Optimal Predictive Coding of 2 D Fields*

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We discuss coding of 2D data using a recursive framework for *noncausal* Gauss Markov random fields (GMRF) defined on finite lattices. This framework exploits to advantage the structure of GMRFs providing the means to achieve recursive optimal processing, while preserving the *noncausality* of the field.

The compression scheme uses noncausal prediction coupled to vector quantization (VQ). The noncausal prediction fits first a noncausal GMRF to the data, then whitens the data by an inverse filtering type operation, and finally vector quantizes the prediction error field. In this paper, we explain the details of the noncausal prediction. Lack of space prevents us to discuss the parameter estimation algorithm that is needed to fit a 2D model to the data, see [1].

GMRF Recursive Structure

Important in the coding of GMRFs is the issue of parameterization. This leads to the question of when is a positive definite matrix the covariance of a GMRF? Partial answers are available only in very special cases. In general, for GMRFs on finite lattices, it is not possible to answer the question directly. It turns out that the right way to pose it is in terms of the inverse of the covariance matrix which we refer to as the potential matrix, see [2] for details.

Let $\{x_{i,j}\}$, $1 \leq i,j \leq N$, represent the 2D field on a finite lattice (taken as a square, for simplicity.) Woods [3]'s minimum mean square error representation of a homogeneous first order GMRF (nearest neighbors) is

$$x_{i,j} = \beta_h(x_{i,j-1} + x_{i,j+1}) + \beta_v(x_{i-1,j} + x_{i+1,j}) + e_{i,j}, \quad (1)$$

where β_h and β_v are the strengths of the neighbor horizontal and vertical field interactions, respectively. We call these the field potentials. Collecting all N^2 equations, taking care of boundary conditions (b.c.) (which here we assume Dirichlet zero boundary conditions, see [2] for general b.c.,) we get

$$A\mathbf{x} = \mathbf{e} \quad (2)$$

where the potentials are collected in the matrix $A = I \otimes B + H \otimes C$, and $\otimes$ is the Kronecker product. The N^2 vector $\mathbf{x} = \text{vec}[\mathbf{x}_i]$, where the N vectors $\mathbf{x}$ collect the intensities of the pixels of the ith - row. I is the N^2 identity matrix, $B = I_N - \beta_h H_N$ and $C = -\beta_v I_N$, H is an N^2 matrix of zero entries, except the upper and lower diagonal (all ones,) and I_N and H_N are like I and H but of dimension N.

The noise $\mathbf{e}$ has correlation $\Sigma_e = \sigma^2 A$. Apart the normalizing factor of σ^2, the covariance $\Sigma_{\mathbf{x}}$ of $\mathbf{x}$ is then the potential matrix A.

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By Cholesky factorization, $A = U^T U$. Equation2) gives

$$U\mathbf{x} = \mathbf{w} \quad (3)$$

where the covariance of $\mathbf{w}$ is $\sigma^2 I$. The Cholesky factor U is not a full matrix. It is block diagonal with band $N+1$. The diagonal and the upper diagonal blocks of U are obtained from the iterates of a Riccati type equation. In [2], the convergence behavior of this iterative scheme is studied. For practical purposes, one may stop it after less than 10 iterations, considerably reducing the associated computational effort.

2D Coding

To code 2D data, we need the field parameter values $\beta_h, \beta_v, \sigma^2$. In [1], we analyze the parameter space of GMRFs and study their maximum likelihood (ML) estimation.

We have used this to code two dimensional data. The basic structure of the (lossy) codec is the following: (i) The global mean is subtracted from the 2D data, which is then input to an ML – estimator; (ii) a Cholesky factorization of A leads to the unilateral representation of the field; (iii) the field is whitened leading to the error field; (iv) the error field is vector quantized; (v) lossless entropy type coding can be used to achieve further compression. When applied to image data, we have verified that we can get over a factor of 3 – 10 of more compression ratio than DCT based techniques. This procedure and modifications to it are presently under study.

References

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[2] José M. F. Moura and Nikhil Balram. Recursive structure of noncausal Gauss Markov random fields. **IEEE Transactions on Information Theory**, IT-38(2):334-354, March 1992.

[3] J. W. Woods. Two-dimensional discrete Markovian fields. *IEEE Trans. Inform. Theory*, IT-18:232-240, 1972.