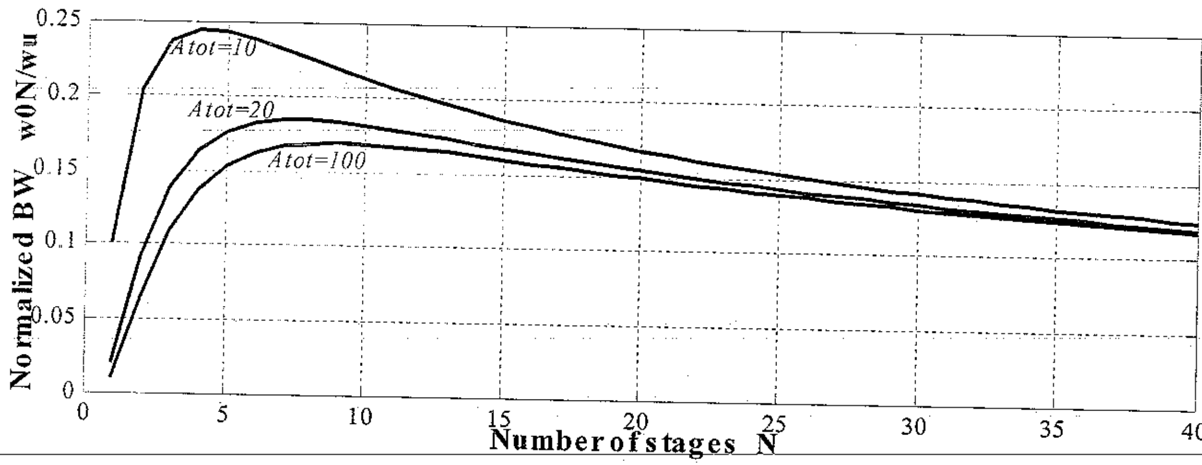
$$\Rightarrow \quad \boxed{A_{0,opt} = \sqrt{e}}$$

⇒ Find optimum BW:

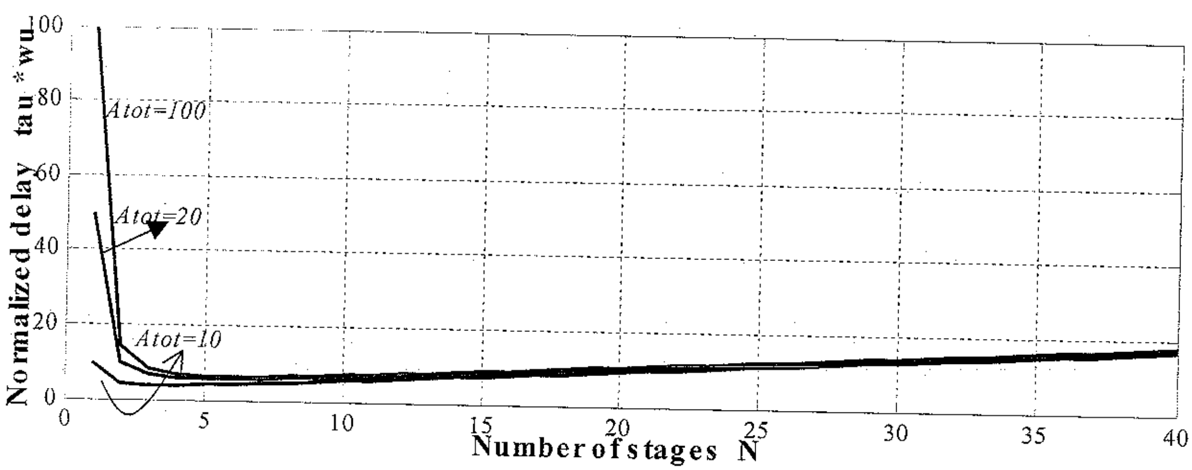
$$\omega_{0N,opt} = \frac{\omega_u}{A_{tot}^{1/N}} \sqrt{2^{1/N_{opt}} - 1} \simeq \frac{\omega_u}{\sqrt{e}} \left[\exp\left(\frac{1}{N_{opt}} \ln 2\right) - 1\right]^{1/2}$$

$$\boxed{\omega_{0N,opt} \simeq \omega_u \sqrt{\frac{\ln 2}{2e \ln A_{tot}}}}$$

⇒ Overall BW trades-off as square-root of total gain!!



Normalized BW w0N/wu
0.25
0.2
0.15
0.1
0.05
0
Atot=10
Atot=20
Atot=100
0
5
10
15
20
25
30
35
40
Number of stages N



Normalized delay tau*wu
100
80
60
40
20
0
Atot=100
Atot=20
Atot=10
0
5
10
15
20
25
30
35
40
Number of stages N