Maximal Scheduling in a Hypergraph Model for Wireless Networks

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***Abstract*— We introduce a hypergraph based interference model for scheduling in wireless networks. As a generalization of the graph model, hypergraph considers the conflicts caused by sum interference. We show in an arbitrary network, the successful transmissions under any graph model can be improved by a hypergraph. In some networks, a hypergraph can double the uniform throughput compared to the disk graph. We then analyze the capacity region of maximal scheduling in the hypergraph, where a linear programming (LP) based lower bound is formulated and proven to be tight. We also show that the maximal scheduling in hypergraph can guarantee a certain fraction of the capacity region. Simulation results show that maximal scheduling in hypergraph can achieve about 40% more uniform throughput than in graph for random networks.**

Keywords: Hypergraph, maximal scheduling, capacity region, wireless networks, MAC.

I. Introduction

In wireless networks, user nodes communicate with each other using a *shared wireless spectrum*. This resource sharing introduces co-channel interference, which may cause severe deterioration to the communication quality of a communication link. The co-channel interference is often controlled by appropriate scheduling in the MAC layer, where a scheduler allocates resources among single hop communication flows so that there is no "conflict", which is defined by a specified interference model and is assumed to correspond to communication failures in realistic networks.

In past research, a flow contention graph is used to describe the interference between communication links. In such a graph $G_c = (V_c, E_c)$, where V_c is the node set consisting of communication links and E_c is the edge set, two links i, j are allowed to share the same resource if and only if $(i, j) \notin E_c$. Implicitly it is assumed that interference is "pairwise", i.e., the failure of a link can be explained completely by another link. Thus, any independent set $I \in V_c$ can transmit successfully together. This model is popular due to its simple structure for analysis and the existence of efficient graph algorithms in applications. However, a graph is neither accurate nor efficient in describing the interference: 1) *Inaccuracy:* the graph gives no performance guarantee. Although no communication failure occurs because of a single link, it is still possible because of the sum interference, i.e., the *cumulative* interference from co-channel transmitters. 2) *Inefficiency:* The resource is not fully utilized in the case of "conservatively" constructed edges in a graph, i.e., in order to reduce the sum interference, a pair of links are not allowed to transmit together even if they are able to in certain scenarios. In this paper we try to tackle these problems by generalizing the pairwise model to the hypergraph, where the sum interference is considered. A hypergraph has the same the node set as the graph, but can have hyperedges including 2 or more links. Note that a hypergraph model has been proposed in [1] for cellular networks. However, to the best of our knowledge, this is the first paper that applies a hypergraph model to wireless ad hoc networks. We show the graph model can be improved by a hypergraph in both cases mentioned above. First, the hypergraph is proved to be more accurate than the graph, in that given an arbitrary network, the throughput achieved by any scheduler using the graph can be improved by a new scheduler using an appropriate hypergraph. Second, under certain scenarios a hypergraph can yield a factor of 2 gain in terms of uniform throughput over the "disk" graph, where an edge is formed between two links if and only if the distance between one link's transmitter to the other's receiver is less than a fixed threshold.

Convinced of the value of the hypergraph, we then analyze the theoretical capacity region under maximal scheduling in a hypergraph model. This region is defined as the arrival rate vector which can be stabilized by *any* maximal scheduler. A maximal schedule was defined and analyzed for graph model in [4], see also [5]. In a hypergraph model, similar to the previous definition, a maximal schedule corresponds to the situation where if a link i wants to transmit, either i is scheduled or there exists a set of scheduled links $\{i_1, \dots, i_k\}$ such that $\{i, i_1, \dots, i_k\}$ is not allowed by a hyperedge. Note the optimal scheduler has been proposed in [2]. However, it is centralized and can have exponential complexity, and therefore is not suitable for realistic networks. Maximal scheduling, although suboptimal, is attractive due to its low complexity and distributed implementation. In this paper we give an LP based lower bound of the capacity region under maximal scheduling. We also show that this lower bound is tight in the sense that there exists a family of networks, in which for any point outside the lower bound region, there exists an arrival process and a maximal scheduler that makes the network unstable. We also generalize the performance guarantee from [4] by showing that maximal scheduling in a hypergraph can guarantee a fraction of the capacity region.

The organization of this paper is as follows: in Section II we describe the scheduling problem and introduce the hypergraph model. In Section III we show the improvement by using hypergraph. Section IV discusses the capacity region of maximal scheduling in hypergraph. Section V gives the simulation results and Section VI concludes this paper.

II. Hypergraph Model

We consider the scheduling problem in an arbitrary wireless network. The topology is described by a directed graph $G = (V, E)$, where V is the set of user nodes and E is the set of communication links. A link $i = (u, v) \in E$ can exist only if node v is in the transmission range of node u, which is determined by the transmission power and channel propagation. We assume a TDMA system. In each time slot, the transmission of a scheduled link i is successful if and only if the signal to interference plus noise ratio (SINR) at its receiver satisfies the following constraint:

$$\frac{p_i g_{ii}}{\sum_{j \in \mathcal{T}_i} p_j g_{ji} + N_0} \geq \gamma_i \tag{1}$$

where γ_i is the threshold determined by the modulation and coding scheme and the detection method, $\mathcal{T}_i$ is the set of links transmitting together with link i, g_{ji} is the channel gain from the transmitter of link j to the receiver of link i, and N_0 is the noise power. Otherwise we say that the transmission of link i is in *outage*.

We consider a single hop network where each transmitter is associated with an external source and an infinite buffer. The arrival process for the transmitter of link i is modeled as $A_i(n)$, which is the cumulative number of packets arrived during the first n slots. The only requirement on the arrival process is that the strong law of large numbers, i.e., $\lim_{n\to\infty} A_i(n)/n = a_i$ with probability 1 (w.p.1). We assume that the packets arrive at the beginning of each time slot. In a time slot, a scheduler chooses a "conflict free" set of links with nonempty buffers for transmission based on certain interference model. The dynamics of the queue length at link i can be described as

$$Q_i(n) = Q_i(0) + A_i(n) - D_i(n). \tag{2}$$

where $D_i(n)$ is the cumulative number of departed packets in the first n time slots and $Q_i(n)$ is the queue length at the transmitter of link i at the end of slot n.

Usually the interference is modeled as a graph $G_c = (V_c, E_c)$, where V_c is the set of links and E_c is the set of conflicts, i.e., $(i, j) \in E_c$ means the concurrent transmission of link i and j is not allowed. We define the neighbor set of a link i as $\mathcal{N}_i = \{j : (i, j) \in E_c\}$ and its interference set as $\mathcal{I}_i = \{e \in E_c : i \text{ appears in } e\}$. Note that $\mathcal{N}_i$ is the set of nodes and $\mathcal{I}_i$ is the set of edges. In a maximal schedule, a link i with nonempty buffer is allowed for transmission if and only if no link in $\mathcal{N}_i$ is scheduled.

We propose a directed hypergraph model $H = (V_c, E_H)$ for scheduling. A hypergraph $H = (V_c, E_H)$ is a generalization of the graph, where V_c is the same as in G_c (the set of links)

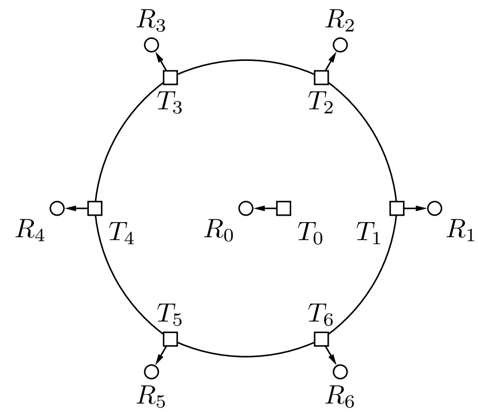

Fig. 1. A "star" network with the transmitters of the 6 outside links lying on a hexagon centered at link 0's receiver. The squares denote the transmitters, and the circles deonte the receivers.

and E_H is the hyperedge set such that that for any two link sets $S, T \subseteq V_c$, $(S, T) \in E_H$ only if

1) *(Validity)* When the links $S \cup T$ are scheduled all the links in S fail and none of T fails;
2) *(Minimality)* If any link in $S \cup T$ is removed, no failure occurs if only the remaining links are scheduled.

Denote the neighbors of link i as $\mathcal{N}_i = \{j : \exists e \in E_H \text{ such that both } i \text{ and } j \text{ appears in } e\}$ and the interference set of link i as $\mathcal{I}_i = \{e \in E_H : i \text{ appears in } e\}$. We define a hyperedge $e = (S, T)$ in $\mathcal{I}_i$ as "active" if $(S \cup T)\backslash\{i\}$ are scheduled. In a maximal schedule, given any link i with nonempty buffer, either it is scheduled, or there exists an active hyperedge $e \in \mathcal{I}_i$.

Example: Consider the network in Fig. 1, where the only failure event is that link 0 fails when all of the links are transmitting. In this case the hypergraph is $H = \{V_c, \{e\}\}$ where $V_c = \{0, 1, 2, 3, 4, 5, 6\}$ and $e = (\{0\}, \{1, 2, 3, 4, 5, 6\})$. Each link has the same neighbor set $\mathcal{N} = \{1, 2, 3, 4, 5, 6\}$ and interference set $\mathcal{I} = \{e\}$. Note $\{0, 1, 2, 3, 4, 5, 6\}$ is not valid because of hyperedge e. Both $\{0, 1, 2, 3, 4, 5\}$ and $\{1, 2, 3, 4, 5, 6\}$ are maximal schedules, where in the later case the hyperedge $e \in \mathcal{I}_0$ is active. If the buffers of link $5, 6$ are empty, $\{0, 1, 2, 3, 4\}$ is a maximal schedule.

We are interested in the theoretical capacity region under maximal scheduling in the hypergraph, $\mathcal{A}_H^M$, which is defined as the set of stable arrival rate vectors under any maximal scheduler. Here the stability corresponds to rate stability [4], i.e., $\lim_{n\to\infty} D_i(n)/n = a_i$ for all i. We denote the theoretical capacity region under the optimal scheduling [2] as $\mathcal{A}_H$. In the next section, we will show that the hypergraph has better performance than the graph in terms of accuracy and efficiency.

III. Improvements due to Hypergraph Model

A graph model is either inaccurate or inefficient due to its pairwise nature. First, if the graph is constructed such that an edge exists only if it is guaranteed to result in outage, it is inaccurate because the sum interference may still cause outage even if no pairwise interference does. For example, consider a star shaped network in Fig. 1 with a center link and 6 peripheral links. All the links have length 0.25 normalized by the radius of the hexagon. We assume the signal to noise ratio (SNR) is 20 dB and $\gamma = 10$ dB. If all the nodes are simultaneously transmitting, the SINR at the center receiver is 9.84 dB, and the minimum SINR among outside receivers is 12.84 dB, which means the outage occurs only at the center link. If one of the outside link is silent and all the others are transmitting, the SINR at the center receiver is 10.55 dB, i.e., successful reception. Thus a graph based on pairwise conflicts leads to $E_c = \emptyset$ because no pairwise transmission causes outage. This means that a scheduler will choose all the links with nonempty queues to transmit. In this case the center fails whenever all the other links have packets. On the other hand, a hypergraph can prevent this by adding a hyperedge $e = (\{0\}, \{1, 2, 3, 4, 5, 6\})$. In this case, a schedule is successful if and only if it is valid in the hypergraph. Formally, we prove that for an arbitrary network, the number of successful transmissions can be increased by adding hyperedge constraints.

Theorem 1: For an arbitrary network, given any graph G_c and scheduler π, there is a hypergraph H and scheduler π' such that 1) H has at most one more hyperedge than G_c and 2) the sum of successful transmissions under π' is not less than that of π.

Proof: Given a graph G_c, we assume that at least one independent set in G_c causes outage, since otherwise we let $H = (G_c, E_H = E_c), \pi' = \pi$ and the result holds. Suppose the set $\{i_1, i_2, \ldots, i_k\}$ causes transmission failure at link i_1. Let $H = (V_c, E_c \cup \{e\})$ with $e = (\{i_1\}, \{i_2, \ldots, i_k\})$ be the hypergraph. Also define the scheduler π' as follows: in each time slot, given the schedule $\pi(n)$, 1) check whether the links in e are scheduled; if so, remove i_1 from $\pi(n)$; 2) check whether any links with empty queues are in $\pi(n)$; if so remove them.

For any time slot n, if not all the links in e are scheduled in $\pi(n)$, no modification is done after (1). Otherwise, by removing a link in outage, we are not decreasing the number of successful transmissions in this time slot. Now consider (2), where any empty queue is the result from at least one more transmission failure in G_c than in H. Therefore after removing the links with empty queues, the sum of successful transmission in H is still not less than in G_c. ■

Second, if the graph is constructed conservatively to suppress the sum interference, i.e., disallowing concurrent transmissions of certain links even if no outage is caused by pairwise interference, the resource may not be fully utilized. We will show this by an example. Consider the star network mentioned above, where an edge is formed whenever the distance from one link's receiver to another link's transmitter is below a certain threshold. The resulting graph is a star with 6 edges, whereas the hypergraph consists only one hyperedge. Therefore the capacity region for the graph is $a_1 + \max_{i=2}^{7} a_i < 1$, and for the hypergraph it is $\sum_{i=1}^{7} a_i < 6$. If we consider the uniform throughput, the hypergraph yields a uniform rate of $6/7$, while the graph only has $1/2$. Thus the hypergraph will have a gain $12/7$ over the graph. It can be verified that under the same physical layer specifications, the number of peripheral links in the network can be more than 10 with a small modification of link length. Thus hypergraph can have a gain of approximately 2 over graph in terms of uniform throughput.

IV. Maximal Scheduling in Hypergraph

Seeing Section III motivated the application of the hypergraph model by showing that a) it is never worse than the graph model and b) it can give a substantial improvement over the graph model in certain examples, in this section we will analyze its theoretical capacity region under maximal scheduling. Note that this is an idealized situation, i.e., we assume that in any maximal schedule the transmissions are successful in the network.

The following theorem gives a lower bound on the capacity region which generalizes the corresponding lower bound for the graph model in [4] and [5].

Theorem 2: The network is rate stable under an arrival rate vector a if the following LP in variables $\{x_{ij}\}$ is feasible:

$$a_i + \sum_{j \in \mathcal{N}_i} a_j x_{ij} < 1 \qquad \forall i \in V_c \tag{3}$$

$$\sum_{j \in \mathcal{N}_i \cap e} x_{ij} \geq 1 \qquad \forall i \in V_c, e \in \mathcal{I}_i \tag{4}$$

$$x_{ij} \in [0, 1], \qquad \forall i, j \in V_c \tag{5}$$

Note when the hypergraph is a graph, for each edge (i, j), we have $x_{ij} = 1$ from constraint (4), and the result is the same as [4].

We first discuss the intuition behind the proof. In any maximal schedule, either a link i with nonempty buffer is scheduled or an edge in $\mathcal{I}_i$ is active. If there is a weight assignment in each i's neighborhood such that, in any time slot, the weighted sum of departures in $\mathcal{N}_i \cup \{i\}$ is more than 1, and the corresponding weighted sum of average arrivals is less than 1, the network is stable.

Proof: We will use the fluid limit to prove this theorem. Note similar proofs have been derived in [3],[4], we only highlight the changes due to the hypergraph model. For details about the convergence to the fluid limit and conditions for rate stability please refer to these papers and the references therein.

Consider a fluid limit $\{\overline{A}(t), \overline{D}(t), \overline{Q}(t)\}$ of the network. We claim that if $\overline{Q_i}(0) = 0$, we have $\overline{Q_i}(t) = 0$ w.p. 1 for all t and i. Suppose this is not true, then there exist a time t_1 and i such that $\overline{Q_i}(t_1) = x > 0$, which implies $\overline{Q_i}(t_1) + \sum_{j \in \mathcal{N}_i} x_{ij} \overline{Q_j}(t_1) = y > 0$. Without loss of generality we assume that $\overline{Q_i}(t_1) + \sum_{j \in \mathcal{N}_i} x_{ij} \overline{Q_j}(t) < y$ for $0 < t < t_1$. Because $\overline{Q_i}(t)$ is continuous, there exists $0 < t_0 < t_1$ such that $\overline{Q_i}(t) > x/2$ for every $t \in (t_0, t_1]$, which means for sufficiently large k we have $Q_i^{r_{n_k}}(t) > x/4$ and $r_{n_k} x/4 > 1$ for every $t \in (t_0, t_1]$. Therefore we have $Q_i(r_{n_k} t_0) > 1$, i.e., the queue i is not empty, and there exists $\epsilon > 0$ such that

$$\overline{Q_i}(t_1) - \overline{Q_i}(t_0) + \sum_{j \in \mathcal{N}_i} x_{ij} \big(\overline{Q_j}(t_1) - \overline{Q_j}(t_0)\big) \geq \epsilon. \tag{6}$$

For k large enough, we have

$$Q_i^{r_{n_k}}(t_1) - Q_i^{r_{n_k}}(t_0) + \sum_{j \in \mathcal{N}_i} x_{ij}\big(Q_j^{r_{n_k}}(t_1) - Q_j^{r_{n_k}}(t_0)\big) \triangleq \Phi_1 - \Phi_2$$

where

$$\Phi_1 = \sum_{j \in \mathcal{N}_i} x_{ij}[A_j^{r_{n_k}}(t_1) - A_j^{r_{n_k}}(t_0)] + A_i^{r_{n_k}}(t_1) - A_i^{r_{n_k}}(t_0)$$

$$\Phi_2 = \sum_{j \in \mathcal{N}_i} x_{ij}[D_i^{r_{n_k}}(t_1) - D_i^{r_{n_k}}(t_0)] + D_i^{r_{n_k}}(t_1) - D_i^{r_{n_k}}(t_0).$$

Note in a maximal schedule if the queue in link i is not empty, either link i will transmit or one of the hyperedges in $\mathcal{I}_i$ is active. From Eqn. (4) and (5), the weighted departure in each time slot is more than 1. Therefore we have $\Phi_2 \leq (t_1 - t_0)$ and by taking limit we get

$$\begin{aligned}
& \overline{Q_i}(t_1) - \overline{Q_i}(t_0) + \sum_{j \in \mathcal{N}_i} x_{ij}\big(\overline{Q_j}(t_1) - \overline{Q_j}(t_0)\big) \\
\leq \quad & a_i(t_1 - t_0) + \sum_{j \in \mathcal{N}_i} a_j x_{ij}(t_1 - t_0) - (t_1 - t_0) \\
= \quad & (a_i + \sum_{j \in \mathcal{N}_i} a_j x_{ij} - 1)(t_1 - t_0)
\end{aligned}$$

which is negative and contradicts Eqn. (6). Therefore we have $\overline{Q_i}(t) = 0$ w.p. 1 for all t. Therefore the rate stability holds following the argument in [3],[4]. ■

This lower bound region $\mathcal{A}_H^{ML}$, i.e., the rate vectors for which the LP is feasible, is not convex. To show this, consider a hypergraph $H = (V_c, E_H)$ where $V_c = \{1,2,3\}$ and $E_H = \{(\{1\},\{2,3\})\}$. In this hypergraph, an outage occurs if and only if all of the links transmit simultaneously. Consider the arrival rate vectors $a = (1,1,0), a' = (1,0,1)$ and $a'' = (0,1,1)$. Clearly each of them can be stabilized by any maximal scheduler. Also it can be verified that all of them are on the boundary of $\mathcal{A}_H^{ML}$. However, their convex combination $\hat{a} = 1/3(a + a' + a'') = (2/3, 2/3, 2/3)$ does not belong to $\mathcal{A}_H^{ML}$. Actually, this non-convexity is due to the definition of the capacity region under maximal scheduling, which requires that the arrival rate be stable under *any* maximal scheduling. In this example, if we fix $a = (2/3, 2/3, 2/3)$ and consider an arrival process where the packets arrive in the first two of every three times slots for all the links, a maximal scheduler which gives higher priority to link $2,3$ will make the network unstable because link 1 only get one time slot for transmission in every three time slots.

In the following theorem we show the tightness of the proposed lower bound region. Particularly, we show that there exists a family of hypergraphs in which the LP feasibility is both sufficient and necessary for the network to be stable under maximal scheduling.

Theorem 3: There exists a hypergraph H such that for any arrival rate vector a outside $\mathcal{A}_H^{ML}$, the network is unstable under certain maximal scheduler π and arrival process with rate a.

Proof: If the hypergraph purely consists of edges, i.e., the graph model, we have $x_{ij} = 1$ in the LP for any $j \in \mathcal{N}_i$. This is the same situation as in [4], which has proved it correct. Now we consider a hypergraph with some hyperedges of cardinality more than 2. Particularly consider the following "star" hypergraph with n links and m hyperedges. All the hyperedges have only one common link, the center link 1. Thus they are pairwise disjoint if we remove link 1 from each hyperedge. Now assume that there is an arrival rate vector a such that the LP is not feasible. Denote the link with minimum rate except link 1 in hyperedge e_k as i_k and the set of such links as M. We claim that $a_1 + \sum_{k=1}^m a_{i_k} \geq 1 + \epsilon$ for some $\epsilon > 0$. Otherwise we can choose $x_{1i_k} = 1, 1 \leq k \leq m$ and $x_{1j} = 0$ else, which satisfies Eqn. (4) and (5), then our claim holds from the assumption that the LP is not feasible.

Consider a maximal scheduler that assigns lowest priority to link 1. Let the arrival process be such that at most one packet arrives in at most one of the links in M in each time slot. Further, in a give time slot, if $i_k \in M$ gets a packet, then all links in the hyperedge of k also get at least one packet, so that link 1 can not transmit. Thus, the fraction of time allocated for link 1 is $1 - \sum_{k=1}^m a_{i_k} < a_1$ by our claim, which makes link 1 unstable. ■

Similar to the graph model, the maximal scheduling in hypergraph gives a guaranteed performance ratio compared to the optimal scheduling. The following theorem gives an optimization based formulation which generalizes the theorem derived in [4] for the graph model:

Theorem 4: For any $a \in \mathcal{A}_H$, we have $a/K \in \mathcal{A}_H^{ML}$, where $K = \max(K_1, K_2, \ldots, K_n)$ with K_i the optimal value of the following problem (for fixed i):

$$\begin{aligned}
\max \quad & x_i + \sum_{j \in \mathcal{N}_i} x_j \\
\text{subject to} \quad & \sum_{j \in e} x_j \leq \mathbf{card}(e) - 1 \qquad \forall e \in E_H \\
& x_j \in \{0, 1\}
\end{aligned}$$

where $\mathbf{card}(e)$ means the cardinality of e.

Note when the hypergraph is a graph, the optimal value of the above problem is the "interference degree" of link i [4], which is the cardinality of the maximum independent set in link i's neighborhood. Also a lower bound is easily computable by relaxing it to an LP.

Proof: Denote $a \in \mathcal{A}_H$ as any rate vector stable under the optimal scheduling. We have $0 \leq a_j \leq 1$ for all j. For any hyperedge e, the number of transmitting links in e in a time slot should be less than its cardinality, and according to the optimization formulation above, the number of transmitting links in $\mathcal{N}_i \cup \{i\}$ should be less than K_i. By taking the time average we get $a_i + \sum_{j \in \mathcal{N}_i} a_j \leq K_i \leq K$ for all $i \in V_c$, where, by dividing K in both sides we have $\frac{a_i}{K} + \frac{\sum_{j \in \mathcal{N}_i} a_j}{K} \leq 1$. It can be seen that the LP in Theorem 2 is feasible for a/K (choose $x_{ij} = 1$) and hence $a/K \in \mathcal{A}_H^{ML}$. ■

V. Simulation Results

In this section, we compare the performance of a graph based model and a hypergraph based model in a realistic

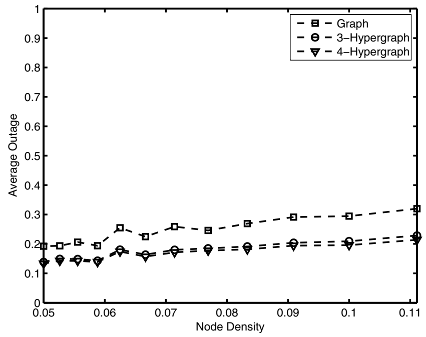

Fig. 2. The outage probability in a single time slot with all nodes contending for transmission.

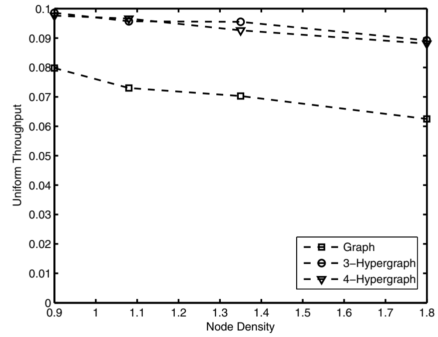

Fig. 3. The maximum uniform throughput in graph and hypergraph under outage constraint 0.1.

network using a simulation. Both models are constructed based on thresholds. For example, in the graph, an edge (i,j) is formed if and only if $g_{ii} \leq \gamma_g g_{ji}$, and in the hypergraph, a hyperedge $(\{k\},\{i,j\})$ is formed if and only if $g_{kk} \leq \gamma_h(g_{ik}+g_{jk})$ and no subset forms an edge. To limit complexity, the hyperedges are constructed only from nodes in link i's neighborhood defined by $\mathcal{N}_i = \{j : g_{ii}/g_{ij} \leq \gamma_n\}$. $\gamma_n, \gamma_g, \gamma_h$ are fixed thresholds chosen carefully. We assume all links have constant length 1, and SNR$=20$dB.

A. Outage Probability

We simulated the outage probability in a single time slot with all the nodes contending for transmission in a network consists of 100 links. Usually the threshold is chosen based on heuristics (e.g. [6]), which use global information about the network (such as density) and various approximations. In the case where no extra information is known, one can choose $\gamma_g = \gamma$ as guided by physical layer considerations ($\gamma_g = \gamma$ is optimum in a sparse network). We show that in this case the hypergraph can give better performance guarantee by considering the sum interference. We set $\gamma_g = \gamma_h = \gamma = 10$dB, $\gamma_n = 17$dB and adjust node density by changing the network area. Fig. 2 compares the outage probability in both models. The result is averaged over 1000 time slots and 20 random networks. In the figure, "4-Hypergraph" refers to a hypergraph whose hyperedge size is at most 4. Thus, a graph is a "2-Hypergraph". It can be observed that the outage probability in the hypergraph model is approximately 30% lower than that of graph. Note that the hypergraph is constructed locally to reduce the complexity, and by increasing the γ_n we can get a lower outage probability. For this locally formed hypergraph, size 3 hyperedge is sufficient to describe the interference.

B. Throughput

We simulate the maximum uniform throughput in $\mathcal{A}_H^{ML}$ with target outage probability less than 0.1. We generate a network of 60 nodes in each simulation and assume that global network topology can be used to change the graph and hypergraph. We first perform a binary search for the optimal γ_g for the graph model, then set $\gamma_n = \gamma_g$ and search for the hypergraph threshold. The maximum uniform throughput is calculated by solving the LP in Theorem 2. Fig. 3 shows the average uniform throughput over 40 random networks. It can be observed that even a 3-Hypergraph is more efficient than a graph since it can achieve the same outage probability with significantly larger throughput. For example, in the case of density 1.8, the 3-hypergraph has about 40% more throughput than the graph.

VI. Conclusion

In this paper, the hypergraph is introduced as a new interference model for scheduling in wireless networks. Compared to the traditionally used graph model, it is shown that hypergraph can improve the performance by considering the sum interference. Also the capacity region under maximal scheduling is analyzed. A lower bound is obtained and shown to be tight. It is also proved that in hypergraph, maximal scheduling can achieve a guaranteed fraction of the capacity region, which generalizes the "interference degree" defined in the graph.

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