Generating Platform-Adapted DSP Libraries Using SPIRAL

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www.ece.cmu.edu/~spiral

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Automates the

Implementation

- cuts development costs
- coding less error-prone

Optimization

- code manipulation techniques like, e.g., unrolling cannot be done by hand in reasonable time
- allows systematic exploration of alternatives both at algorithmic level and code optimizations

Platform-Adaptation

- takes advantage of architecture specific features
- porting without loss of performance

of DSP algorithms

- are performance critical

→ **A library generator for highly optimized, platform-adapted signal processing transforms**

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Organization

- **SPIRAL approach**
- SPIRAL system
- Some experimental results
- Recent work

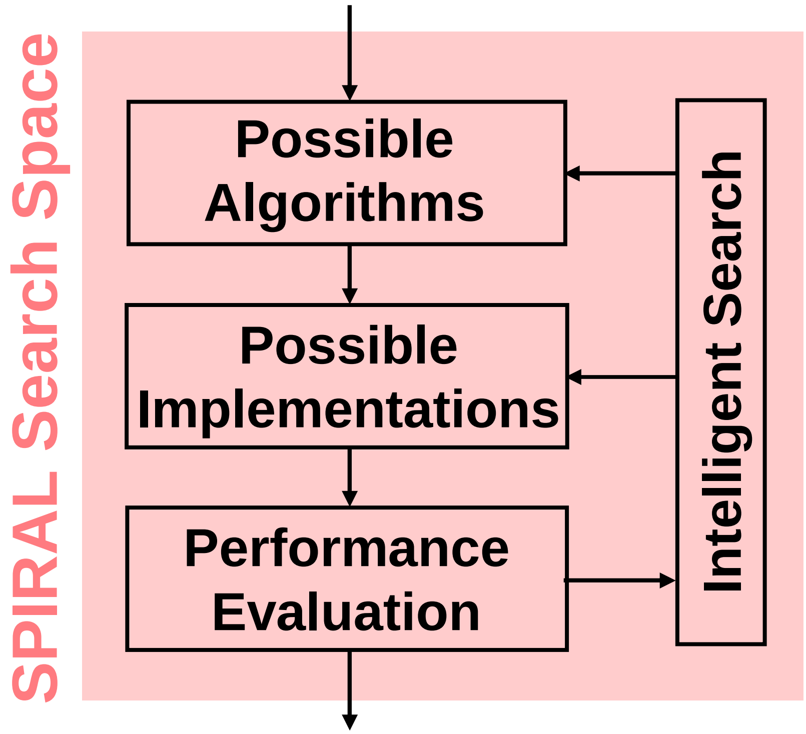
Key Observations

- For every DSP transform there are exponential many different algorithms, which do not differ in arithmetic cost
- The best algorithm is highly platform dependent
- The best algorithm is hard to determine

SPIRAL Methodology

given → **DSP Transform**

(DFT, DCT, Wavelets etc.)



→ **adapted implementation**

given → **Computer Architecture**

Uniprocessor:

- Pentium
- SUN
- Alpha

- Multiprocessor
- Hardware

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DSP Algorithms: Example 4-point DFT

Cooley/Tukey FFT (size 4):

Fourier transform | Diagonal matrix (twiddles)

$$DFT_4 = (DFT_2 \otimes I_2) \cdot T_2^4 \cdot (I_2 \otimes DFT_2) \cdot L_2^4$$

Kronecker product | Identity | Permutation

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} = \left[\begin{array}{cc|cc} 1 & & 1 & \\ & 1 & & 1 \\ \hline 1 & & -1 & \\ & 1 & & -1 \end{array}\right] \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & j \end{bmatrix} \left[\begin{array}{cc|cc} 1 & 1 & & \\ 1 & -1 & & \\ \hline & & 1 & 1 \\ & & 1 & -1 \end{array}\right] \left[\begin{array}{cc|cc} 1 & & & \\ & 0 & 1 & \\ \hline & 1 & 0 & \\ & & & 1 \end{array}\right]$$

- **product of structured sparse matrices**
- **mathematical notation**

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Transforms, Rules, & Formulas

DSP transform DFT_{nm} **a matrix**

Rule

$$DFT_{nm} \rightarrow \left(DFT_n \otimes I_m\right) \cdot D \cdot \left(I_n \otimes DFT_m\right) \cdot P$$

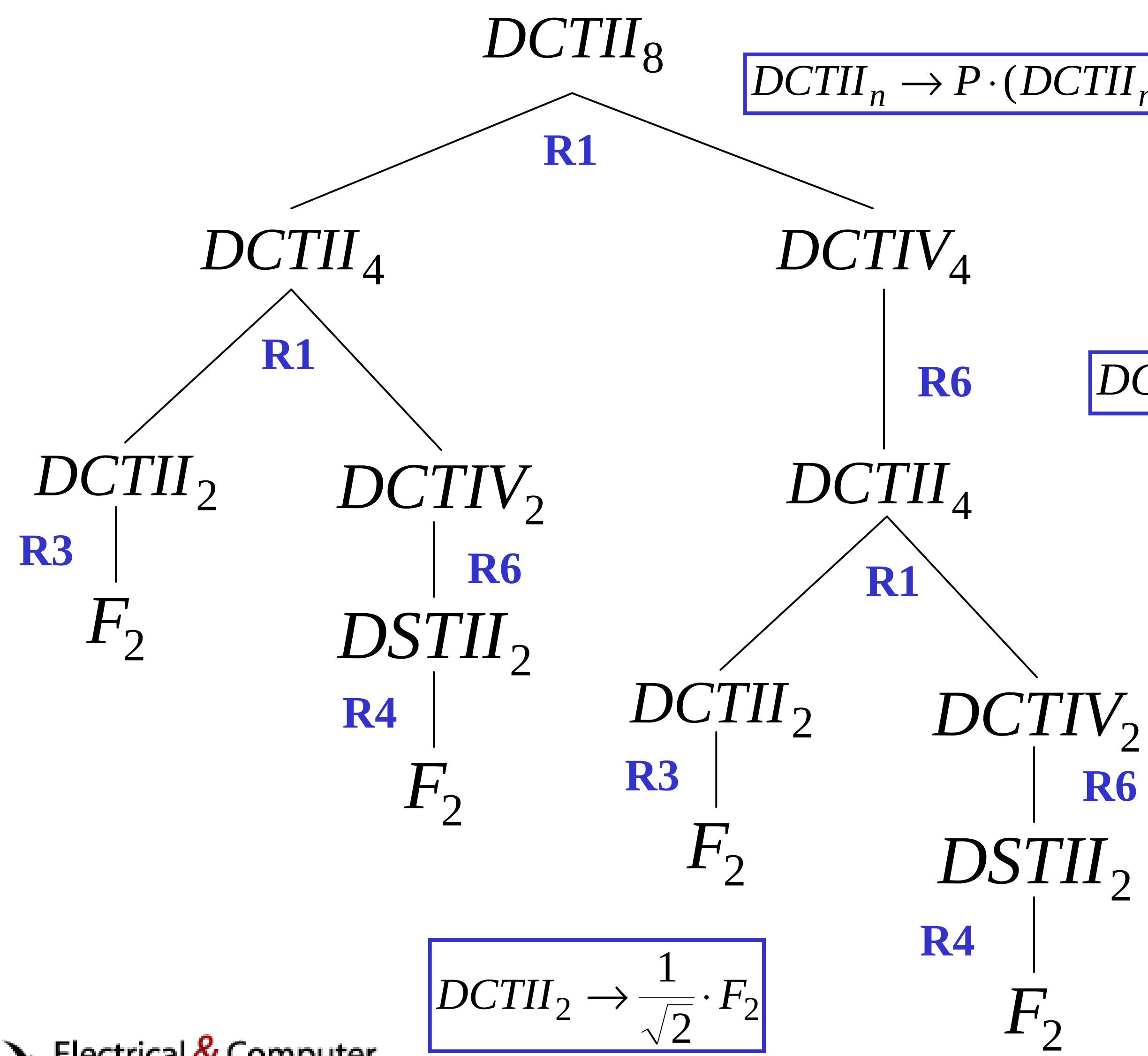
- a breakdown strategy
- product of sparse matrices

Formula

$$DFT_8 = \left(F_2 \otimes I_4\right) \cdot D \cdot \left(I_2 \otimes \left(I_2 \otimes F_2 \cdots\right)\right) \cdot P$$

- arises from recursive application of rules
- product of sparse matrices
- uniquely defines an algorithm

Algorithms = Ruletrees = Formulas



$$DCTII_n \rightarrow P \cdot (DCTII_{n/2} \oplus DCTIV_{n/2}) \cdot (F_2 \otimes I_{n/2})$$

$$DCTIV_n \rightarrow P \cdot DCTII_n \cdot S$$

$$DCTII_2 \rightarrow \frac{1}{\sqrt{2}} \cdot F_2$$

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Number of Formulas/Algorithms

Currently 12 transforms and 31 rules:

k	# DFTs, size 2^k	# DCTIVs, size 2^k
1	1	1
2	6	10
3	40	126
4	296	31242
5	27744	1924443362
6	162570361280	7343815121631354242
7	~1.01 • 10^27	~1.07 • 10^38
8	~2.31 • 10^61	~2.30 • 10^76
9	~2.86 • 10^133	~1.06 • 10^153

➡ **exponential search space**

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Formulas in SPL

• • • •

```
( compose
     ( diagonal ( 2*cos(1/16*pi) 2*cos(3/16*pi) 2*cos(5/16*pi) 2*cos(7/16*pi) ) )
     ( permutation ( 1 3 4 2 ) )
     ( tensor
       ( I 2 )
       ( F 2 )
     )
     ( permutation ( 1 4 2 3 ) )
     ( direct_sum
       ( compose
         ( F 2 )
         ( diagonal ( 1 sqrt(1/2) ) )
       )
       ( compose
         ( matrix
           ( 1 1 0 )
           ( 0 (-1) 1 )
         )
         ( diagonal ( cos(13/8*pi)-sin(13/8*pi) sin(13/8*pi) cos(13/8*pi)+sin(13/8*pi) ) )
         ( matrix
           ( 1 0 )
           ( 1 1 )
           ( 0 1 )
         )
         ( permutation ( 2 1 ) )
```

• • • •

SPL Compiler, 4-point FFT

fast algorithm as formula as SPL program

```
(compose (tensor (F 2) (I 2)) (T 4 2)
  (tensor (I 2) (F 2)) (L 4 2))
```

#codetype

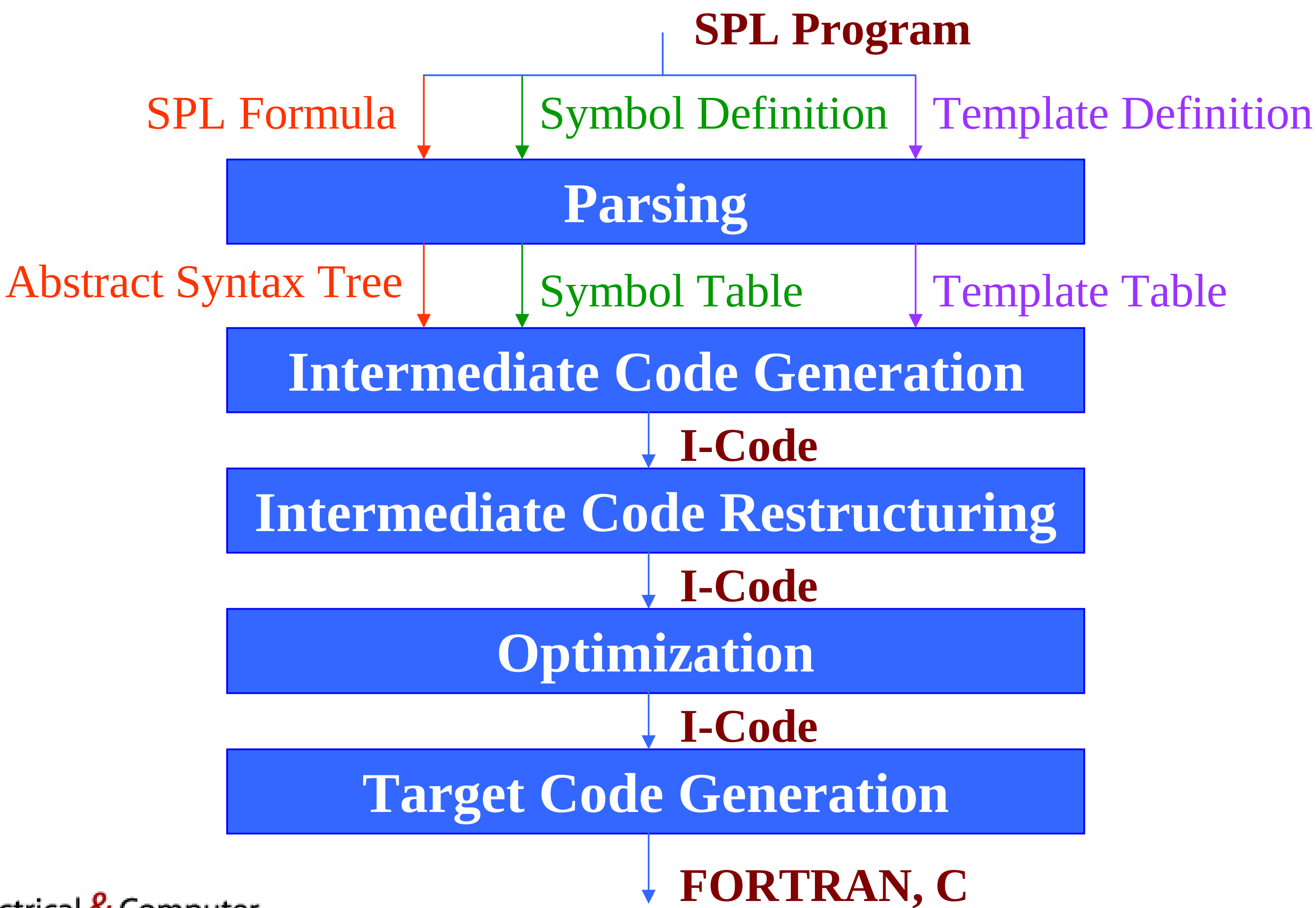
complex

```
f0   = x(1) + x(3)
f1   = x(1) - x(3)
f2   = x(2) + x(4)
f3   = x(2) - x(4)
f4   = (0.00d0,-1.00d0)*f(3)
y(1) = f0 + f2
y(2) = f0 - f2
y(3) = f1 + f4
y(4) = f1 - f4
```

real

```
r0   = x(1) + x(5)
r1   = x(1) - x(5)
r2   = x(2) + x(6)
r3   = x(2) - x(6)
r4   = x(3) + x(7)
r5   = x(3) - x(7)
r6   = x(4) + x(8)
r7   = x(4) - x(8)
y(1) = r0 + r4
y(2) = r1 + r5
y(3) = r0 - r4
y(4) = r1 - r5
y(5) = r2 + r7
y(6) = r3 - r6
y(7) = r2 - r7
y(8) = r3 + r6
```

The SPL Compiler



Search Methods Available in SPIRAL

- **Exhaustive Search**
- **Dynamic Programming (DP)**
- **Random Search**
- **STEER (similar to a genetic algorithm)**

	Possible Sizes	Formulas Timed	Results
Exhaust	Very small	All	Best
DP	All	10s-100s	Good
Random	All	User decided	Poor to fair
STEER	All	100s-1000s	Very good

- **Search over new user-defined transforms and breakdown rules**
- **Search over formulas and options to SPL compiler**

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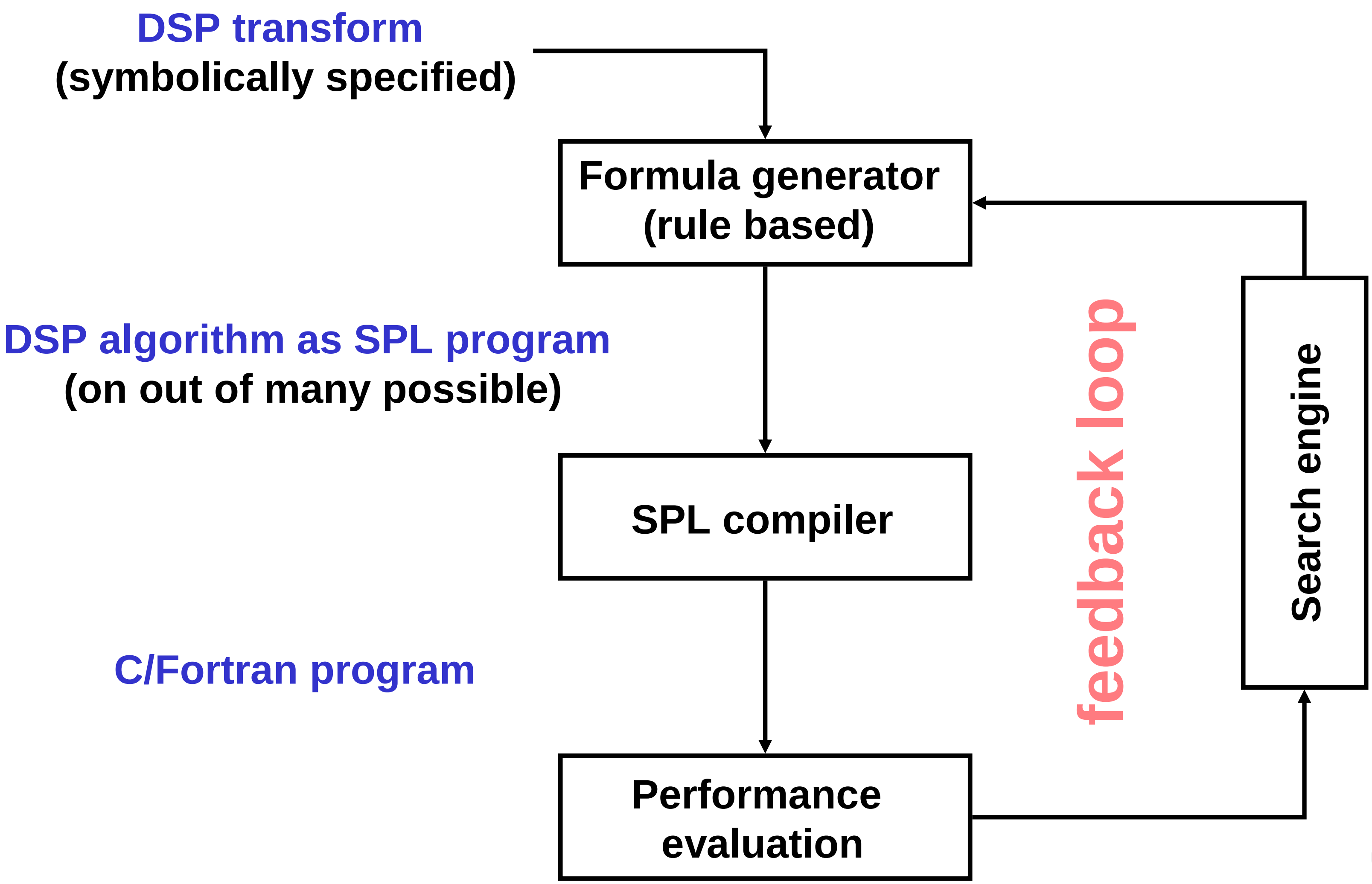
Summary: SPIRAL Architecture

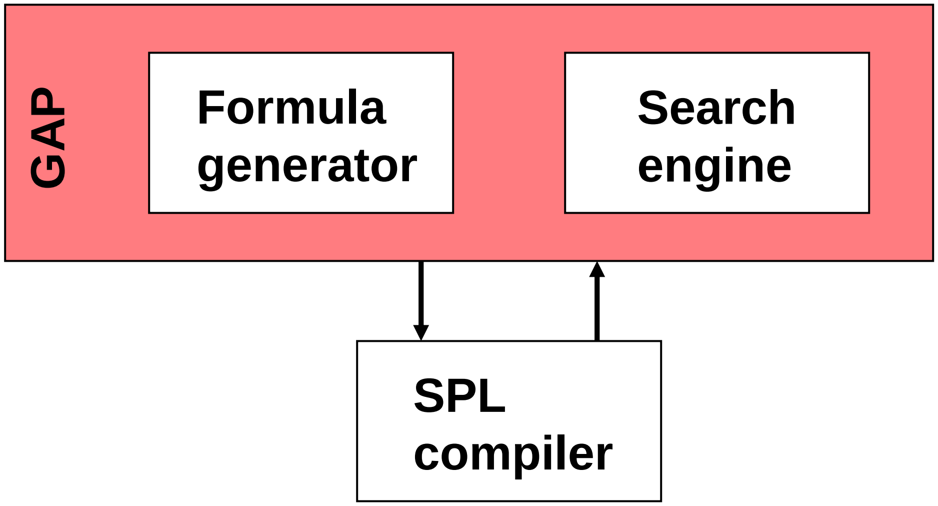


Organization

- SPIRAL approach
- **SPIRAL system**
- Some experimental results
- Recent work

The SPIRAL System: Implementation

- Infrastructure of SPIRAL is based on the computer algebra system and language GAP (`http://www-gap.dcs.st-and.ac.uk/~gap/`)
  - ➢ command line interface
  - ➢ symbolic (exact) computation with DSP formulas
  - ➢ full-fledged programming environment
- Formula generator and search engine implemented in GAP
- SPL compiler implemented in C



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The SPIRAL System: Main Features

- Easy installation from one source on
  - ➢ Unix based systems (configure – make)
  - ➢ native Windows systems (Visual C/Intel compiler make)
- DSP transforms: DFT, DCTs, DSTs, WHT, Haar transform, …
- new transforms can easily be included
- multi-dimensional transforms automatically supported
- composed DSP transforms supported
- verification of generated code
- programming environment included (GAP)
- online documentation

download at: `www.ece.cmu.edu/~spiral`

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SPIRAL System Examples I

Implementing a DFT of size 1024 in C:

SPIRAL command prompt

transform size

```
spiral> S := Transform("DFT", 1024);
spiral> Implement(S, rec(search := "DP",language := "c”));
```

search method: dynamic programming

target language

➡ C function in working directory

SPIRAL System Examples II

Implementing an 8 x 8 DCT of type 2 in Fortran:

```
spiral> S := Transform("DCT2", 8);
spiral> S1 := TensorSPL(S, S);
spiral> Implement(S1, rec(search := “STEER",
                          language := "f77”));
```

search method: STEER

SPIRAL System Examples III

Implementing a composed transform in C:

```
spiral> S1 := Transform("DFT", 8);
spiral> S2 := DiagSPL([1, 2, 4, 2, 3, 5, 1, -2]);
spiral> S3 := Transform(“DCT3”, 8);
spiral> S := S1 * S2 * S3;
spiral> Implement(S, rec(search := “TimedSearch",
                         timeLimit := 30,
                         language := “c”));
```

search method:
timed search 30 minutes

a DCT type 3 followed by scaling followed by a DFT

Organization

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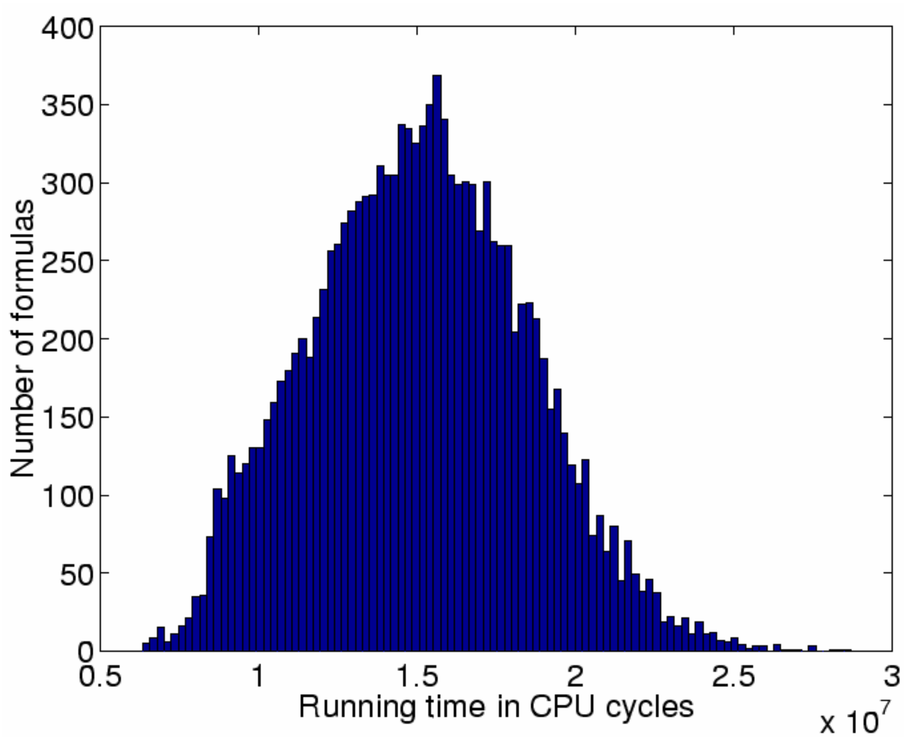
Search Space and Varying Performance

WHT(2^{10}): 51,819 (binary) ruletrees = formulas



- **large spread in runtime**
- **not due to arithmetic cost**
- **good ones are rare**

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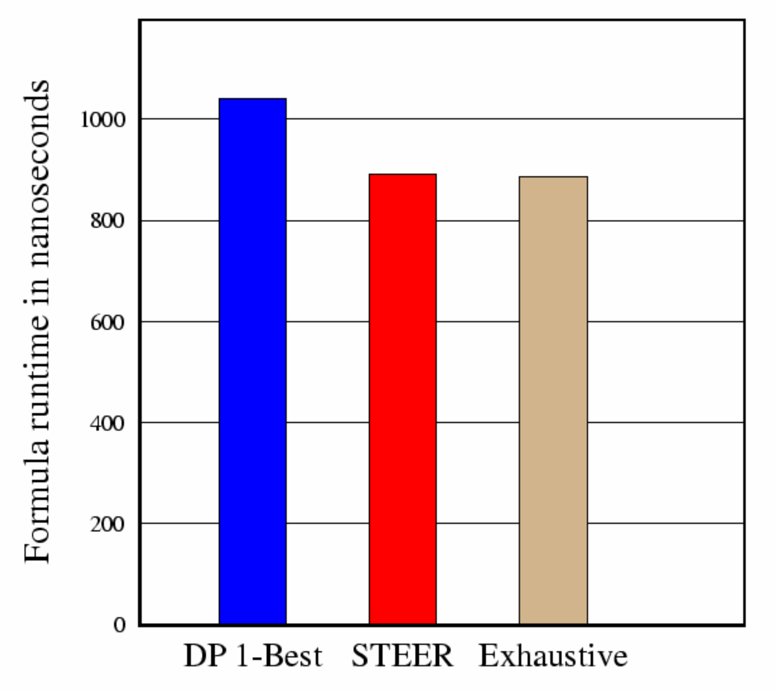
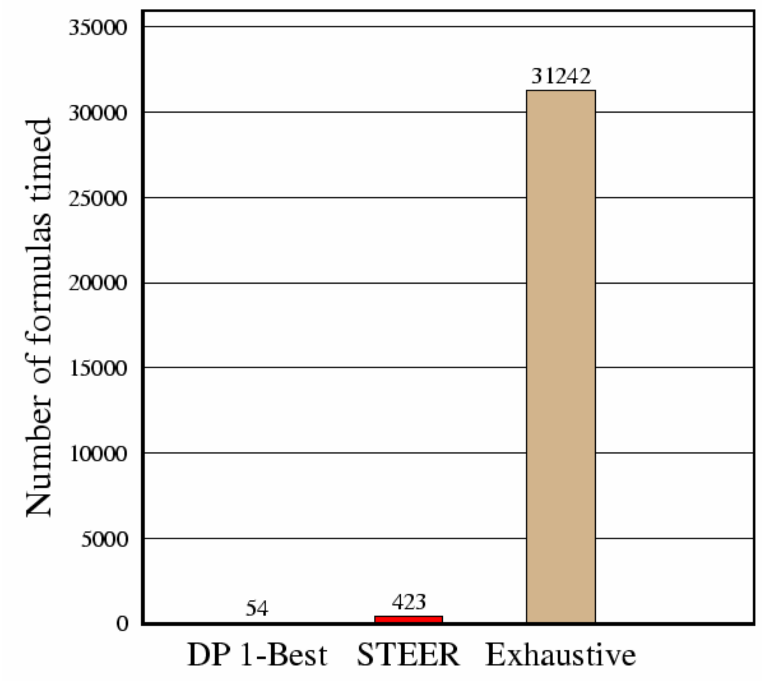
Comparison Search Methods I

DCT, type IV, size 16

Fastest Found Formulas



Number of Formulas Timed



DP and STEER perform well

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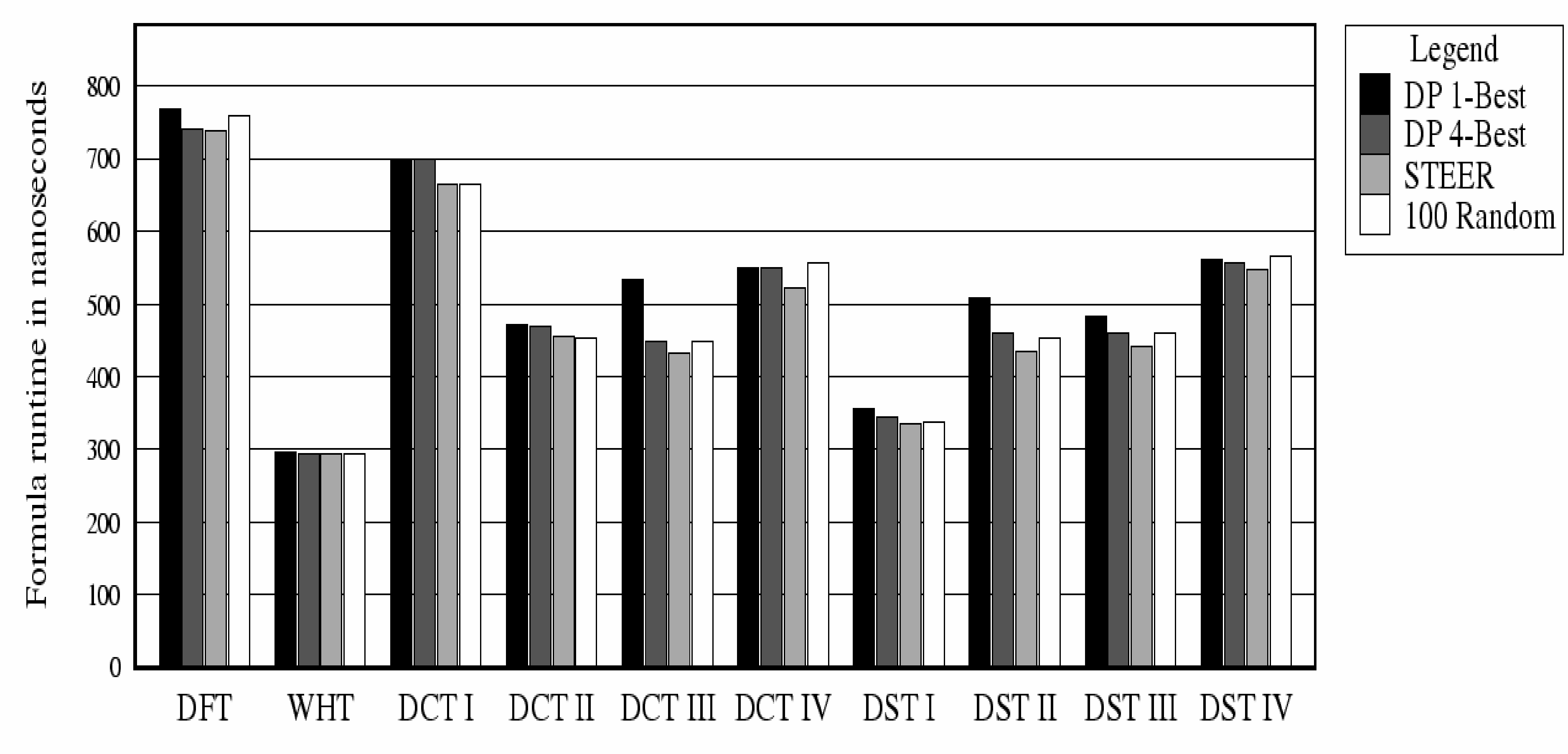
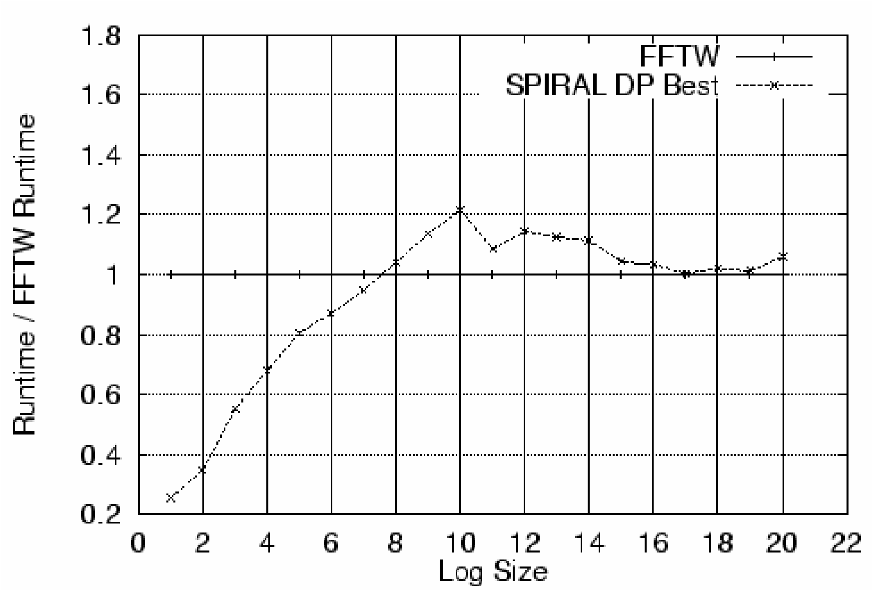
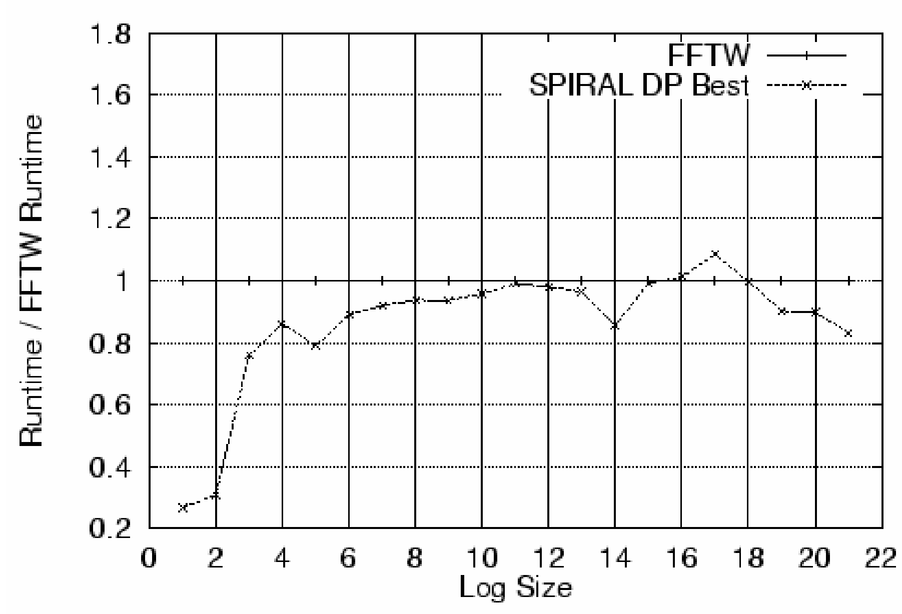
Comparison Search Methods II

across transforms of size 16

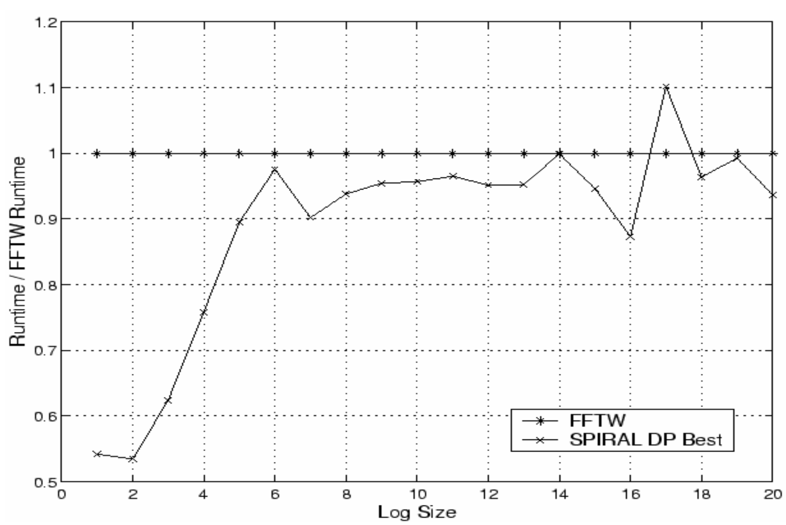




Pentium III/Linux/gcc



Athlon/Linux/gcc



Pentium III/Win2000/Intel compiler

SPIRAL vs. FFTW (lower = better)

comparable performance

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Learning instead of Searching

- **Method:**
  - **Runs a number of formulas of one size**
  - **Analyzes the cache misses caused by different parts of the formulas**
  - **Then design fastest formulas of different sizes, even larger sizes!**
- **Designs fast formulas of sizes that it has never even timed before**
- **Designed fastest known formulas for WHT!**

Fast Formula Generation Results

Size	Number of Formulas Generated	Generated Included the Fastest Known	Top N Fastest Known Formulas in Generated
2^{12}	101	yes	77
2^{13}	86	yes	4
2^{14}	101	yes	70
2^{15}	86	yes	11
2^{16}	101	yes	68
2^{17}	86	yes	15
2^{18}	101	yes	25
2^{19}	86	yes	16
2^{20}	101	yes	16

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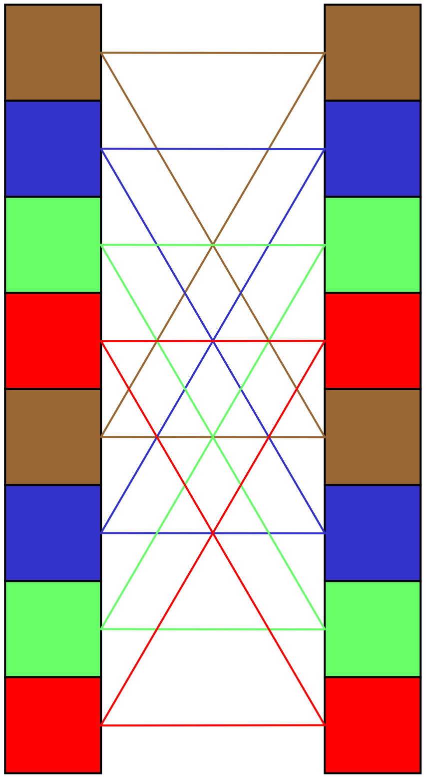
SPIRAL SIMD

joint work with
Franz Franchetti, Christoph Űberhuber,
Technical University Vienna

$$DFT_2 \otimes I_4$$



- **Portable SIMD Support (SSE; planned: SSE2, AltiVec),**
  based on Compiler Support
- **Handle $A \otimes I_n$ and $I_n \otimes A$**
- **Support for Diagonals and Permutations**
- **Unrolled code and loop code**

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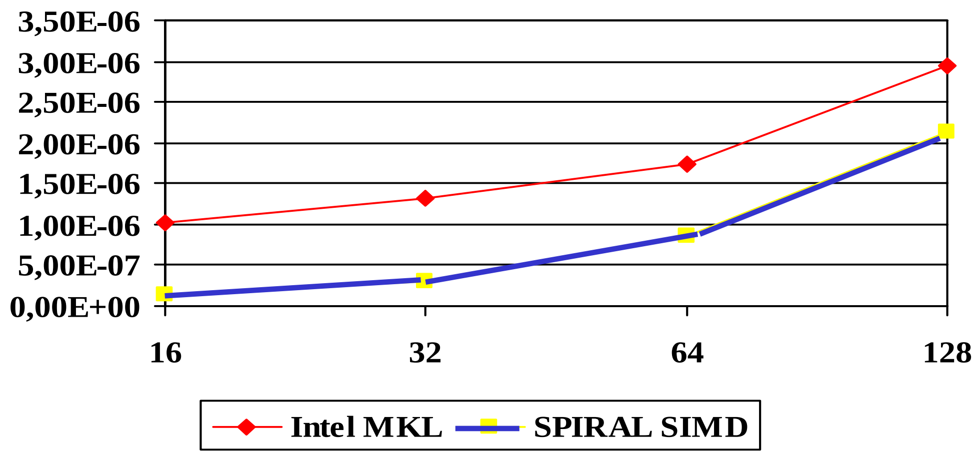
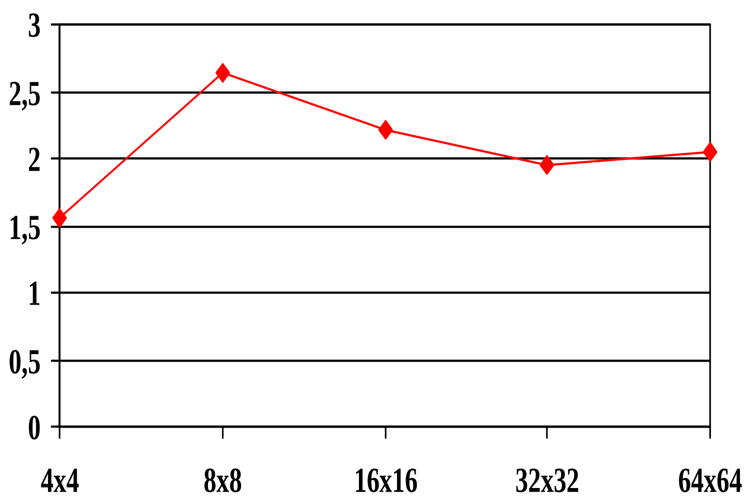
Experimental Results

Pentium4
SSE - float
Windows 2000
Intel C++ Compiler 5.0
Spiral 3.1

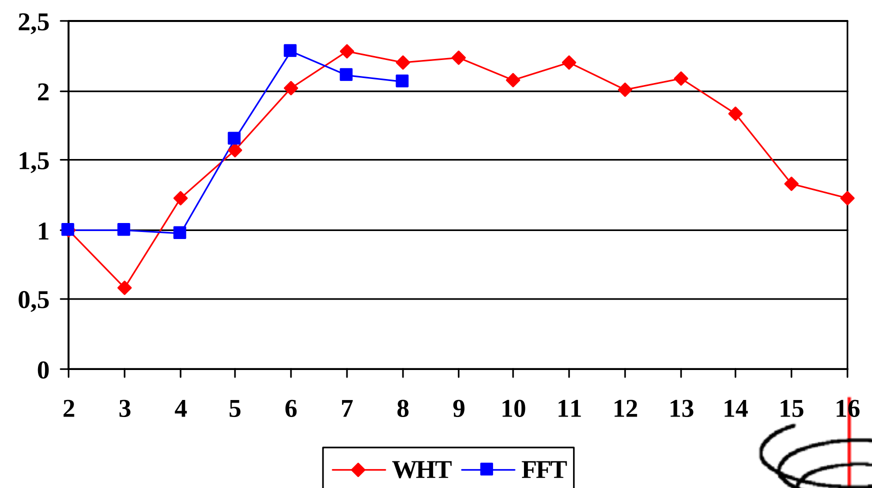
FFT: Benchmark



DCT2xDCT2: Speed-up



Speed-up



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Summary

- SPIRAL
  - ➢ generates platform-adapted code for linear DSP transforms
  - ➢ is extensible to include new transforms
  - ➢ easily installs on a variety of platforms
- The generated code is verified and very competitive

download at: `www.ece.cmu.edu/~spiral`

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