ENVIRONMENTAL LIMITS TO SOURCE LOCALIZATION

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ABSTRACT

Extension of the operational range of underwater location techniques has been related to the incorporation of reliable acoustic propagation models, that closely predict oceans' behaviour. In this paper, the performance of ocean tomography and passive localization of underwater acoustic sources is studied, by analyzing the coupling of performance degradations imposed by modeling mismatches on the ability to estimate the position of acoustic sources.

1. INTRODUCTION

Source localization uses parametric models of the underwater propagation, to invert the acoustic field received from a distant source. The correctness of these models is of paramount importance, these systems being particularly sensitive to errors on the parameters related to the description of the sound speed profile. Acoustic tomography solves a dual problem: using a receiver/transmitter pair at fixed known positions, the ocean parameters are determined by fitting the received field to that predicted by the parametric model.

In this paper we analyze the performance of acoustic source localization under modeling errors arising from imprecise physical knowledge. The coupling between the localization and the tomography problems is captured by considering that the propagation model used for source localization is tuned using parameters derived from an independent tomography experience. In paper, θ_a denotes the true source location, γ_a the true ocean parameters estimated by the tomography experience, and $\hat{\theta}$ and $\hat{\gamma}$ the corresponding estimates.

We base our study in the tool for global performance analysis under model mismatch presented in [5], see also [6]. This tool is based on the Kullback-Leibler divergence, and describes the increase in dispersion and eventual introduction of biases that can be expected due to use of wrong *a priori* physical knowledge:

$$\mathcal{S}(\theta : \theta_a | \hat{\gamma}, \gamma_a).$$

This function is the ambiguity between the true source location θ_a and a distinct location θ, when the propagation model is tuned for parameters $\hat{\gamma}$, while the true ocean parameters are γ_a.

The study is carried out for a deep water scenario, using a bilinear approximation to the sound speed profile. The sound speed in the lower layer is assumed to be known, and the parameter defining the upper layer is estimated by a tomography experience, and then fed to the localization device. Both source localization and ocean tomography are based on fully coherent fitting operations.

The paper is organized as follows. In Section 2 the sensitivity measure is shortly reviewed. In Section 3 the problem of characterizing the errors in the ocean parameters generated by the tomography experiment is addressed. We outline two possible approaches: the first leads to the determination of the exact error probability distribution under finite data; the second is based on an asymptotic analysis and yields a characterization of systematic biases due to model ambiguities. Finally, in Section 4, we show the result of applying this analysis to a deep water scenario for a tomography/localization system with imperfect knowledge of antenna depth.

2. SENSITIVITY MEASURE

The sensitivity analysis presented in this paper is based on the definition of ambiguity proposed in [5]. In this section, we briefly present the definition of the function of interest to this study, referring the interested reader to [5], where a detailed presentation of the tool is given.

Statistically motivated parametric estimation methods are based on knowledge of a family of conditional pdf's, that describes the dependency of the observed data (r) distribution on the parameter of interest (γ): $\mathcal{G}_\gamma = \{p(r|\gamma), \gamma \in \Gamma\}$.

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When $\mathcal{G}_\gamma$ perfectly reflects the behaviour of the physical environment, the ambiguity function defined in [5] can be used to predict the global performance of the parametric estimation mechanism, identifying possible large errors or configurations of poor observability of the parameter γ. Motivated by the close relationship between Maximum Likelihood estimators and the Kullback-Leibler directed divergence between probability density functions (pdf's) for exponential distributions. That function defines ambiguity between two values $\gamma_0, \gamma \in \Gamma$ as a normalized version of the Kullback-Leibler distance between the corresponding elements of $\mathcal{G}_\gamma$.

In this study, we are interested in quantifying the impact of imperfect world knowledge, i.e., the fact that the true world's behaviour is described by a given family of distributions $\mathcal{G}_\gamma^0 = \{p^0(r|\gamma), \gamma \in \Gamma\}$, while a distinct model is used in the fitting operation done at the receiver site: $\mathcal{G}_\gamma \neq \mathcal{G}_\gamma^0$. The observed data r is governed by a single member of $\mathcal{G}_\gamma^0$, that we denote by $p^0 = p^0(r|\gamma_0)$. The ability to correctly predict γ_0 using r under model mismatch can be analyzed studying a normalized version of the Kullback-Leibler distance between p^0 and the elements of $\mathcal{G}_\gamma$:

$$\mathcal{S}(\gamma) \triangleq 1 - \frac{I(p^0 : p(r|\gamma))}{I_{sup}(p^0)} \tag{1}$$

In this equation $I(p : q) = \mathrm{E}_p \{\ln(p/q)\}$ is the Kullback-Leibler directed divergence between pdf's p and q, where $\mathrm{E}_p\{\cdot\}$ is expectation with respect to the pdf p, and $I_{sup}(p^0)$ is an upper bound on $I(p^0 : p(r|\gamma))$. This sensitivity index is proposed in [5], where its relation to optimal estimation procedures is discussed.

The normalization in (1) is suitable to analyze the performance of a single method. However, when comparing several methods, it is more convenient to work directly with the Kullback directed divergence $I(p^0 : p(r|\gamma)$. In this way, we can not only compare the global performance, but also local performance, making use of the relation between the gradient of the Kullback divergence and the Fisher information matrix (see [2]). Ideally, $I(p^0 : p(r|\gamma))$ is zero for $\gamma = \gamma_0$, and has large values for all $\gamma \neq \gamma_0$. Modeling errors induce systematic biases in the estimation procedure, that are flagged by the fact that $I(p^0 : p(r|\gamma)$ has its minimum at an erroneous value $\gamma^* \neq \gamma_0$.

3. STATISTICAL CHARACTERIZATION OF OCEAN PARAMETERS

In this section we outline two possible approaches to the statistical characetrization of the estimates of the ocean parameters generated by coherent tomography processing. The first approach (subsection 3.1) eads to an estimate of the error density of the estimates over finite time horizons, and is based on non-linear filtering tools. The second approach (subsection 3.2) is based on the use of our sensitivity measure, and allows the identification of systematic biases (which must correspond to the important lobes of the pdf determined by the first approach). It is an asymptotic analysis, considering large observation intervals. The price paid for the fineness of the first method is the increased numerical complexity. In the examples presented in this paper, we consider the effects of biases in the estimation of the ocean parameter of interest (gradient in the upper layer of a bilinear velocity profile), using the approximate approach of subsection 3.2.

3.1. Density Estimation

The procedure outlined in this section was originally proposed [1, 3] to compute the error probability density in non-linear filtering problems. It has a numerical complexity orders of magnitude lower than standard Monte-Carlo techniques, by directly assessing, in each Monte Carlo run, a realization of the error density. Consider the following (discrete-time) model of the tomography antenna outputs: $r_n = s(n, \gamma_n) + w_n$, where w_n is (white Gaussian) observation noise, and $s(n, \gamma)$ is the information bearing signal, corresponding to the propagation of the (known) signal $(s(\cdot))$ emitted by a distant source (at a known location) to the receiving antenna: $s(n, \gamma) = (h_\gamma(\cdot) * s(\cdot))(t_n)$. In this last equation, h_γ is the medium's impulse response, which we consider to be of the ideal multipath type

$$h_\gamma(t) = \sum_{p=1}^{P(\gamma)} a_p(\gamma)\delta(t - \tau_p(\gamma)).$$

The propagation model describes the dependency of P, a_p and τ_p on γ.

The set of physical parameters to be estimated (γ) are modelled using the following degenerated state equation:

$$\gamma_{n+1} = \gamma_n.$$

The initial condition for this state equation, γ_0, is assumed to have a fixed distribution on a given interval Γ: $\gamma_0 \sim p_{\gamma_o}(\gamma_0)$. This initial pdf models prior information that may result, for example, from previous measurements.

The error probability estimation technique of [1] can be applied to the non-linear estimation of γ from observations following the previous model to compute $p_\epsilon(\epsilon|\gamma_a)$. This technique uses the equations of non-linear filtering that establish the conditional density of the state (γ_n) given the observations received up to time t_n: $r^n = \{r_\ell, \ell \leq n\}$. We denote this density by $p(\gamma_n|r^n, \gamma_a)$, to emphasize the fact that the information bearing component of the observations is generated using a fixed vaue of the oceans' parameters γ_a. For each possible realization of r^n, a new realization of this conditional density is obtained. The conditional density of the state (ocean parameter) is obtained by averaging over r^n: $p_\gamma(\gamma_n|\gamma_a) = \mathrm{E}_{r^n}\{p(\gamma_n|r^n, \gamma_a)\}$. To compute the error density at time t_n, we have simply to translate this pdf by the known true value γ_a: $p_\epsilon(\epsilon_n|\gamma_a) = p_\gamma(\gamma_n - \gamma_a|\gamma_a)$. This conditional error density may then used to compute a mean sensitivity for the source localization problem

$$\overline{\mathcal{S}}(\theta : \theta_a|\gamma_a) \triangleq \mathrm{E}_{\hat{\gamma}}\{\mathcal{S}(\theta : \theta_a|\hat{\gamma}, \gamma_a)\},$$

that reveals the impact of environmental uncertainties on localization performance.

3.2. Important Biases

Instead of trying to describe exactly the evolution of the information about the environemental parameters as observation interval increases, we can aternatively evaluate the impact of those errors that are ecpected to occur more often. The sensitivity measure eq. (1) yields exactly this information. To analyze coherent receivers, as the ones assumed in this paper, requires the computation of the Kullback directed divergence between the data records themselves. Invoking a large observation interval assumption, we use the asymptotic characterization of the directed divergence between stationary Gaussian processes derived in [4], that expresses $I(p^0 : p(r|\gamma)$ in terms of the power spectral densities of the observations:

$$\overline{I}(p^0 : p(r|\gamma)) = \lim_{T\to\infty} \frac{1}{T} I(p^0 : p(r|\gamma)) \quad (2)$$
$$= \frac{1}{2}\int \left[\mathrm{tr}\left[S^0(\lambda)S_\gamma(\lambda)^{-1}\right] - K - \ln\frac{|S^0(\lambda)|}{|S_\gamma(\lambda)|}\right] d\lambda.$$

4. EXAMPLES

The model used at receiver assumes that the ocean is horizontally stratified, with two distinct homogeneous layers. In the superficial layer (up to depth $y_{duct} = 914$m), the sound speed decreases linearly with depth (with rate $g_0 = -.035ms^{-1}$), increasing linearly from y_{duct} until the ocean bottom. The ocean boundaries are perfectly flat, with reflection coefficients depending on the grazing angle. The duct and ocean depths (equal to 914 m and 4 Km, respectively) and the sound speed gradient in the deep layer ($g_1 = .013ms^{-1}$)

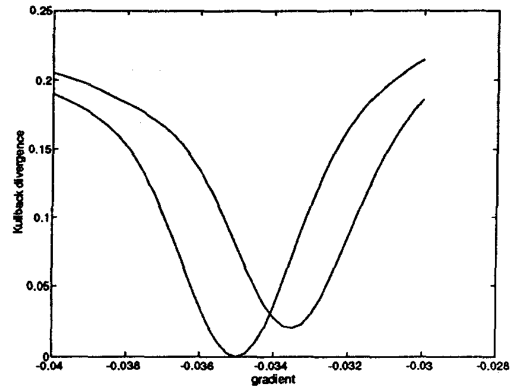

Figure 1: Kullback divergence, gradient estimation with wrong array depth.

are assumed to be perfectly known at the receiver, as well as all other parameters relating to receiver array (geometry, localization, gain). For all surfaces shown, the distance between the tomography source and the receiving array is 6 Km, and source and receiver immersion are $200m$. The array is vertical, linear, uniform, with $K = 30$ sensors, and sensor spacing is half wavelength at the higher frequency of analysis. The source signal spectrum is flat in the band [3.5, 4.5] KHz.

We present next examples of the sensitivity surfaces that result from a small mismatch on knowledge of the antenna immersion. Figure 1 shows the ambiguity function (Kullback divergence) for estimation of the velocity gradient in the first layer, under a small error on the value of the antenna immersion (receiver is using, as a perfectly known value, a depth of 190 m, when antenna is actually placed at 200 m). We can see that this small mismatch in antenna depth results in a small bias in the estimation of g_0: $\hat{g}_0 \simeq -.0335s^{-1}$ instead of the true value of $g_0 = -.035$. The dashed curve in Figure 1 illustrates the performance of this tomography experiment under no mismatches, i.e., for a correct value of antenna immersion. We can see that aside the introduction of a small bias, the variance of the estimate is not significantly affected. The next figures illustrate the performance degradation due to the accumulated effect of wrong antenna immersion (the same erroneous vaue of 190 m is used in all cases) and resulting wrong estimate of the velocity gradient in the upper layer (using the biased value at which the Kullback divergence in Figure 1 is centered). Four source positions around the position of the emitter used in the tomography experiment are considered: Figure 2: (7 Km, 150 m); Figure 3: (7 Km, 200 m); Figure 4: (5 Km, 200 m) and Figure 5: (5 Km, 250 m). All plots show the same rectangular area centered at the position of the tomography source: [4 Km, 8 Km]×[100 m, 300 m]. The errors are larger for the second ex-

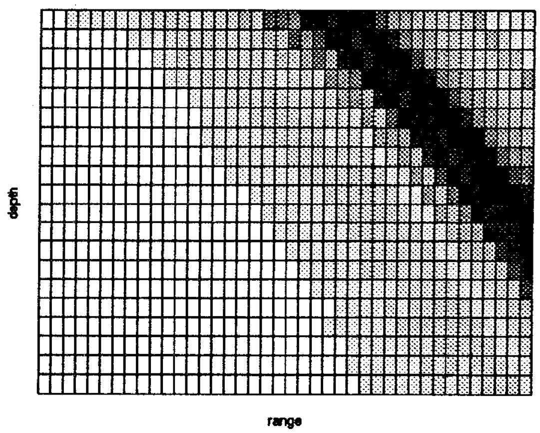

Figure 2: Source at (7 Km, 150 m).

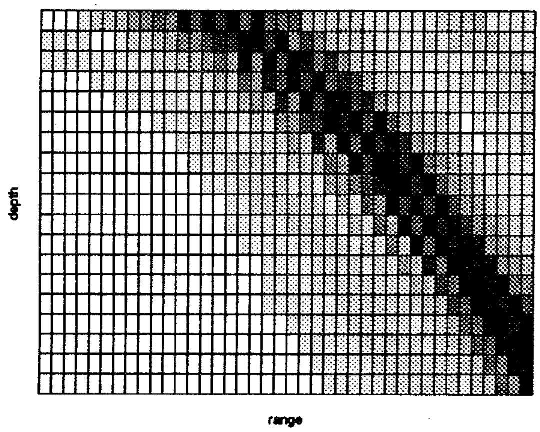

Figure 3: Source at (7 Km, 200 m).

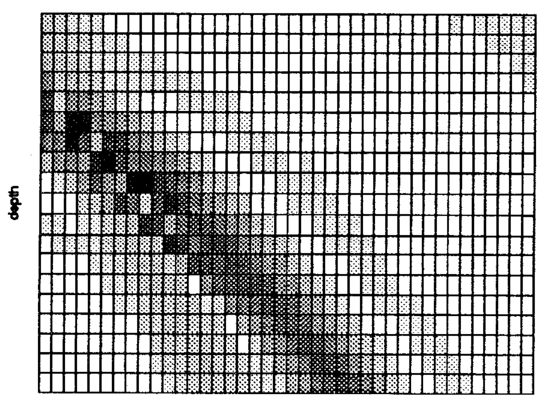

Figure 4: Source at (5 Km, 150 m).

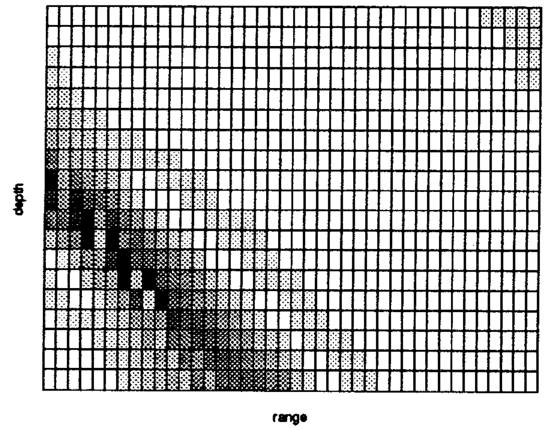

Figure 5: Source at (5 Km, 250 m).

ample, Figure 3, about 1.2 Km in range and 80 m in depth. In the other three cases, the errors are approximately equal, in the order of 500 m in range and 50 m in depth.

5. REFERENCES

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