Uncertainty Modeling for Ocean Tomography Systems

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Abstract

In ocean tomography acoustical means are used to infer characteristic parameters of the underwater medium such as temperature profile and current velocity. These techniques are based on the functional dependency between the propagation and the parameters that must be estimated. Accuracy of this modelisation is critical to the performance:a mismatched model may preclude corrrect inference, either by imposing biases or by affecting the variance of the estimates. In this paper, we apply to this problem a global performance prediction tool showing its ability to predict system's sensitivity to modeling errors.

1 Introduction

An important problem in the design of tomography systems is to predict performance, both expected accuracy and parameter observability. Since this systems are based on knowledge of the functional dependency of the propagation/observation operator on the desired parameters, actual performance is highly dependent on the correctness of all the prior information concerning the physical medium and experimental conditions. In this paper, we show how an ambiguity function, initially developed for localization problems, may predict expected global performance of a given tomography system under modeling inaccuracies.

The ambiguity function is a global analysis tool that accounts for large errors rather than the local errors taken into consideration by the Cramér-Rao bounds. Based on the information provided by the geometric properties of the probabilistic manifold that describes the observed data, this function describes the statistical observability of the parameters being estimated, incorporating in an integrated manner the invertibility of the transmission operator, the effect of the noise, and the impact of uncertainties. As we will see, the ambiguity index introduced in [3] is defined directly in terms of the Kullback directed divergence between the conditional distributions associated with a parametrized family modeling the observed data and can thus be directly applied to many estimation problems, in particular to ocean tomography.

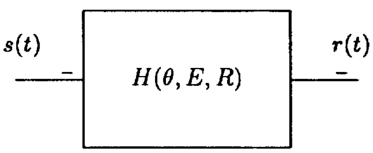

Figure 1: Inverse problems in underwater signal processing.

In the paper, we demonstrate the use of this tool, by considering a trivial case-study, where two parameters describing a simple bilinear propagation model are to be infered from the observations produced on an aray of sensors. We show ambiguity plots, that demonstrate the sensitivity of the analyzed system to several parameters: source positiom, bottom depth, velocity gradient in the lower layer.

In the next section we formulate the problem. Then, we introduce the ambiguity function. In the last section, using a very simple case study, we demonstrate the application of the ambiguity function to the analysis of ocean tomography.

2 Problem Formulation

Ocean tomography and sonar systems share a very important characteristic, namely they utilize the same probabilistic model to describe the observed data, only its parametrization is different. The diagram of Figure 1 illustrates this point. In the diagram, $s(t)$ denotes a vector of source signals which corresponds to multiple sources in source location problems or to several emitters in tomography; $r(t)$ is the signal observed by an array of sensors; E and R are the parameters describing the geometry and location of the emitting and receiving antennas, respectively; θ describes the physical medium parameters; and $H(\theta, E, R)$ is the matrix transmission operator that combines the directional characteristics of the emitters and of the receivers with the propagation effects of the channel. The transmission operator $H(\theta, E, R)$ is parametrized by the emitter and the receivers' location parameters E and R, respectively, as well as by the channel physical parameters. With ocean tomography, we are interested in estimating θ assuming that E and R are known. With localization problems, it is θ and R that are assumed to be known while it is E that is to be estimated. It is in this sense that tomography and localization are said to be inverse problems.

To demonstrate the validity of the approach, the paper considers the simple case of an horizontally stratified medium, with perfectly flat boundaries. The ocean is divided in two horizontal layers where the velocity gradient is constant: in the upper layer, sound speed decreases linearly with depth, in the lower layer there is a positive constant gradient (ducted propagation). This simple bilinear model has the advantage of preserving a certain degree of analyticity, exibiting, at the same time, the multipath propagation which is commonly present in vast ocean areas, with a number of distinct rays between any two given points being present. In [3], we studied the performance of location systems using this tool, showing the potential advantage of explicitly modeling the temporal (inter-path) delay structure of the observations.

We assume that the bottom depth, the sound speed at the surface, and the gradient of the sound speed profile (SVP) at the lower layer are known and that it is the duct's depth and the gradient in the upper layer that are to be estimated. We consider that the emitting and receiving antennas are fixed (i,e,, their position do not change with time). The emitter is a point source radiating a wideband pseudo-random signal with known power density, and the receiving antenna consists of several sensors arranged in a uniformly spaced vertical linear array.

For this simple scenario, we use our ambiguity function to study performance sensitivity to incorrect prior knowledge, i.e., how does erroneous information regarding pior parameters affects the ambiguity structure associated with the estimated parameters.

3 Ambiguity Function

Consider a family $\mathcal{G}_\alpha$ of density functions, indexed by a parameter $\alpha \in A$:

$$\mathcal{G}_\alpha \triangleq \{p(x|\alpha), \quad \alpha \in A\}.$$

The Kullback-Leibler number (also called Kullback directed divergence or cross-entropy) between two members of $\mathcal{G}_\alpha$ is [1]:

$$I(\alpha_1, \alpha_2) \triangleq \mathrm{E}_{\alpha_1}\left\{\ln \frac{p(x|\alpha_1)}{p(x|\alpha_2)}\right\}.$$

In this equation, E_{α_1} is expectation with respect to the probability density function $p(x|\alpha_1)$. This functional was introduced by Kullback [1] in the framework of information theory. Although it has some distance-like properties, it is not, in fact, a distance. As it can be easily seen, it is not symmetric and it does not satisfy, in general, the triangular inequality. However, $I(\alpha_1, \alpha_2) \geq 0$, with equality iff $\alpha_1 = \alpha_2$. Note that

$$I(\alpha_1, \alpha_2) = \mathrm{E}_{\alpha_1}\{\ln p(x|\alpha_1) - \ln p(x|\alpha_2)\},$$

i.e., $I(\cdot, \cdot)$ is the mean value of the difference between the values of the log-likelihood function for two points in the parameter space, for observations x, conditioned on one of those points. The value of $I(\cdot, \cdot)$ depends, naturally, on the size of the observation interval. Here, we consider only the asymptotic case of very long observation interval.

Heuristically, $I(\alpha_1, \alpha_2)$ is a measure of the resemblance, or proximity, of the two models described by $p(x|\alpha_1)$ and $p(x|\alpha_2)$. The values of α_2 that yield small values of $I(\alpha_1, \alpha_2)$ indicate possible erroneous estimates of α when the true value of the parameter is α_1.

Based on these arguments, ambiguity between two points (α_1, α_2) in the parameter space is defined as

$$\mathcal{A}(\alpha_1, \alpha_2) \triangleq \frac{I_{MAX}(\alpha_1) - I(\alpha_1, \alpha_2)}{I_{MAX}(\alpha_1)} \qquad (1)$$

where $I_{MAX}(\alpha_1)$ denotes an upper bound on the value of $I(\alpha_1, \alpha_2)$ over $\alpha_2 \in A$. Since $I(\cdot, \cdot)$ is not symmetric, $\mathcal{A}(\alpha_1, \alpha_2)$ is not, in general, a symmetric function of its two arguments.

Consider that the observations' power spectrum is described by

$$R_\theta(\omega) = S(\omega) h_\theta(\omega) h_\theta(\omega)^H + \sigma^2(\omega) I_K$$

where we assume that the observation noise is spatially incoherent, with known power density $\sigma^2(\omega)$.

In the previous equation, $\mathcal{S}(\omega)$ is the ***unknown*** source spectral density and $h_\theta(\omega)$ is the resultant vector, that describes the coherent combination of the steering vectors corresponding to the P replicas received, see [2, 3] for further details.

The resultant vector can be decomposed as

$$h_\theta(\omega) = D(\theta)b(\theta)$$

where the $K \times P$ matrix $D(\theta)$ describes the spatial structure of the individual replicas, depending only on the inter-sensor delays for each received path, and $b(\theta)$ is a P dimensional vector that depends only on their temporal alignment.

Using the relation

$$\mathcal{R}_\theta(\omega)^{-1} = \frac{1}{\sigma^2(\omega)}\left(I - \frac{\mathcal{S}(\omega)}{E_\theta(\omega)} h_\theta(\omega) h_\theta(\omega)^H\right)$$

where the scalar $E_\theta(\omega)$ is defined by

$$E_\theta(\omega) = \sigma^2(\omega) + \mathcal{S}(\omega)\|h_\theta(\omega)\|^2,$$

leads to

$$\begin{aligned}\overline{I}(\theta_0 : \theta) \;&=\; \frac{1}{2}\int \left[\frac{\mathcal{S}(\omega)}{\sigma^2(\omega)}\|h_{\theta_0}(\omega)\|^2 - \frac{\mathcal{S}(\omega)}{E_\theta(\omega)}\|h_\theta(\omega)\|^2\right.\\ &-\; \frac{\mathcal{S}(\omega)^2}{\sigma^2(\omega)E_\theta(\omega)}|h_{\theta_0}(\omega)^H h_\theta(\omega)|^2\\ &+\; \left.\ln\frac{E_\theta(\omega)}{E_{\theta_0}(\omega)}\right] d\omega.\end{aligned}$$

Ambiguity is computed using this equation in the general definition given before, see [4] for a complete discussion.

4 Case Studies

We present in this section ambiguity plots for the estimation of surface layer velocity gradient and duct depth in a deep ocean area, considering a bilinear approximation to a velocity profile typical of the North Atlantic with the following nominal values: gradient above duct (g_0): -0.0035 s^{-1}, gradient below duct (g_1): .013 s^{-1}, duct depht: 950 m, sound speed at the surface: 1500 m^{-1}, ocean depth. $4000m$.

The receiving antenna is a 10 element uniform vertical linear array, with inter-element spacing $d = \ldots.m$, and top-most element at an immersion $100m$. Horizontal distance from the source to the receiving array is $60Km$, and source is placed near the duct axis, at an immersion of $600m$. The source signal has a flat spectrum in the bandwidth $400 - -500Hz$.

All figures presented show gray-scale density plots of the ambiguity surfaces, white areas corresponding

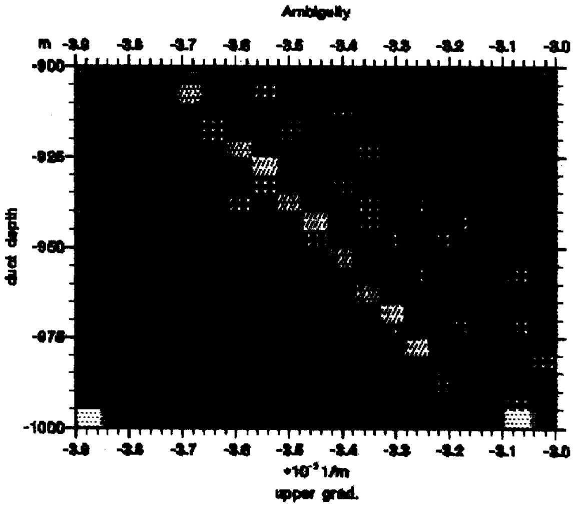

Figure 2: Perfect knowledge.

to small ambiguity values (close to zero) and dark regions corresponding to values close to one. Horizontal axis is upper gradient (on a grid between $-.039m^{-1}$ and $-.03m^{-1}$), vertical axis being duct depth (on a grid from $-1000m$ to $-900m$).

The first plot, Fig. 2, shows the ambiguity surface in the ideal case, where perfect knowledge of all the modeling parameters is asumed, except of those being estimated. The diagonal orientation of the ambiguity lobes of this plot displays a strong correlation between the two parameters, showing that a lower value of upper sound gradient can be partially compensated for by an increase in duct depth.

The second group of plots, Fig. 3 through Fig. 5 show the sensitivity of the tomography systems to uncertainty on the parameters that are treated as being known (source position, ocean bottom, deep layer gradient). Comparing these plots to the corresponding ideal one Fig. 2, we see that utilization of nominal erroneous parameter values results in the introduction of biases (the ambiguity curves no longer peak at the right values) and/or lead to deformation of the original structure.

Fig. 3 shows that an error of 10 meters in source immersion has a drastic effect on the ability to estimate the two parameters of interest. The most striking feature of this plot is the existence of two peaks, both at wrong (upper gradient, duct depth) coordinates indicating abias and ambiguity problem.

In Fig. 4 we see that an error of 300 meters on bottom depth has no significant influence on system's performance on the grid of analysis, except for the region of large values of duct depth and upper gradient. This fact seems natural, since the receiving antenna

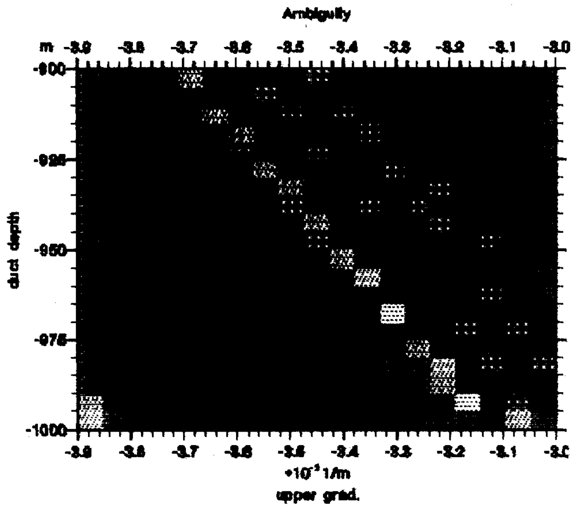

Figure 3: Wrong value of source immersion (600 m).

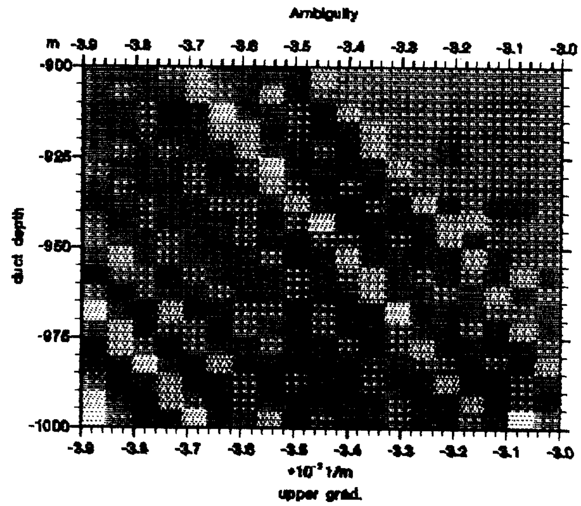

Figure 4: Wrong value of ocean bottom (4300 m).

is placed in the first convergence zone, where the importance of bottom reflected rays is negligible.

The last figure (5) corresponds to a mismatch in the velocity gradient of the lower layer. In this case, the diagonal structure is still present, but no relevant peak exists in the vicinity of the actual value. This is due to the fact that with a wrong value for the lower layer gradient, the location of the convergence zone is displaced, leading to a large deviation from the actual values of the unknwon parameters in order to fit the model to the actual observations.

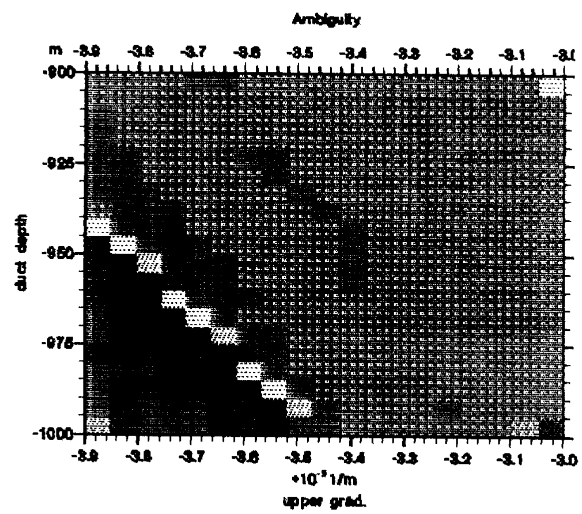

Figure 5: Wrong value of bottom layer gradient ($g_1 =$ .014 s^{-1}.

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