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PARTITION-AND-SHIFT LDPC CODES FOR HIGH DENSITY MAGNETIC RECORDING

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Introduction

High-rate low-density parity-check (LDPC) codes are the focus of intense research in magnetic recording because, when decoded by the iterative sum-product algorithm, they show decoding performance close to the Shannon capacity. LDPC codes can be described by a bipartite graph called Tanner graph. The length g of the shortest cycle in a Tanner graph is referred to as the *girth* g of the graph. Since large girth leads to more efficient decoding and large minimum distance, LDPC codes with large girth are particularly desired. We propose a class of structured LDPC codes with large girth and flexible code rates, called *Partition-and-Shift LDPC codes* (PS-LDPC).

Code Construction

Let V_c be the set of all check nodes and V_b the set of all bit nodes in a Tanner graph. Divide V_c into N_c disjoint subsets of equal size provided that the code block length $n = N_c \cdot p$ where p is a natural number. We index the check nodes in each subset from 0 to $p-1$. Similarly, partition V_b into N_b disjoint subsets of equal size and index the bit nodes in each subset from 0 to $p-1$.

PS-LDPC codes satisfy the following assumptions:

1. **Check nodes** Each check node is connected to k bit nodes in k different bit node subsets.
2. **Bit nodes** Each bit node is connected to j check nodes in j different check node subsets.
3. **Shifts** Every check node, indexed by X in the α-th check node subset is connected to the bit node indexed by W in the β-th bit node subset, where $W = X \overset{p}{\oplus} S_{\alpha,\beta}$ and $0 \le S_{\alpha,\beta} \le p-1$.

The operator $\overset{p}{\oplus}$ in assumption 3 represents modulo- p addition. The parameters $S_{\alpha,\beta}$, $1 \le \alpha \le N_c$, $1 \le \beta \le N_b$, in assumption 3 are called *shifts*. We collect all the shifts $S_{\alpha,\beta}$ in an $N_c \times N_b$ matrix called *the shift matrix* $\mathbf{S} = [S_{\alpha,\beta}]$. For example, Fig.1 (c) shows a 4×6 shift $\mathbf{S}$ matrix. Fig.1 (a) shows the Tanner graph for a PS-LDPC code with 3 check node subsets and 4 bit node subsets.

Cycles & Shifts

Theorem 1 (2t-CYCLES) The Tanner graph for a PS-LDPC code contains at least one $2t$-cycle if and only if there exists a closed path of length $2t$ in the shift matrix $\mathbf{S}$ such that its $2t$ corners $S_{\alpha_1,\beta_1}, S_{\alpha_2,\beta_2}, \ldots, S_{\alpha_{2t},\beta_{2t}}$ satisfy the condition $\oplus_{i=1}^{2t} (-1)^{i+1} S_{\alpha_i,\beta_i} = 0$.

$$(\oplus_{i=1}^{2t} (-1)^{i+1} S_{\alpha_i,\beta_i} = S_{\alpha_1,\beta_1} \overset{p}{\oplus} (-S_{\alpha_2,\beta_2}) \overset{p}{\oplus} \Lambda \overset{p}{\oplus} S_{\alpha_{2t-1},\beta_{2t-1}} \overset{p}{\oplus} (-S_{\alpha_{2t},\beta_{2t}}))$$

We illustrate Theorem 1 with an example. As shown in Fig.1 (c), $S_{2,5}$, $S_{3,5}$, $S_{3,4}$, $S_{4,4}$, $S_{4,6}$, and $S_{2,6}$ are the six corners of a closed path of length 6 in the shift matrix $\mathbf{S}$. If we choose $S_{2,5} \overset{p}{\oplus} (-S_{3,5}) \overset{p}{\oplus} S_{3,4} \overset{p}{\oplus} (-S_{4,4}) \overset{p}{\oplus} S_{4,6} \overset{p}{\oplus} (-S_{2,6}) \neq 0$, then by Theorem 1, the 6-cycles shown in Fig.1 (b) does not exist. Similarly, by choosing shifts $S_{\alpha,\beta}$ for $\alpha = 1,2,\Lambda\, N_c$ and $\beta = 1,2,\Lambda\, N_b$ that violate the condition in Theorem 1, we can design PS-LDPC codes with arbitrary girth.

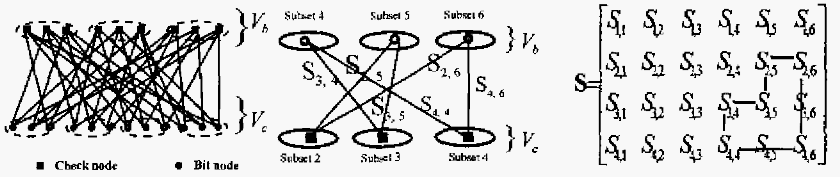


Fig.1 (a) Tanner graph Fig.1 (b) 6-cycle Fig.1 (c) Closed path in $\mathbf{S}$

As an illustration, Figure 2 shows a (6075, 3, 27) PS-LDPC code with rate $r = 8/9$, free of cycles shorter than 8 and whose structure is described by the 675×6075 parity check matrix $\mathbf{H}$.

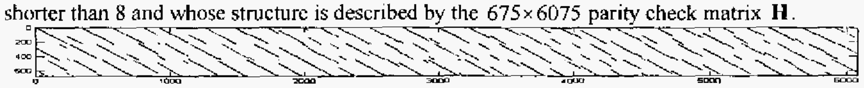
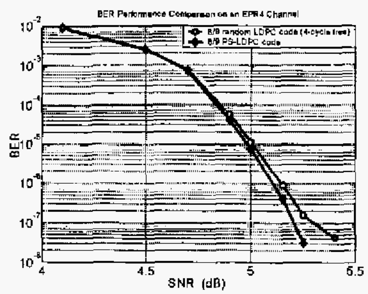


Fig.2 Parity check matrix for a (6075, 3, 27) PS-LDPC code with girth 8

Simulation Results

We compare by simulation the bit error rate of a girth 8 PS-LDPC code with the BER of a randomly constructed LDPC code that is free of 4-cycles in an EPR4 channel. Both codes have column weight 3, block length 6075, and code rate 8/9.



In the high SNR, the PS-LDPC code outperforms the random LDPC code (free of 4-cycles) by SNR = .15 dB at BER=3×10^{-8} where the performance of the random code has bottomed while the PS-LDPC code has not yet reached the error floor at this BER.

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