Grouping-and-shifting Designs for Structured LDPC Codes with Large Girth[1]

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Abstract — **We introduce a method to design structured LDPC codes with large girth and flexible code rates. The method is simple to explain: we divide the nodes in the Tanner graph into groups and connect nodes in these groups according to a set of parameters called shifts. We derive a general theorem on the shifts to prevent small cycles. Simulations show that these codes, GS-LDPC codes, outperform random LDPC codes.**

I. Introduction and construction

Low-density parity-check (LDPC) codes can be described by a bipartite graph called Tanner graph [1]. The length of the shortest cycle in a Tanner graph is referred to as its *girth* g. Since large girth leads to more efficient decoding and large minimum distance d_{min}, LDPC codes with large girth are particularly desired. We propose a class of structured LDPC codes with large girth and flexible code rate, called *grouping-and-shifting based LDPC codes* (GS-LDPC).

Let V_c be the set of all check nodes and V_b the set of all bit nodes. Divide V_c into N_c disjoint subsets of equal size provided that the code block length $n = N_c \cdot p$ where p is a natural number. We call each subset a *group* and index the check nodes in each group from 0 to $p-1$. Similarly, partition V_b into N_b disjoint groups of equal size and index the bit nodes in each group from 0 to $p-1$.

GS-LDPC codes satisfy the following conditions:

1.1 **Condition 1** Each check node is connected to k bit nodes that belong to k different groups.

1.2 **Condition 2** Each bit node is connected to j check nodes that belong to j different groups.

1.3 **Condition 3** The check node indexed by X in the y^{th} group in V_c is connected to the bit node indexed by $X \overset{p}{\oplus} S_{y,z}$ in the z^{th} group in V_b where $0 \leq S_{y,z} \leq p-1$. (The parameters $S_{y,z}$ are named *shifts* and $\overset{p}{\oplus}$ represents modulo-p addition.)

II. Results and conclusions

We derive a general rule to relate $2l$-cycles ($l \in \mathbb{N}$) to shifts.

Theorem 1 (2l-CYCLES) *The Tanner graph for a GS-LDPC code contains at least one $2l$-cycle if and only if there exist $2l$ shifts $S_{y_1,z_1}\ S_{y_2,z_2}\ \ldots\ S_{y_{2l},z_{2l}}$ that satisfy the following conditions:*

2.1 **Index Condition 2.1** $y_{2t} = y_{2t+1}$, $t = 1, 2, 3, \ldots, l-1$ and $y_{2l} = y_1$ and $z_{2t-1} = z_{2t}$, $t = 1, 2, 3, \ldots, l$

2.2 **Index Condition 2.2** $y_{2t-1} \neq y_{2t}$, $t = 1, 2, 3, \ldots, l$ and $z_{2t} \neq z_{2t+1}$, $t = 1, 2, 3, \ldots, l-1$ and $z_{2l} \neq z_1$

2.3 **Shift Condition 2.3** $\oplus_{t=1}^{2l}(-1)^{t-1} S_{y_t,z_t} = S_{y_1,z_1} \overset{p}{\ominus} S_{y_2,z_2} \overset{p}{\oplus} \cdots \overset{p}{\oplus} S_{y_{2i-1},z_{2i-1}} \overset{p}{\ominus} S_{y_{2i},z_{2i}} \overset{p}{\oplus} \cdots \overset{p}{\oplus} S_{y_{2l-1},z_{2l-1}} \overset{p}{\ominus} S_{y_{2l},z_{2l}} = 0$ *($\overset{p}{\ominus}$ represents modulo-p subtraction)*

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When $N_c = j$ and $N_b = k$ where j is the column weight of the parity-check matrix and k is the row weight of the parity-check matrix, by choosing shifts $S_{y,z}$ for $y = 1, \cdots, j$ and $z = 1, \cdots, k$ that violate the conditions in Theorem 1, we can design GS-LDPC codes with girth $g \leq 12$. For larger N_c and N_b, we can generate GS-LDPC codes with higher girth. As an illustration, Figure 1 shows a $(4500, 3, 9)$ GS-LDPC code with rate $r = 2/3$, free of cycles shorter than 10 and whose structure is described by the 1500×4500 matrix $\mathbf{H}$ constructed using Theorem 1.

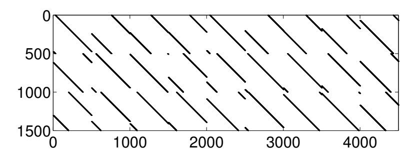

Fig. 1: $\mathbf{H}$ for a $(4500, 3, 9)$ GS-LDPC code with rate $2/3$ and girth 10.

We compare by simulation the bit error rate (BER) of a grith 8 GS-LDPC code with the BER of a randomly constructed LDPC code that is free of 4-cycles [2] in an AWGN channel. Both codes have column weight 3, block length 4536, and code rate 7/8. We adopt the rate-adjusted signal to noise ratio (SNR) defined in [2]: SNR $= 10\log_{10}\left[E_b/\left(2r\sigma^2\right)\right]$ where r denotes the code rate.

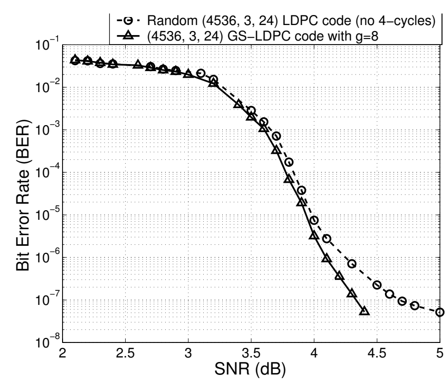

Fig. 2: BER performance: GS-LDPC code, column weight 3, girth 8, and rate 7/8 vs. same size random LDPC code free of 4-cycles.

In the high SNR, the GS-LDPC code outperforms the random LDPC code (free of 4-cycles) by SNR = .6 dB at BER= 5×10^{-8} where the performance of the random code has bottomed while the GS-LDPC code has not yet reached the error floor at this BER.

References

[1] C. E. Shannon, "A recursive approach to low complexity codes," *IEEE Trans. Inform. Theory*, vol. IT-27, no. 5, pp. 533–547, 1981.

[2] D. J. C. Mackay, "Good error-correcting codes based on very sparse matrices," *IEEE Trans. Inform. Theory*, vol. 45, no. 2, pp. 399–431, 1999.