Expedient Media Noise Modeling: Isolated and Interacting Transitions

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Abstract— **We propose the triangle zig-zag transition (TZ-ZT) model as a faster alternative to micromagnetic media modeling. We present equations that relate the statistics of the TZ-ZT model to recording process parameters. Both isolated and interacting transitions are modeled. Simulation results show the performance of the model.**

I. Introduction

There is renewed interest in finding expedient media noise models. Micromagnetic modeling (MM) [1], [2] is probably the most accurate model for media (transition) noise. Although accurate, MM is too expensive when generation of thousands of bits (transitions) is required (e.g., error rate predictions). Generating a single MM transition on a 50×50 grain sample takes 10-20 minutes on an average workstation, which is prohibitive for signal processing purposes.

Alternative models can be deterministic, e.g., [3], or statistical. We consider statistical zig-zag models. These models capture the statistical essence of the random zig-zag line (wall) that separates oppositely magnetized regions of the media. Arnoldussen and Tong [4], and Middleton and Miles [5] suggest zig-zag patterns where the peak-to-peak distances are the independent random variables of the model. This leads to instability (a down-track wall drift) since the model is then an independent increment random process [6]. Reference [7] fixes this by deconvolving the peak-to-peak probability density function (pdf) to obtain a zero-to-peak pdf. In this paper, we circumvent this problem by presenting a model that involves only zero-to-peak distances in the form of triangle heights, rather than peak-to-peak distances. Thus, we solve the instability issue by avoiding altogether an independent increment random process. Furthermore, the geometry of our model allows us to find unique relationships between the model defining quantities and the recording process parameters (transition profile and cross-track correlation width). We call our model the triangle zig-zag transition (TZ-ZT) model. TZ-ZT modeling is 10^4 times faster than MM. We show results comparing the TZ-ZT to MM. We also propose a modification to the TZ-ZT model to incorporate high density nonlinearities.

II. Triangle Zig-zag Transition Modeling

The triangle zig-zag transition (TZ-ZT) model is a stochastic model of the zig-zag line that separates two oppositely magnetized regions of the magnetic medium. The TZ-ZT model (see Figure 1) places side-by-side isosceles triangles of alternating orientations on the line representing the nominal transition center. The triangle heights $(h_1, h_2, \ldots)$ are independent random variables drawn from a pdf $f_H(h)$. The vertex angle θ is chosen to be constant. This makes it possible to find a relationship between θ and the cross-track correlation width, which we actually use to determine θ for a given medium.

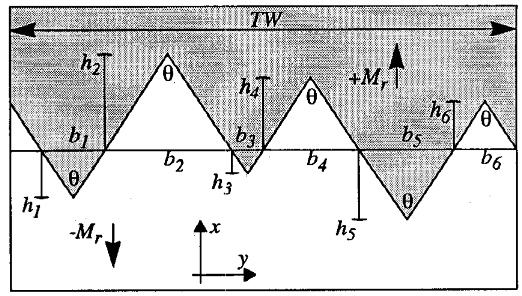

Fig. 1. The triangle zfig-zag transition model.

III. Isolated Transitions

A. Transition Profile and Jitter Noise Modeling

Denote by $M_x(x)$ the average down-track magnetization profile, where x denotes the down-track direction. We are assuming that for $x > 0$, $M_x(x) > 0$ and that $M_x(x)$ is odd, e.g., $M_x(x) = \frac{2M_r}{\pi} \cdot \mathrm{atan}\left(\frac{x}{a}\right)$ or $M_x(x) = M_r \cdot \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)$, where M_r is the remanent magnetization.

Theorem 1 *The average transition profile $M_x(x)$ is related to the pdf of TZ-ZT heights $f_H(h)$ for an isolated transition through*

$$f_H(h) = -\frac{M_x''(x)}{M_x'(0)} \quad \text{for} \quad h \geq 0, \tag{1}$$

where $M_x''(x) = \frac{d}{dx}\left[M_x'(x)\right] = \frac{d^2}{dx^2}\left[M_x(x)\right]$.

The proof of Theorem 1 escapes the length constraints of this paper, see [6]. The tricky part in the proof is to relate the zig-zag patterns to renewal theory. To avoid errors made in previous attempts to derive similar relationships, we need to recognize the paradox of residual life in renewal theory [8]. Due to this paradox, the pdf $p(w)$ in equation (18) of reference 15 in [5] should be replaced by $w \cdot p(w)/\mathrm{E}[w]$. This will change the relationship between the magnetization profile and the sawtooth pdf $p(w)$ in Equation (5) in [5] to its correct form. See [6] for details.

The cross-track correlation width s (see [9]) is correlated with the jitter in the readback signal. We find an expression for the cross-track correlation width s_{TZZT} of the TZ-ZT model.

Theorem 2 *Let B be the random variable that represents the bases of the zig-zag triangles used in the TZ-ZT model. Then*

$$s_{TZZT} = \frac{\mathrm{Var}(B)}{\mathrm{E}[B]}, \tag{2}$$

where $\mathrm{E}[B]$ is the mean and $\mathrm{Var}(B)$ is the variance of B.

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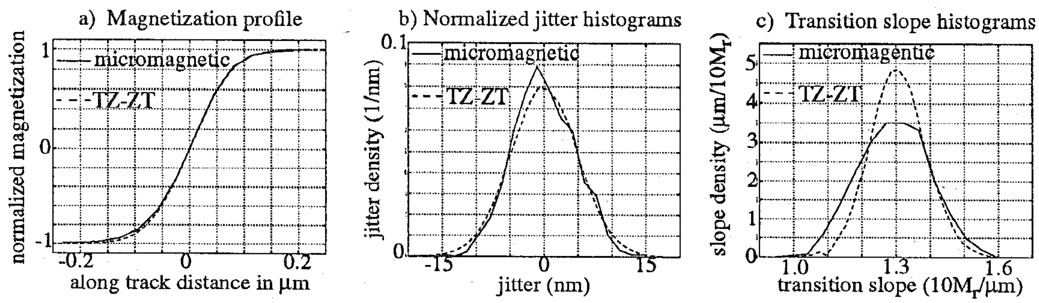

Fig. 2. Comparison of TZ-ZT and micromagnetic statistics.

We relegate the proof to [10]. Equation (2) in Theorem 2 holds for other types of models too. In particular, in [10], we apply Theorem 2 to the microtrack model of [11] to show that the cross-track correlation width, as computed by (2), equals the microtrack width. We interpret Equation (2). Let B be the random variable representing the cross-track size of a cluster. Let the magnetization of adjacent clusters point in opposite down-track directions, both with intensity M_r. Then Equation (2) gives the cross-track correlation width. Since the cross-track correlation width is correlated with the jitter noise level, to have low jitter noise we either reduce the cluster size variance $\mathrm{Var}(B)$ or increase the average cluster size $\mathrm{E}[B]$. The latter, however, is not an option in high density recording because it requires increasing the track width.

Corollary 2.1 *The TZ-ZT modeled medium and the thin film medium being modeled have the same cross-track correlation width s if the TZ-ZT vertex angle θ is*

$$\theta = 2\mathrm{atan}\left[\frac{s \cdot \mathrm{E}[H]}{2 \cdot \mathrm{Var}(H)}\right], \tag{3}$$

where H represents the random TZ-ZT heights.

Proof: From Figure 1, the bases b_i are related to the heights h_i as $b_i = h_i \cdot 2\tan\frac{\theta}{2}$. If we substitute $B = H \cdot 2\tan\frac{\theta}{2}$ into (2) and solve the resulting equation for θ, we get (3).

B. Modeling Results

We modeled a magnetic thin-film with the following characteristics: remanent magnetization $M_r = 625\mathrm{emu/cm}^3$, coercivity $H_c = 1670\mathrm{Oe}$, media thickness $\delta = 400\text{Å}$, orientation ratio O.R.= 1.3. The chosen track width was $TW = 4.8\mu\mathrm{m}$. We used a Karlqvist writing head with gap length $g = 0.28\mu\mathrm{m}$ and flying height (magnetic spacing) $d = 0.1\mu\mathrm{m}$.

Using the micromagnetic model we obtained 30 independent isolated magnetization profiles (normalized to M_r). We found their average to be $M_x(x) \approx \mathrm{erf}\left(\frac{x}{\sqrt{2}\sigma}\right)$, with $\sigma = 610\text{Å}$. Applying Theorem 1 to an erf magnetization profile, we find the triangle heights pdf $f_H(h)$ to be a Rayleigh pdf

$$f_H(h) = -\frac{M_x''(x)}{M_x'(0)} \approx \frac{h}{\sigma^2}\exp\left(-\frac{h^2}{2\sigma^2}\right) \quad \text{for} \quad h \geq 0. \tag{4}$$

The shape of a Rayleigh pdf matches well the experimental findings [4]. To find the vertex angle θ of the TZ-ZT model, we first calculate the sample magnetization variance $\hat{\sigma}_M^2(x)$ from the same 30 isolated micromagnetic profiles. The magnetization variance σ_M^2 is related to the normalized magnetization M through the equality $\sigma_M^2 = \frac{s}{TW}\left(1 - M^2\right)$, where s is the cross-track correlation width [12]. By least-squares fitting a parabola to the curve of $\hat{\sigma}_M^2(x)$ versus $M_x(x)$, we obtain $s = 197\text{Å}$. Corollary 2.1 yields then $\theta = 50.7°$, where we used $\mathrm{E}[H] = \sigma\sqrt{\pi}/\sqrt{2}$ and $\mathrm{Var}(H) = (2 - \pi/2)\sigma^2$ for a Rayleigh distributed random variable H as in (4).

Figure 2 compares the statistical properties for isolated pulses of the TZ-ZT model with those of the MM. The plots are based on 500 independent runs of the MM and 50,000 runs of the TZ-ZT model. The transition shape and jitter histograms match almost perfectly, see Figures 2-a and 2-b. The transition slope variance shows a 30% mismatch, see Figure 2-c. This is a second order effect with little impact on the total noise power since the amplitude (slope) variations are weaker than the jitter for isolated pulses [13].

IV. Interacting Transitions

A. Nonlinear Bit Shift and Amplitude Loss Modeling

We model nonlinear bit shift (NLBS) and partial signal erasure (percolation) by modifying the existing TZ-ZT model for isolated transitions. Our model for interacting transitions relies on results derived by Bertram [9]. Equation (9.7) in [9] shows that the bit shift is

$$\Delta x = \frac{4M_r\delta\,(d + \delta/2)^3}{\pi Q H_c B^3}, \tag{5}$$

where B is the nominal separation between the current and the previous transition and Q is the head-field gradient factor. We apply this formula by writing the current transition closer to the previous one by Δx.

A second effect observed at closely separated transitions is that the down-track position of the transition varies with a variance that is dependent on the distance between neighboring transitions. This variance is (equation (12.33) in [9])

$$\sigma_B^2 = \left(1 - \frac{8M_r\delta d^2}{\pi B_1^3 Q H_c}\right)^{-1}\sigma_\infty^2. \tag{6}$$

Here, σ_∞^2 is the down-track position variance of an isolated transition and σ_B^2 is the variance of a transition whose nearest neighboring transition is separated by B_1, where B_1 is the distance between neighboring transitions after the NLBS adjustment in (5), i.e., $B_1 = B - \Delta x$. Applying (6) to the TZ-ZT

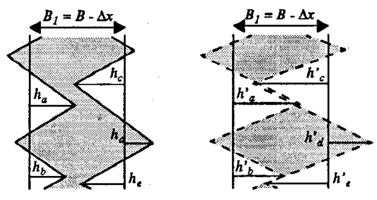

Fig. 3. Left: two isolated transitions. Right: transitions with elongated triangle heights (marked by the superscript ') according to (7).

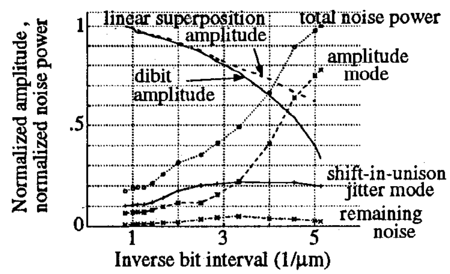

Fig. 4. Normalized dibit amplitude and normalized media noise mode powers as a function of inverse transition spacing.

model is fairly easy. Suppose that we created a TZ-ZT transition with triangle heights $h_1, h_2, \ldots$. We write this transition at a distance $B_1 = B - \Delta x$ from the previous transition, where the distance B is user defined and Δx is calculated from (5). The triangles of the created transition will have bases $b_1, b_2, \ldots$ that correspond to the heights $h_1, h_2, \ldots$ through the angle θ given in (3), see also Figure 1. In order to change the variance according to (6) and at the same time preserve the cross-track correlation width s, we need to elongate the heights $h_1, h_2, \ldots$, while keeping the bases $b_1, b_2, \ldots$ as they were for an isolated transition. Thus, we set the elongated triangle heights to

$$h'_i = \left(1 - \frac{8M_r\delta d^2}{\pi B_1^3 Q H_c}\right)^{-1/2} h_i, \quad i = 1, 2, \ldots \tag{7}$$

A side effect of this elongation is that the angle θ changes for interacting transitions, but that does not mean that the cross-track correlation width changed since the triangle bases remain equal to those generated for an isolated transition. Equation (7) needs to be applied to both transitions located at each end of their separation length B_1. After the adjustments described in equations (5) and (7), given a small enough $B_1 = B - \Delta x$, we will observe that the zig-zag patterns of the neighboring transitions overlap at certain portions of the track. In these portions of the track, we consider that the magnetization has percolated, leaving islands of oppositely magnetized regions in between the two closely separated transitions, see Figure 3. We use this as our model for partial erasure.

B. Modeling Results

We wrote TZ-ZT dibits and read it with a Lindholm head for different transition spacings. For each transition separation, we obtained 500 independent dibit waveforms. After subtracting their mean, we obtained the dibit media noise waveforms, and calculated their empirical correlation function. We decomposed the correlation function into its principal components (modes) using the Karhunen-Loeve decomposition (KLD) [13]. The KLD revealed that two basic noise modes (amplitude variation mode and shift in unison jitter mode) dominate the dibit media noise. Their relative contribution to the total noise power changes with transition separation, as shown in Figure 4. At large transition separations, jitter is the dominant noise mode, while at short spacings the amplitude variations dominate. This plot, obtained with TZ-ZT modeling, is similar to experimental results obtained in [13]. Figure 4 also shows the dibit amplitude as a function of inverse transition separation and the amplitude obtained by linear pulse superposition.

V. Conclusion

We presented a computationally efficient media and media noise model. Magnetization transitions are represented as portions of a zig-zag line across the track. The zig-zag line is a random process obtained by placing isosceles triangles on a common basis line. Formulas are presented that link the defining quantities of the model to recording parameters. For isolated transitions, the transition shape and jitter noise are accurately modeled. For interacting transitions, nonlinear amplitude loss and high density media noise are also incorporated. The model can be applied to generate signals with high density nonlinearities and media noise, and is therefore useful for statistical studies (error rate studies) of readback subsystems.

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References

[1] J.-G. Zhu and N. H. Bertram, "Recording and transition noise simulations in thin film media," *IEEE Trans. Magn.*, vol. 24, pp. 2706–2708, Nov. 1988.

[2] P.-L. Lu and S. Charap, "Angular variations of coercivity in thin-film media by micromagnetic model," *IEEE Trans. Magn.*, vol. 28, pp. 986–989, March 1992.

[3] X. Y. Zhang, H. Suhl, and P. K. George, "Relationship between the transition width and the zigzag wavelength," *J. Appl. Phys.*, vol. 63, pp. 3257–3259, April 1988.

[4] T. C. Arnoldussen and H. C. Tong, "Zigzag transition profiles, noise, and correlation statistics in highly oriented longitudinal media," *IEEE Trans. Magn.*, vol. 22, pp. 889–891, Sept. 1986.

[5] B. K. Middleton and J. J. Miles, "Recorded magnetization distributions in thin film media," *IEEE Trans. Magn.*, vol. 27, pp. 4954–4959, Nov. 1991.

[6] A. Kavčić and J. M. F. Moura, "Triangle zig-zag transition modeling," tech. rep., DSSC, ECE Dept., Carnegie Mellon University, Pittsburgh, PA, 1995. 54 pages.

[7] Y.-S. Tang and L. Osse, "Zig-zag domains and metal film disk noise," *IEEE Trans. Magn.*, vol. MAG-23, pp. 2371–2373, Sept. 1987.

[8] L. Kleinrock, *Queueing Systems*. NY: J. Wiley & Sons, 1975.

[9] H. N. Bertram, *Theory of Magnetic Recording*. Cambridge: Cambridge University Press, 1994.

[10] A. Kavčić and J. M. F. Moura, "Statistical study of zig-zag transition boundaries in magnetic recording." in preparation.

[11] J. Caroselli and J. K. Wolf, "A new model for media noise in thin film magnetic recording media," in *Proc. SPIE: Coding and Signal Processing for Information Storage*, vol. 2605, (Philadelphia), pp. 29–38, Oct. 1995.

[12] H. N. Bertram and X. Che, "General analysis of noise in recorded transitions in thin film recording media," *IEEE Trans. Magn.*, vol. MAG-29, pp. 201–208, 1993.

[13] J.-G. Zhu and X.-G. Ye, "Impact of medium noise on various partial response channels," *IEEE Trans. Magn.*, vol. 31, pp. 3087–3089, Nov. 1995.