A Polynomial-time Approximation Algorithm for Weighted Sum-rate Maximization in UWB Networks

Gyouhwan Kim Qiao Li Rohit Negi
gyouhwan@cmu.edu qiaoli@cmu.edu negi@ece.cmu.edu
Department of Electrical and Computer Engineering, Carnegie Mellon University

***Abstract*— Scheduling in an ad hoc wireless network suffers from the non-convexity of the cost function, caused by the interference between communication links. In previous optimization theoretic analysis, the weighted sum-rate maximization (WSRM) which inherits the non-convexity has been identified as a core problem of the hard scheduling problem.**

In this paper, we propose a polynomial-time approximation algorithm with guaranteed accuracy for WSRM under an Ultra-wide band (UWB) assumption. The algorithm is obtained by an appropriate adaptation of the 'shifting' strategy (a well-known approximation technique for some geometric problems) for the wireless broadcast environment. The worst case accuracy and complexity of the algorithm are analyzed by utilizing the quadratic link rate function derived in previous research, under the assumption of a large bandwidth, as is typical in UWB networks. The average case performance of the algorithm is investigated by simulations on random ad hoc networks.

Keywords: scheduling, MAC, wireless ad hoc networks, ultra wide band, shifting strategy.

I. Introduction

Transmission over a wireless medium causes interference to unintended receivers. Thus, in a wireless network with multiple links (pairs of transmitters and receivers), the *scheduling* of transmission of links is of extreme importance, to prevent strong interference between links, so as to maximize the utilization of the limited wireless resource. However, an optimal schedule is, in general, hard to obtain since link rate is non-convex in transmission powers, due to interference [1], [2]. This motivates using simple models of a wireless network to model interference, such as a disk graph model to allow polynomial-time scheduling algorithms.

In a *disk graph model*, each link is represented by a disk and any pair of links with intersecting disks are considered to interfere with each other. This geometric graph representation of a wireless network is useful in using the machinery of graph theory, such as algorithms related to scheduling. For example, there is a polynomial time heuristic that achieves a graph coloring within six times of the optimum [3]. Such algorithms exploit the fact that the graph has been obtained from an underlying geometric formulation. However, the performance achieved in a disk graph may not be preserved when the solution is mapped back to the original wireless network. This is because the disk graph distorts or ignores many underlying physical layer (PHY) aspects. In a disk graph, the interference between links is modeled in a *pairwise* manner (by the intersection of disks) and also the transmitter and the receiver of a link are assumed to be co-located [4]. In particular, for systems operating at low signal to interference plus noise ratio (SINR) thresholds such as CDMA and Ultra-wide band (UWB) systems [5], it has been shown that the number of channels required by coloring heuristics can be much larger than six times of the optimum [4]. This result has been derived by considering a more sophisticated PHY model which explicitly accounts for the SINR threshold, thus demonstrating the crucial importance of accurate PHY modeling.

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In order to overcome the above mentioned disadvantage of the disk graph, a more precise way of dealing with the scheduling problem is attempted in [1]. The *Shannon capacity* formula is used to quantify the data rate on each link as a better PHY model, since this account for total interference, and the scheduling problem is formulated as an optimization problem. In that work, an interesting observation has been made. That is, the scheduling problem can be viewed as a series of simpler problems requiring maximization of the sum-rate of links with a given weight vector, i.e., the *weighted sum-rate* ($\sum_i \lambda_i c_i$) *maximization* (WSRM). WSRM also appears in the optimization theoretic dual problems of various wireless networking problems with a different objective (e.g., max-min rate or log utility maximization) and a wider scope of the problem (e.g., a joint scheduling and routing) [2], [6]. The significance of WSRM is observed even when the input traffic or channel state is dynamic. For a network with time varying input traffic, WSRM is required in every slot to achieve the maximum throughput, while keeping the system stable [7]. Under the assumption of independent block fading on each link, WSRM is again the key sub-problem to be solved in each slot to exploit multi-user diversity [8]. Unfortunately, however, WSRM is inherently non-convex in the link transmission powers, and thus, in general, computing an exact solution to it is computationally prohibitive. Thus, WSRM is a bottleneck in optimally solving various wireless networking problems that involves scheduling.

This paper contributes to the study of the hard scheduling problem by presenting an approximation algorithm for WSRM in UWB networks, where the performance of a typical disk graph coloring heuristic can be much worse than the optimum (due to the low SINR threshold of operation). This algorithm achieves the weighted sum-rate to within any factor $\rho = 1 - \epsilon$ of the optimum with a complexity $\epsilon^{-2} n^{O(\epsilon^{-2-\frac{2}{\alpha-2}})}$, where n is the number of links and $\alpha > 2$ is the path loss exponent. The positive number $\rho < 1$ is referred to as the performance ratio. The algorithm is distinct from existing heuristics that have weaker or no guarantees on their accuracy, as well as from approximation algorithms that rely on inaccurate graph-based representations of wireless networks.

The algorithm is devised by an adaptation of the shifting strategy, which was introduced in [9] for geometric packing and covering problems. The adaptation is meant to take advantage of the quadratic approximation of link capacity valid in UWB networks [1], [6], [11]. The shifting strategy is meant to bound the performance ratio of a divide-and-conquer algorithm that solves *independent* smaller *local* problems defined by a certain geometric partitioning, instead of solving a large problem directly [9]. This shifting strategy is the motivation for the algorithm presented in this paper. However, a direct application of the shifting strategy is not possible, due to the following fundamental difference. The interference between radio links is a *network-wide* phenomenon while the interaction between geometric objects (e.g., disks) is local. A disk can intersect another disk only within its radius and the intersection is determined by the pair of disks, without taking into account the positions of other disks. However, in a wireless network, a transmitter affects *all* receivers. Further, the *total* interference is important to decide whether communication is feasible, not just individual (maximum) interference. Thus, in a wireless network, the coupling between local problems is inherent and no geometric partitioning results in completely independent local problems. In view of this fact, we present an adapted shift algorithm and a shifting lemma for WSRM. We also prove that the loss due to the dependency of local problems can be bounded by utilizing the special structure of the quadratic link rate approximation. Combined with the shifting lemma, this analysis yields the overall performance ratio of the presented algorithm. The structure of the quadratic link rate approximation is also used to show that the complexity of the algorithm is polynomial in n, which is not impossible, in general, for the original Shannon capacity formula. These results demonstrate the importance of PHY properties in dealing with the hard scheduling problem.

The outline of the paper is as follows:

In Section II, we review the quadratic link rate approximation of a UWB link [1], which will play a key role in analyzing the accuracy and complexity of the algorithm.

In Section III, we adapt the shifting strategy [9] to a wireless network by introducing a spatial partition that keeps space between local areas. This is to prevent strong interference from outside a local area. The inaccuracy caused by this spacing is then bounded by a method similar to one used in [9], [10].

In Section IV, an upper bound on the interference from outside a local area (which unlike a pure geometric graph, remains even after the spacing) is derived, and incorporated into the performance ratio ρ. We also show that the complexity of the algorithm is polynomial in n. This completes the design of the polynomial-time algorithm for WSRM in UWB networks with a guaranteed performance ratio of ρ.

In Section V, some simulation results are presented to discuss the effect of the algorithm and network parameters on the practical accuracy and complexity of the algorithm.

II. Weighted sum-rate maximization for UWB networks

In this section, we briefly review the UWB link rate approximation developed in [1], [6], [11] and the algorithm (denoted 'A') which solves WSRM for a small network under this approximation. Algorithm A will be used as a 'local' algorithm (i.e., to solve a local problem) in the next section.

Consider a wireless network with n links sharing a given bandwidth W. Assuming each link operates at Shannon capacity, the rate of link i normalized by the slot size t and a constant (unit) bandwidth W_0 is as below,

$$c_i = \frac{1}{x}\ \log\left(1 + \frac{p_i g_{ii}}{\frac{1}{x} + \sum_{j\neq i} p_j g_{ji}}\right). \tag{1}$$

x is the ratio W/W_0 referred to as the 'UWB parameter' and $p_i \in [0,1]$ is the transmission power normalized by $N_0 W_0$, where N_0 is the thermal noise power spectral density. Note that $x \to 0$ implies that the links use a large bandwidth. We assume that the gains follow a simple path loss model with the exponent $\alpha > 2$, i.e., the gain from T_j (the transmitter of link j) to R_i (the receiver of link i) $g_{ji} = d(T_j, R_i)^{-\alpha}$, where $d(T_j, R_i)$ is the distance between T_j and R_i. The distance between the transmitter and receiver of a link is assumed to range between d_{min} and 1. The quadratic link rate approximation that will be used throughout the paper is obtained from the first-order Taylor series approximation of c_i about $x = 0$ (equivalently $W = \infty$), i.e.,

$$\tilde{c}_i \doteq p_i g_{ii} - x p_i g_{ii}\Big[\frac{1}{2} p_i g_{ii} + \sum_{j\neq i} p_j g_{ji}\Big]. \tag{2}$$

$\tilde{c}_i \leq c_i$, and so $\tilde{c}_i$ is a lower bound to c_i as shown in [1]. The UWB parameter x determines the accuracy of the approximation $\tilde{c}_i$. As x decreases (i.e., more UWB-like), the lower bound $\tilde{c}_i$ increases to the upper bound $p_i g_{ii}$, and so, the approximation becomes more accurate. Refer to [1] for prior work that has evaluated this UWB approximation. Under this approximation, WSRM becomes

$$\mathcal{I}^*_{loc} = \max_{0\leq \mathbf{p} \leq\ 1} \ \boldsymbol{\lambda}^T \tilde{\mathbf{c}} = \max_{\mathbf{0}\leq\mathbf{p}\leq\mathbf{1}} \mathbf{h}^T\mathbf{p} - \frac{x}{2}\mathbf{p}^T\mathbf{A}\mathbf{p}\,, \tag{3}$$

where $A_{ij} \doteq \lambda_i g_{ii} g_{ji} + \lambda_j g_{jj} g_{ij}$, $A_{ii} \doteq \lambda_i g_{ii}^2$ and $h_i \doteq \lambda_i g_{ii}$. $\mathcal{I}^*_{loc}$ is a quadratic optimization problem, and although usually non-convex, is simpler to solve than WSRM with true rate c_i. For a small network, the optimal solution $\mathbf{p}^*$ to $\mathcal{I}^*_{loc}$ may be obtained by the following algorithm.

Algorithm A: 1) Choose a subset of links s_m for which the corresponding sub-matrix of $\mathbf{A}$, $\mathbf{A}_m$, is positive definite, i.e., $\mathbf{A}_m \succ 0$. Let $\mathbf{h}_m$ be the corresponding sub-vector of $\mathbf{h}$. 2) Of the remaining links, choose a subset s_u to operate at the limit ($p_i = 1$) and the rest to remain silent ($p_i = 0$). Let $\mathbf{A}_u$ and $\mathbf{h}_u$ be the sub-matrix and sub-vector corresponding to s_u. Also denote the sub-matrix of $\mathbf{A}$ consisting of rows of s_m and columns of s_u as $\mathbf{A}_{mu}$. 3) If $\mathbf{0} < \frac{\mathbf{A}_m^{-1}\mathbf{h}'_m}{2} < \mathbf{1}$ (here, $\mathbf{h}'_m \doteq \mathbf{h}_m - \frac{x}{2}\mathbf{A}_{mu}\mathbf{1}$), evaluate $Val(s_m, s_u) \doteq \mathbf{h}^T\mathbf{p} - \frac{x}{2}\mathbf{p}^T\mathbf{A}\mathbf{p}$. 4) $\mathcal{I}^*_{loc} = \max\ Val(s_m, s_u)$ is achieved for index sets s^*_m and s^*_u, at the optimal $\mathbf{p}^*$. Here $\mathbf{p}^*_{s^*_m} = \frac{\mathbf{A}_m^{-1}\mathbf{h}'_m}{2}$, $\mathbf{p}^*_{s^*_u} = \mathbf{1}$ and $\mathbf{p}^* = \mathbf{0}$ otherwise. (The proof is in [11].)

Algorithm A, in general, requires 3^n evaluations for the possible $\mathbf{p}$ vectors when there are n links, since each p_i has 3 choices ($p_i = 0$, $p_i = 1$ or $0 < p_i < 1$). Thus, the complexity becomes prohibitive as n increases. This prevents the application of the algorithm to a large network. However, we can still use algorithm A for each local problem (which will be defined in the shift algorithm in the following sections) when the number of links is limited. Considering such an application of algorithm A to a specific local problem, $\tilde{c}_i$ in (2) can be written as below,

$$\tilde{c}_i(\mathbf{p}) = p_i g_{ii} - x p_i g_{ii}\Big[\frac{1}{2} p_i g_{ii} + \sum_{j\in\mathcal{L}(i)\setminus i} p_j g_{ji} + I^{(i)}_{out}\Big], \tag{4}$$

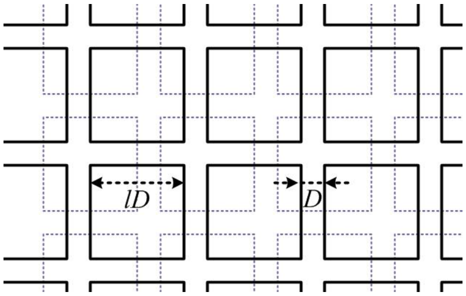

Fig. 1. An example spatial partition with shifting parameter $l = 4$

where $\mathcal{L}(i)$ is the set of links that have their receivers in the local area (corresponding to the local problem) and $I_{out}^{(i)}$ is the interference from outside of the local area (i.e., from the links in $\mathcal{L}(i)^c$). If $I_{out}^{(i)} \leq \beta$, then a lower bound on $\tilde{c}_i$ is

$$c_i'(\mathbf{p}) = p_i g_{ii} - x p_i g_{ii} \Big[\frac{1}{2} p_i g_{ii} + \sum_{j \in \mathcal{L}(i) \setminus i} p_j g_{ji} + \beta\Big]. \quad (5)$$

Setting $I_{out}^{(i)} = 0$, an upper bound on $\tilde{c}_i$ is

$$c_i''(\mathbf{p}) = p_i g_{ii} - x p_i g_{ii} \Big[\frac{1}{2} p_i g_{ii} + \sum_{j \in \mathcal{L}(i) \setminus i} p_j g_{ji}\Big]. \quad (6)$$

These bounds will be used in the following sections.

III. An adapted shifting strategy for weighted sum-rate maximization

The shifting strategy was first introduced by Hochbaum and Maass [9] for approximation algorithms for NP-hard geometric packing and covering problems and extended to weighted versions (such as maximum weight independent set problem and minimum weight vertex cover problem in intersecting graphs) by Erlebach et al. in [10]. The basic idea is to bound the performance ratio of a simple divide-and-conquer algorithm that is applied separately to each local area, obtained from a spatial partition. The bound is obtained by using the algorithm repeatedly for all possible partitions and then selecting the most favorable partition [9]. In this section, we first describe how to adapt this shifting strategy to WSRM, which has been identified as a core problem of various wireless networking problems. Second, but as importantly, we point out the fundamental difference between WSRM and geometric graph problems, which will motivate the analysis in the following sections. We borrow the notation from [9] for clear comparison between the original shifting strategy and the one for WSRM.

Consider an ad hoc network of n links in some area. As shown in Fig. 1, consider the set of squares of area $(lD)^2$ separated by strips of width D. By applying horizontal and vertical shifts of integer multiples of D, there are $(l+1)^2$ different positions of this set of squares. We denote such a shift by a pair of shift indices (h, v) and the set of squares obtained by shift (h, v) by $S_{(h,v)}$. Fig. 1 shows the set $S_{(0,0)}$ (solid lined) and $S_{(3,2)}$ (dashed lined). Let $A(S_{(h,v)})$ be the algorithm that applies algorithm A of Section II to each square in $S_{(h,v)}$ (more precisely, to the set of links whose receivers are in the square) with $I_{out} = 0$, and delivers the union of selected (i.e., $p_i > 0$) links (selected by optimization (3)). Thus, in this section, we use c_i'' in (6) as the link rate ignoring I_{out} and each square of area $(lD)^2$ corresponds to the local area mentioned in the previous section. Since the interference from outside a square is ignored in this step, we call the application of algorithm A a 'local algorithm'. The shift algorithm S_A then delivers the set of links of maximum weighted sum-rate among all $(l+1)^2$ sets obtained by $A(S_{(0,0)}), ..., A(S_{(h,v)})$. The weighted sum-rate of the set of links obtained by S_A is denoted by $\mathcal{I}_{S_A}$.

Lemma 1: $\mathcal{I}_{S_A} \geq \left(1 - \frac{1}{l+1}\right)^2 \mathcal{I}^*$, where $\mathcal{I}^*$ is the optimal solution to the global WSRM problem.

Proof: A shifting argument similar to the ones in [9], [10] applies as outlined below (Due to lack of space, the complete proof is in [12]). Consider horizontal strips of width lD which are equally spaced by distance D. Vertical shifting of the set of strips by D defines $l+1$ different sets of strips, $S_0, ..., S_l$. Suppose that we solve WSRM locally for every strip in S_v and let $\mathcal{I}_v$ be the weighted sum-rate of the union of selected links. Since each S_v covers $\frac{l}{l+1}$ of the entire area, at least one among all S_v's, say S_{v^*}, must include a subset of the optimal set of links with weighted sum-rate not less than $\frac{l}{l+1}\mathcal{I}^*$. Also, the interference across strips is ignored. Thus, the maximum $\mathcal{I}_{v^*}$ among all $\mathcal{I}_v$'s is at least $\frac{l}{l+1}\mathcal{I}^*$. This procedure can be repeated for horizontal shifting of vertical strips, over the chosen set of horizontal strips S_{v^*}. It is easily seen that the repetition introduces the same factor $\frac{l}{l+1}$ again to the weighted sum-rate for the finally chosen set of squares. Therefore, *Lemma 1* holds for S_A, which finds the most favorable set of squares by both horizontal and vertical shifting. ■

Note that in the above shifting strategy, we have spacing by distance D between squares, which is different from the partitioning used in [9], [10]. This spacing is to limit the interference I_{out} from the transmitters outside a square (which have been ignored by the local algorithm) to the receivers in the square. However, the solution obtained by S_A cannot achieve $\mathcal{I}_{S_A}$ because the true weighted sum-rate will be further reduced by I_{out} even after the spacing. In contrast, in the case of geometric graphs [9], [10], the union of local solutions is a feasible global solution with guaranteed accuracy. This crucial difference between the two cases is due to the broadcast nature of the wireless medium. Therefore, what remains is to modify S_A to obtain an algorithm that provides a solution with guaranteed accuracy and analyze its complexity.

IV. The Shift algorithm in the presence of I_{out}

In this section, we consider the effect of I_{out}, which was ignored in the previous section. In Section IV-A, we use the shift algorithm S_A with c_i' in (5) that includes I_{out} as an upper bound $I_{out} \leq \beta$, where β is a constant independent of n, and calculate the performance degradation. In Section IV-B, we show that the bound on I_{out} does indeed hold, for all selected links. In Section IV-C, we prove that the complexity of the algorithm is polynomial in n for a fixed performance ratio. It should be noted that all proofs are essentially based on the special structure of the quadratic link rate approximation (2), and thus, the algorithm is valid only in the case of a UWB network. Details of the proofs can be found in [12].

A. Effect of I_{out} on the accuracy of the algorithm

Apply the shift algorithm S_A in Section III with the link rate equal to c_i' in (5), i.e., the lower bound on $\tilde{c}_i$ obtained by assuming $I_{out} \leq \beta$. We can show the following:

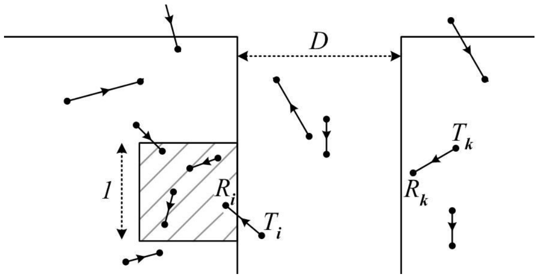

Fig. 2. A unit square for the interference analysis

Lemma 2: If $I_{out} \leq \beta$ for every selected link in the network, the shift algorithm S_A that uses c_i' solves WSRM with the weighted sum-rate $\mathcal{I}'_{S_A} \geq \left(1 - \frac{1}{l+1}\right)^2 (1 - x\beta)^2 \mathcal{I}^*$.

Proof: Consider a power vector $\mathbf{p}''$ obtained from the shift algorithm S_A which assumes that the link rate is c_i'' ignoring I_{out}, as was done in Section III. By the definitions of c_i' and c_i'' (in (5) and (6)), $c_i'((1-x\beta)\mathbf{p}'') = (1-x\beta)^2 c_i''(\mathbf{p}'')$. Now, taking the weighted sum of both sides for all links shows that $\boldsymbol{\lambda}^T \mathbf{c}'((1-x\beta)\mathbf{p}'') = (1-x\beta)^2 \mathcal{I}_{S_A}$, where $\mathcal{I}_{S_A}$ is as in *Lemma 1*. On the other hand, $\mathcal{I}'_{S_A} \geq \boldsymbol{\lambda}^T \mathbf{c}'((1-x\beta)\mathbf{p}'')$ since $\mathcal{I}'_{S_A}$ is the maximum weighted sum-rate attainable when using c_i'. Therefore, $\mathcal{I}'_{S_A} \geq (1-x\beta)^2 \mathcal{I}_{S_A}$. Combining this result with *Lemma 1* leads to *Lemma 2*. ■

B. Upper bound on I_{out}

Here, we validate the assumption in Section IV-A, that $I_{out} \leq \beta$ for all selected links. Suppose that $\mathbf{p}'$ is the solution to the shift algorithm S_A that uses c_i' in (5). Then,

$$\frac{\partial c_i'}{\partial p_i} |_{\mathbf{p}=\mathbf{p}'} = g_{ii} - x g_{ii} \Big[\sum_{j \in \mathcal{L}(i)} p_j' g_{ji} + \beta \Big] \geq 0 \qquad (7)$$

for every selected link i. Otherwise, there exists $p_i < p_i'$ which increases $\mathcal{I}'_{S_A}$ by increasing c_i' while not decreasing c_j'. This is because we can find $p_i < p_i'$ to c_i' when $\frac{\partial c_i'}{\partial p_i} |_{\mathbf{p}=\mathbf{p}'} < 0$ and such p_i does not decrease any c_j' since it causes less interference. Since $\beta \geq 0$, (7) implies $\sum_{j \in \mathcal{L}(i)} p_j' g_{ji} \leq 1/x$. Now, consider a unit square inside the square containing R_i as shown in Fig. 2 and denote the set of links that have their receivers in the unit square as $\mathcal{L}_U(i)$. Clearly $\sum_{j \in \mathcal{L}_U(i)} p_j' g_{ji} \leq 1/x$ since $\mathcal{L}_U(i) \subseteq \mathcal{L}(i)$. Also, for any link $j \in \mathcal{L}_U(i)$, $d(T_j, R_i) \leq \sqrt{2} + 1$ and $d(T_j, R_k) \geq D - 1$, where R_k is a receiver in the square next to the square containing R_i (Fig. 2). Thus, we have $\sum_{j \in \mathcal{L}_U(i)} p_j' g_{jk} \leq \frac{1}{x} \left(\frac{\sqrt{2}+1}{D-1} \right)^\alpha$. Now, considering a set of unit squares that cover the network area outside the square with R_k, we can show that $I_{out} \leq \beta \doteq \frac{\kappa_1}{x(D-2)^{\alpha-2}}$ for α, $D > 2$, where the constant $\kappa_1 = \frac{6\pi(\sqrt{2}+1)^\alpha}{\alpha - 2}$ [12]. Clearly, $\beta \to 0$ as $D \to \infty$. From this and *Lemma 2*, it follows that the effect of I_{out} can be bounded by a constant by introducing an appropriate spacing D between squares, regardless of the number of links n.

C. Complexity of the algorithm

The complexity of the algorithm A is, in general, 3^n when there are n links (Section II). However, when no more than a constant c among n links can be selected, the complexity reduces to less than n^c selections, which is polynomial in n. In the following, we show that for each local problem (scheduling in a square of area $(lD)^2$), the number of links in the optimal solution is bounded by a number c, which is independent of n.

First, consider only the selected links with $p_j = 1$. The inequality $\sum_{j \in \mathcal{L}_U(i)} p_j g_{ji} \leq 1/x$ in the previous section still holds in this case, since excluding $p_j < 1$ can only reduce the sum. Further, the sum of g_{ji} cannot be more than $1/x$ since $p_j = 1$. On the other hand, $d(T_j, R_i) \leq \sqrt{2} + 1$ for all $i, j \in \mathcal{L}_U(i)$ and hence, $g_{ji} \geq \frac{1}{(\sqrt{2}+1)^\alpha}$. Therefore, in a unit square, the number of links with $p_j = 1$ should be at most $\left\lceil \frac{(\sqrt{2}+1)^\alpha}{x} \right\rceil$, which is a constant. Therefore, the number of selected links with $p_j = 1$ in a square of area $(lD)^2$ is $O(l^2 D^2)$.

Next, consider only the set of selected links with $0 < p_j < 1$. The principal sub-matrix of $\mathbf{A}$ for this set is positive definite (see the algorithm A in Section II.) Therefore, for any two links in this set, $A_{ii} A_{jj} > A_{ij}^2$. This condition implies $d(R_j, R_i) > (2^{1/\alpha} - 1) d_{min}$ [12]. Now, considering a packing of a square of area $(lD)^2$ with disks of radius $\frac{(2^{1/\alpha}-1)d_{min}}{2}$, it is clear that the number of links with $0 < p_j < 1$ is less than $\frac{(lD)^2}{\kappa_2}$, where κ_2 is the constant area of the disk. Consequently, the number of links with $p_j \neq 0$ is $O(l^2 D^2)$ for each local problem and thus, the complexity of A in S_A is $n^{O(l^2 D^2)}$. Since there is at most n non-empty squares for each set of squares $S_{(h,v)}$ and there are $(l+1)^2$ distinct shifts, the overall complexity of S_A is $(l+1)^2 n^{O(l^2 D^2)}$.

So far, we have calculated the complexity and accuracy in terms of parameters l and D. We can obtain the best trade-off between complexity and accuracy by optimizing over l and D. This results in the choice $l = \lceil 4\epsilon^{-1} - 1 \rceil$ and $D = \left(\frac{4\kappa_1}{\epsilon}\right)^{\frac{1}{\alpha-2}} + 2$ and guarantees a performance ratio and complexity as below.

Theorem 1: Under the quadratic link rate approximation $\tilde{c}_i$ in (2), which is valid in UWB networks, the algorithm S_A which uses c_i' in (5) solves WSRM with any required performance ratio $\rho = (1 - \epsilon)$ at a complexity that is polynomial in n, i.e., $\epsilon^{-2} n^{O(\epsilon^{-2-\frac{2}{\alpha-2}})}$.

V. Simulation results

The worst case complexity of the algorithm S_A in *Theorem 1* appears to be practically very high, although theoretically it is only polynomial in n. In practice, however, there are not too many links concentrated in a small area (i.e., each square in Fig. 1). Further, with a small x (i.e., large bandwidth typical in a UWB network), the effect of I_{out} may be small enough to allow small D, since the thermal noise will be dominant over I_{out}. Thus, in simulations, instead of c_i' in (5), we use c_i'' in (6) ignoring I_{out} when applying S_A to randomly generated networks. Then, based on the obtained results, we calculate I_{out} and discuss its effect in different scenarios. This will provide guidelines on how to practically choose the algorithm parameters D and l to obtain a solution close to the optimum, when a specific value of the system parameter x is given. For this purpose, we consider three different values of (D, l); $(1, 13)$, $(1.4, 9)$ and $(2, 6)$ respectively, keeping the area of the network unchanged. Since we can calculate the approximation error, we allow D close to 1, though the analysis in the previous section that considers the worst case requires $D > 2$.

TABLE I

SIMULATION RESULT FOR $D = 1, l = 13$

x	$\mathcal{I}_{S_A}$	$\mathcal{I}_{LB}$	$\mathcal{I}_{UB}$	$\min \frac{\tilde{c}_i}{c''_i}$	$\min \frac{\tilde{c}_i}{c_i}$
1	0.387	0.355	0.448	-1.150	-5.404
0.1	0.855	0.852	0.992	0.935	0.625
0.01	0.928	0.928	1.077	0.994	0.897
0.001	0.938	0.938	1.088	0.999	0.998

TABLE II

SIMULATION RESULT FOR $D = 1.4, l = 9$

x	$\mathcal{I}_{S_A}$	$\mathcal{I}_{LB}$	$\mathcal{I}_{UB}$	$\min \frac{\tilde{c}_i}{c''_i}$	$\min \frac{\tilde{c}_i}{c_i}$
1	0.368	0.348	0.454	0.132	-0.296
0.1	0.811	0.809	1.002	0.967	0.632
0.01	0.882	0.882	1.089	0.998	0.942
0.001	0.892	0.892	1.101	1.000	0.998

TABLE III

SIMULATION RESULT FOR $D = 2, l = 6$

x	$\mathcal{I}_{S_A}$	$\mathcal{I}_{LB}$	$\mathcal{I}_{UB}$	$\min \frac{\tilde{c}_i}{c''_i}$	$\min \frac{\tilde{c}_i}{c_i}$
1	0.337	0.327	0.459	0.836	0.567
0.1	0.746	0.745	1.016	0.988	0.667
0.01	0.811	0.811	1.104	0.999	0.898
0.001	0.820	0.820	1.116	1.000	0.998

We also consider different values of x ranging from 1 to 0.001 representing different bandwidths. 20 random networks were generated with uniformly distributed $n = 96$ links and $d(T_i, R_i) = 1$. When S_A was run, the maximum number of links for a local problem turned out to be 14. The weight λ_i was set to be $\frac{1}{n}$ for all links (max sum-rate).

Table I, II and III show the simulation results of the 20 sample networks. Recall that c_i, $\tilde{c}_i$ and c''_i are the Shannon capacity, the quadratic approximation and the quadratic approximation with $I_{out} = 0$. To investigate the effect of I_{out}, we evaluated $\tilde{c}_i$ considering I_{out} for every solution obtained by S_A. (Recall that in simulations, S_A uses c''_i, which ignores I_{out}.) Using the evaluated $\tilde{c}_i$, we also evaluated the corresponding weighted sum-rate (denoted by $\mathcal{I}_{LB}$), which is a lower bound on the optimum $\mathcal{I}^*$. $\mathcal{I}_{UB}$ is an upper bound on $\mathcal{I}^*$ computed by using *Lemma 1*, i.e., $\left(1 - \frac{1}{l+1}\right)^{-2} \mathcal{I}_{S_A}$. In the last two columns of the tables, $\min_i \frac{\tilde{c}_i}{c''_i}$ shows the worst-case degradation on link capacity due to I_{out} and $\min_i \frac{\tilde{c}_i}{c_i}$ shows the accuracy of the quadratic link rate approximation $\tilde{c}_i$. For $x \leq 0.01$, $\mathcal{I}_{S_A} \approx \mathcal{I}_{LB}$, i.e., $\frac{\mathcal{I}_{LB}}{\mathcal{I}_{UB}} \approx \left(1 - \frac{1}{l+1}\right)^2$, and $\min \frac{\tilde{c}_i}{c''_i} \approx 1$, showing that I_{out} is negligible. This is because the thermal noise $N_0 W$ is dominant over I_{out} due to the large bandwidth $W = \frac{W_0}{x}$. In contrast, for $x \geq 0.1$, we can clearly see the reduction in the weighted sum-rate and link rate caused by I_{out}. The quadratic link rate approximation also becomes inaccurate in this case. The effect of I_{out} becomes stronger as D decreases showing that for a large x, a large D is required to prevent strong interference across the squares in Fig. 1.

From the analytical result in the previous sections, we know that the complexity of the algorithm is exponential in $(lD)^2$, i.e., the area of each square in Fig. 1. On the other hand, the shifting parameter l and the width of strips D determine the performance ratio ρ by individually affecting the two factors $\left(1 - \frac{1}{l+1}\right)^2$ and $(1 - x\beta)^2$ constituting ρ. Therefore, when there is a restriction on the computational power, the simulation results may be translated into the following guideline for a better choice of l and D. For small x for which the effect of I_{out} is negligible, i.e., $(1 - x\beta)^2 \approx 1$, it is better to choose a small D and a large l while keeping $(lD)^2$ constant, in order to make ρ close to 1 without increasing the complexity. Notice that the maximum $\frac{\mathcal{I}_{LB}}{\mathcal{I}_{UB}}$ was obtained in the case with small x and $D = 1$ (Table I). On the other hand, when x is large, we need to increase D to restrict the effect of I_{out} while balancing the two terms $(1 - x\beta)^2$ and $\left(1 - \frac{1}{l+1}\right)^2$ to maximize the product (Table III).

VI. CONCLUSION

We presented a polynomial-time algorithm for the WSRM problem, which appears as a core problem in several scheduling scenarios. Our algorithm guarantees the accuracy of the solution in cases with large bandwidths. This algorithm can be used as a protocol design tool for UWB systems, considering that there is no other known computationally efficient scheduling method, which has a formally proven guarantee on accuracy.

It is also worth stating that the modified shifting strategy can solve WSRM even for some narrow band systems, when there is a local problem solver available. One important example is the binary power (i.e., $p_i = \{0, P_{max}\}$) case where the complexity of the local solver is a constant. The same shifting argument in Section III applies to this case. This result will be presented in a subsequent paper.

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