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ARMA PROCESSES: ORDER ESTIMATION

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ABSTRACT

The paper studies for an ARMA(p_0,q_0) process, the joint determination from a finite data sample of its structural parameters p_0 and q_0, its AR and MA components, and its innovation power σ^2.

The order estimation algorithm is based upon the minimization of a functional d that measures the mismatch of the assumed model ARMA(p,q) to the data. The functional is evaluated from the estimated reflection coefficient sequence associated with the process. When the orders are decided, the proposed technique simultaneously provides the estimates of the AR and the MA coefficients, as well as σ^2.

1. INTRODUCTION

In any identification problem going from data to a model, the model structure (e.g., the number of poles and zeros) has to be determined prior or within the estimation procedure.

For an ARMA(p_0,q_0) process, the AIC introduced in [1] and the MDL suggested in [2] are well known techniques for order selection. They both select the number of poles, p_0, and the number of zeros, q_0, minimizing a functional that sums the power of the sequence of residues with a term that accounts for the model overparametrization. In [3], [4] those techniques are used in conjunction with an AR and MA estimation procedure.

This paper presents a joint order and coefficient estimation algorithm for ARMA processes, exclusively based on the reflection coefficient sequence associated with the ARMA(p_0,q_0) process. It extends previously reported work on ARMA estimation, (see [5], [6]).

The order selection is based upon the minimization of a functional that measures the mismatch of each assumed ARMA(p,q) model to the data. The functional d is evaluated through the reflection coefficient sequence estimated from the data using the Burg technique, [7]. When the orders are decided, the proposed scheme simultaneously provides the AR and the MA components.

The paper organization is the following. In section 2, the functional d is defined. It is proved that $d = 0$ for the true model, i.e., $p = p_0$ and $q = q_0$. Some properties of d are listed in section 3 as the base for the order selection scheme proposed. Section 4 displays some simulation results. Finally, section 5 concludes the paper.

2. FUNCTIONAL DEFINITION

Let $\{y_n\}$ be a stationary, Gaussian, ARMA(p_0,q_0) process given by

$$y_n + \sum_{i=1}^{p_0} a_i y_{n-i} = e_n + \sum_{i=1}^{q_0} b_i e_{n-i}\ , \ n \in Z \tag{1}$$

where $\{e_n\}$ is a white Gaussian noise with zero mean and variance σ^2. The poles and zeros of (1) lie inside the unit circle. The usual conditions on system minimality are assumed to be verified, thus the coefficients p_0, q_0, $\{a_i, i = 1,\ldots,p_0\}$, $\{b_i, i = 1,\ldots,q_0\}$ and σ^2 fully characterize the model (1).

In this section we define a functional $d(N,p,q)$ that measures the mismatch between each assumed ARMA(p,q) model and the ARMA(p_0,q_0) process. Throughout the section we assume the exact knowledge of the reflection coefficient sequence associated with $\{y_n\}$. We first introduce notation and present important relationships derived in [5], [6] when the orders p_0 and q_0 are known.

Notation

We denote by a_j^N and W_j^N, $0 \le j \le N$, the coefficients of the prediction error filter and innovation filter of order N, ($N \ge 0$), associated with the ARMA(p_0,q_0) process. Let $\mathbf{W}_N$ and $\mathbf{W}_N^{-1}$ be the lower triangular matrices of order $N+1$, with unit diagonal, and nondiagonal entries

$$[\mathbf{W}_N]_{ij} = W_{i-j}^i\ ,\ \ j \le i\ ,\ 0 \le i,j \le N \tag{2}$$

$$[\mathbf{W}_N^{-1}]_{ij} = a_{i-j}^i\ ,\ \ j \le i\ ,\ 0 \le i,j \le N. \tag{3}$$

The matrices $\mathbf{M}_{21}(N,p_0,q_0)$, $\mathbf{M}_{22}(N,p_0,q_0)$, $\mathbf{m}_1^T(N,p_0,q_0)$, and $\mathbf{m}_2^T(N,p_0,q_0)$ are the blocks of $\mathbf{W}_N$ with the orders shown in Fig. 1 for $N \ge p_0 + q_0$.

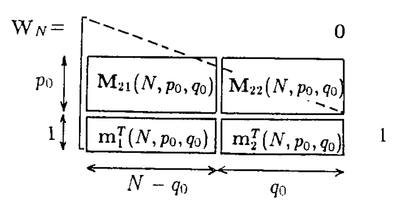

Figure 1: Blocks considered in $\mathbf{W}_N$.

Define

$$\mathcal{M}(N,p_0,q_0) = \left[\mathbf{M}_{21}(p_0+q_0,p_0,q_0) \mid \ldots \mid \mathbf{M}_{21}(N,p_0,q_0)\right] \tag{4}$$

$$m(N,p_0,q_0) = \left[\mathbf{m}_1^T(p_0+q_0,p_0,q_0) \mid \ldots \mid \mathbf{m}_1^T(N,p_0,q_0)\right] \tag{5}$$

Let $\mathbf{N}_{21}(N,p_0,q_0)$, $\mathbf{N}_{22}(N,p_0,q_0)$, $\mathbf{n}_1^T(N,p_0,q_0)$ and $\mathbf{n}_2^T(N,p_0,q_0)$ be defined in correspondence with the matrix $\mathbf{W}_N^{-1}$ in a similar way as defined above for $\mathbf{W}_N$, replacing the roles of p_0 and q_0.

Let

$$\mathbf{a} = [\, a_1\ a_2\ \ldots\ a_{p_0}\,]^T \quad (6)$$

$$\mathbf{b} = [\, b_1\ b_2\ \ldots\ b_{q_0}\,]^T \quad (7)$$

be the AR and MA components of the ARMA(p_0,q_0) process, and

$$\mathbf{\Omega}(N) = [\, \Omega_1(N)\ \Omega_2(N)\ \ldots\ \Omega_{q_0}(N)\,]^T$$

be a conveniently defined vector (see [5], [6]), that converges to the MA component (7) as N goes to infinity, i.e., $\lim_{N\to\infty} \mathbf{\Omega}(N) = \mathbf{b}$.

Define the lower triangular matrices of order $N+1$, $\mathbf{A}(N,p_0,q_0)$ and $\beta(N,p_0,q_0)$ as

$$[\mathbf{A}(N,p_0,q_0)]_{k\bullet} = \left[\mathbf{0}^T_{k-p_0} \mid \mathbf{a}^T \mathbf{J}_{p_0} \mid 1 \mid \mathbf{0}^T_{N-k}\right], \quad (8)$$

$$[\beta(N,p_0,q_0)]_{k\bullet} = \left[\mathbf{0}^T_{k-q_0} \mid \mathbf{\Omega}^T(\mathbf{k})\mathbf{J}_{q_0} \mid 1 \mid \mathbf{0}^T_{N-k}\right], \quad p_0+q_0 \le k \le N, \quad (9)$$

where $[\ ]_{k\bullet}$ denotes the line k of the corresponding matrix, $\mathbf{J}_k$ is the circular permutation matrix of order k and $\mathbf{0}_k$ is the null vector of order k.

The symbol $\|\ \|_2$ will be used to represent the Cartesian norm.

Known Orders

When p_0 and q_0 are known, the increasing order prediction and innovation filter coefficients associated with $\{y_n\}$ are related with the vectors a and $\mathbf{\Omega}(k)$. The following result is proved in [5], [6].

Result 1: The vectors a e $\mathbf{\Omega}(k)$ satisfy,

$$\mathbf{N}^T_{21}(k,p_0,q_0)\,\mathbf{J}_{q_0}\,\mathbf{\Omega}(k) - \mathbf{n}_1(k,p_0,q_0) = \mathbf{0} \quad (10)$$

$$\mathcal{M}^T(N,p_0,q_0)\,\mathbf{J}_{p_0}\,\mathbf{a} + m(N,p_0,q_0) = \mathbf{0} \quad (11)$$

$$\mathbf{M}^T_{22}(k,p_0,q_0)\,\mathbf{J}_{p_0}\,\mathbf{a} + \mathbf{m}_2(k,p_0,q_0) = \mathbf{J}_{q_0}\,\mathbf{\Omega}(k) \quad (12)$$

$$\mathbf{N}^T_{22}(k,p_0,q_0)\,\mathbf{J}_{q_0}\,\mathbf{\Omega}(k) + \mathbf{n}_2(k,p_0,q_0) = \mathbf{J}_{p_0}\,\mathbf{a}, \quad (13)$$

$$p_0+q_0 \le k \le N.$$

□

For $N \ge p_0+q_0$, the vectors a and $\mathbf{\Omega}(k)$ are the solution of the systems of linear equations in (10)-(11). Those equations are decoupled with respect to a and $\mathbf{\Omega}(k)$. On the other hand, the linear algebraic relations (12)-(13) establish a joint function between those vectors and the increasing order prediction error filter and innovation filter coefficients. In a matrix format, Result 1 is expressed as

$$[\mathbf{A}(N,p_0,q_0)]_{k\bullet}\ \mathbf{W}_N = [\beta(N,p_0,q_0)]_{k\bullet} \quad (14)$$

$$[\beta(N,p_0,q_0)]_{k\bullet}\ \mathbf{W}_N^{-1} = [\mathbf{A}(N,p_0,q_0)]_{k\bullet} \quad p_0+q_0 \le k \le N. \quad (15)$$

Unknown Orders

When p_0 and q_0 are not known, we assume that $\{y_n\}$ is an ARMA(p,q) process. The order selection algorithm is based upon a functional $d(N,p,q)$ evaluated for each pair (p,q). Its value is obtained from a set of vectors computed from the coefficients of the prediction and innovation filters associated with $\{y_n\}$ and considered up to order N.

Those vectors are defined as follows. Let $\mathbf{N}_{ij}(N,p,q)$, $\mathbf{n}_i(N,p,q)$, $\mathbf{M}_{ij}(N,p,q)$, $\mathbf{m}_i(N,p,q)$, $i,j=1,2$ and $\mathcal{M}(N,p,q)$, $m(N,p,q)$ be defined for (p,q) in accordance with the corresponding matrices introduced for $p=p_0$ and $q=q_0$.

Definition 1: ${}^1\delta(k,p,q) \in \mathrm{R}^q$ is the minimum norm vector $\mathbf{x}$ that minimizes

$$\left\|\mathbf{N}^T_{21}(k,p,q)\,\mathbf{J}_q\,\mathbf{x} + \mathbf{n}_1(k,p,q)\right\|_2, \quad p+q \le k \le N. \quad (16)$$

□

Definition 2: ${}^1\gamma(N,p,q) \in \mathrm{R}^p$ is the minimum norm vector $\mathbf{y}$ that minimizes

$$\left\|\mathcal{M}^T(N,p,q)\,\mathbf{J}_p\,\mathbf{y} + m(N,p,q)\right\|_2, \quad N \ge p+q. \quad (17)$$

□

With (16) and (17), we build the lower triangular band matrices of order $N+1$, ${}^1\mathbf{A}(N,p,q)$ and ${}^1\beta(N,p,q)$ as,

$$\left[{}^1\mathbf{A}(N,p,q)\right]_{k\bullet} = \left[\mathbf{0}^T_{k-p} \mid {}^1\gamma(N,p,q)^T\,\mathbf{J}_p \mid 1 \mid \mathbf{0}^T_{N-k}\right], \quad (18)$$

$$\left[{}^1\beta(N,p,q)\right]_{k\bullet} = \left[\mathbf{0}^T_{k-q} \mid {}^1\delta(k,p,q)^T\,\mathbf{J}_q \mid 1 \mid \mathbf{0}^T_{N-k}\right] \quad (19)$$

$$p+q \le k \le N.$$

Using the Result 1, note that for $p=p_0$ and $q=q_0$,

$${}^1\delta(k,p_0,q_0) = \mathbf{\Omega}(k), \quad p_0+q_0 \le k \le N, \quad (20)$$

$${}^1\gamma(N,p_0,q_0) = \mathbf{a}, \quad N \ge p_0+q_0, \quad (21)$$

leading to

$$\left[{}^1\mathbf{A}(N,p_0,q_0)\right]_{k\bullet} = [\mathbf{A}(N,p_0,q_0)]_{k\bullet} \quad (22)$$

$$\left[{}^1\beta(N,p_0,q_0)\right]_{k\bullet} = [\beta(N,p_0,q_0)]_{k\bullet}. \quad (23)$$

The above four equalities show that when $p=p_0$ and $q=q_0$, the vectors defined in (17) and (16) coincide with a and $\mathbf{\Omega}(k)$ related with the AR and the MA components of the ARMA(p_0,q_0) model (1). By analogy, we associate ${}^1\gamma(N,p,q)$ and ${}^1\delta(k,p,q)$ with the AR and the MA components of the ARMA(p,q) model assumed for $\{y_n\}$.

Definition 3: The vectors ${}^{2,k}\gamma(k,p,q)$ and ${}^{2,k}\mathbf{e}_{AR}(k,p,q)$ are defined by

$$\mathbf{J}_p\ {}^{2,k}\gamma(k,p,q) = \mathbf{N}^T_{22}(k,p,q)\,\mathbf{J}_q\ {}^1\delta(k,p,q) + \mathbf{n}_2(k,p,q), \quad (24)$$

$${}^{2,k}\mathbf{e}_{AR}(k,p,q) = \mathbf{N}^T_{21}(k,p,q)\,\mathbf{J}_q\ {}^1\delta(k,p,q) + \mathbf{n}_1(k,p,q) \quad (25)$$

$$p+q \le k \le N.$$

□

Definition 4: The vectors ${}^{2,k}\delta(N,p,q)$ and ${}^{2,k}\mathbf{e}_{MA}(N,p,q)$ are defined by

$$\mathbf{J}_q\ {}^{2,k}\delta(N,p,q) = \mathbf{M}^T_{22}(k,p,q)\,\mathbf{J}_p\ {}^1\gamma(N,p,q) + \mathbf{m}_2(k,p,q) \quad (26)$$

$${}^{2,k}\mathbf{e}_{MA}(N,p,q) = \mathbf{M}^T_{21}(k,p,q)\,\mathbf{J}_p\ {}^1\gamma(N,p,q) + \mathbf{m}_1(k,p,q) \quad (27)$$

$$p+q \le k \le N.$$

□

Note that (24) and (26) are similar to (13) and (12) that are verified for the ARMA(p_0,q_0) model (1). The linear algebraic relations (25) and (27) evaluate the error associated with the least-squares minimizations in Definitions 1 and 2.

Once the elements of the matrices ${}^1\mathbf{A}$ and ${}^1\beta$ in (18) and (19) are evaluated using (16) and (17), we build the matrices ${}^2\mathbf{A}$ and ${}^2\beta$ as

$${}^1\mathbf{A}(N,p,q)\ \mathbf{W}_N = {}^2\beta(N,p,q) \quad (28)$$

$${}^1\beta(N,p,q)\ \mathbf{W}_N^{-1} = {}^2\mathbf{A}(N,p,q). \quad (29)$$

Using the Definitions 3 and 4 yields,

$$\left[{}^2\beta(N,p,q)\right]_{k\bullet} = \left[{}^{2,k}\mathbf{e}_{MA}(N,p,q)^T \mid {}^{2,k}\delta(N,p,q)^T\mathbf{J}_q \mid 1 \mid \mathbf{0}^T_{N-k}\right]$$

$$\left[{}^2\mathbf{A}(N,p,q)\right]_{k\bullet} = \left[{}^{2,k}\mathbf{e}_{AR}(k,p,q)^T \mid {}^{2,k}\gamma(k,p,q)^T\mathbf{J}_p \mid 1 \mid \mathbf{0}^T_{N-k}\right], \quad p+q \leq k \leq N.$$

For $p = p_0$, $q = q_0$, replacing the Result 1 and (20)-(21) in the Definitions 3-4 leads to

$$ {}^{2,k}\mathbf{e}_{AR}(k,p_0,q_0) = \mathbf{0}, \quad p_0+q_0 \leq k \leq N, \tag{30}$$

$$ {}^{2,k}\mathbf{e}_{MA}(N,p_0,q_0) = \mathbf{0}, \quad p_0+q_0 \leq k \leq N, \tag{31}$$

$$ {}^{2,k}\gamma(k,p_0,q_0) = \mathbf{a}, \quad p_0+q_0 \leq k \leq N, \tag{32}$$

$$ {}^{2,k}\delta(N,p_0,q_0) = \mathbf{\Omega}(k), \quad p_0+q_0 \leq k \leq N. \tag{33}$$

As a consequence, both ${}^2\mathbf{A}(N,p_0,q_0)$ and ${}^2\beta(N,p_0,q_0)$ given by (28) and (29) have a band struture, beginning on line p_0+q_0. In particular, ${}^1\mathbf{A}(N,p_0,q_0) = {}^2\mathbf{A}(N,p_0,q_0)$ and ${}^1\beta(N,p_0,q_0) = {}^2\beta(N,p_0,q_0)$.

We finally define the functional $d(N,p,q)$.

Definition 5: The functional $d(N,p,q)$ is given by

$$d(N,p,q) = d_{AR}(N,p,q) + d_{MA}(N,p,q), \tag{34}$$

where

$$d_{AR}(N,p,q) = \sum_{k=p+q}^{N} \left\| \left[{}^1\mathbf{A}(N,p,q)\right]_{k\bullet} - \left[{}^2\mathbf{A}(N,p,q)\right]_{k\bullet} \right\|_2^2, \tag{35}$$

$$d_{MA}(N,p,q) = \sum_{k=p+q}^{N} \left\| \left[{}^1\beta(N,p,q)\right]_{k\bullet} - \left[{}^2\beta(N,p,q)\right]_{k\bullet} \right\|_2^2, \tag{36}$$

$$N \geq p+q.$$

□

From (35) and (36) the elements d_{AR} and d_{MA} on the functional d evaluate the mismatch between the last $N+1-p-q$ lines on the pair of matrices $({}^1\mathbf{A}, {}^2\mathbf{A})$ and $({}^1\beta, {}^2\beta)$. For $p = p_0$ and $q = q_0$ those matrices coincide and so $d(N,p_0,q_0) = 0$.

The order selection is based on the values of $d(N,p,q)$ for each pair (p,q) and increasing values of N. From the Definitions 1-5, note that $d(N,p,q)$ is not evaluated for N less than $p+q$. The vectors $({}^1\gamma, {}^{2,k}\gamma, {}^{2,k}\mathbf{e}_{AR})$ and $({}^1\delta, {}^{2,k}\delta, {}^{2,k}\mathbf{e}_{MA})$ that define the functional d, may be evaluated recursively in p, q and N [8].

3. ORDER SELECTION

The order selection is based upon the properties of the functional $d(N,p,q)$ defined in the previous section.

Properties

When there is exact knowledge of the prediction and innovation filter coefficients of increasing order, the following properties hold. Due to space limitation, we will not present the proofs, the reader being refered to [8]. In figures 2 to 4 we sketch the locus, in the (p,q) plane, in correspondence with the properties $P1$ to $P6$. The symbol ////, refers to a non null functional d, while 0 represents the set of pairs (p,q) where d is zero.

P1. $d(N,p,q_0) = 0$ for $p \geq p_0$, $N \geq p+q_0$.

P2. $d(N,p_0,q) = 0$ for $q \geq q_0$, $N \geq p_0+q$.

For $p \geq p_0$, $q = q_0$ and $N \geq p+q_0$, $P1$ is verified because the errors associated with the least-squares minimizations in (16)-(17) are zero and [8].

$$ {}^1\gamma(N,p,q_0) = {}^{2,k}\gamma(k,p,q_0) = \left[\mathbf{a}^T \mid \mathbf{0}^T_{p-p_0}\right]^T, \tag{37}$$

$$ {}^1\delta(k,p,q_0) = {}^{2,k}\delta(N,p,q_0) = \mathbf{\Omega}(k), \quad p+q_0 \leq k \leq N. \tag{38}$$

This means that the assumed ARMA(p,q_0) model ($p \geq p_0$), has p_0 poles at the same locations as the poles of (1), the remaining $p-p_0$ poles being at the origin. Also, from (38) the two models have the same set of zeros.

By duality, similar conclusions are valid for $P2$.

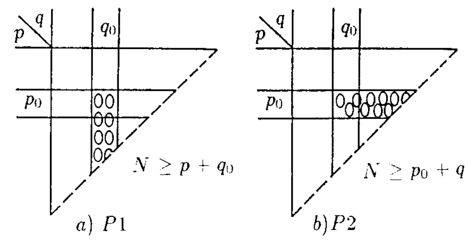

Figure 2: Locus considered in $P1$ and $P2$

P3. If $\mathbf{N}_{21}(p+q,p,q)$ and $\mathbf{M}_{21}(p+q,p,q)$ are nonsingular matrices, $d(p+q,p,q) = 0$.

This property refers to the set of pairs (p,q) such that $N = p+q$, i.e., the pairs located at the diagonal shown in figure 3a. Note that this diagonal is shifted down as N increases.

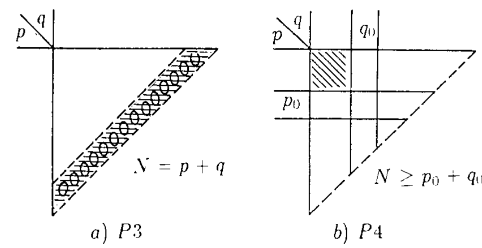

Figure 3: Locus considered in $P3$ and $P4$.

P4. $d(N,p,q) \neq 0$ for $p < p_0$, $q < q_0$, $N \geq p_0+q_0$.

P5. $d(N,p,q) \neq 0$ for $p \geq p_0$, $q < q_0$, $N \geq p+q_0$.

P6. $d(N,p,q) \neq 0$ for $p < p_0$, $q \geq q_0$, $N \geq p_0+q$.

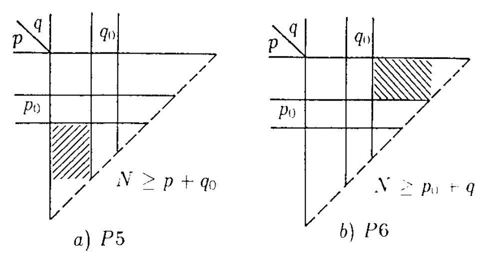

Figure 4: Locus considered in $P5$ and $P6$.

Selection

From $P1$ and $P2$ the range space of $d(N,p,q)$ presents an orthogonal pattern of null zeros, with (p_0,q_0) as its intersection point (see fig. 2). This pattern is established for $N = p_0 + q_0$ and does not change as N increases.

Every ARMA(p,q) model with (p,q) belonging to an orthogonal pattern of null functional has the same spectrum as the one corresponding to its intersection point [8]. For the pattern associated with (p_0,q_0), this is due to $P1$-$P2$. Note that among all the models related to an orthogonal pattern, the intersection point is the one with the smallest number of parameters. Together with $P3$ to $P6$, we then conclude that (p_0,q_0) is the intersection point of the orthogonal pattern with $d = 0$ established for the smallest value of N and that it is "stable" with increasing values of N. The selection algorithm identifies this pattern.

Define

$$I(N,p,q) = \{(p,q) \;:\; d(N,p,q) = 0 \vee N < p+q\} \quad (39)$$

$$\mathcal{L}(p,q) = \bigcap_{N \geq 1} I(N,p,q) \quad (40)$$

where $\bigcap$ stands for set intersection.

For each value of N, the set $I(N,p,q)$,

i) **does not include** the orders (p,q) of ARMA models that definitely are not the correct structure for $\{y_n\}$, i.e., $\{(p,q):\; d(N,p,q) \neq 0,\; N \geq p+q\}$;

ii) **collects** all the models for which there is not enough information for an accept or reject decision, i.e., $\{(p,q):\; d(N,p,q) = 0,\; N \geq p-q\}$ or $\{(p,q):\; N < p+q\}$.

The functional properties yield

$$\{(p,q):\; (p = p_0, q \geq q_0) \vee (p \geq p_0, q = q_0)\} \subset \mathcal{L}(p,q)$$

i.e., the orthogonal pattern with (p_0,q_0) as its intersection point is a subset of $\mathcal{L}(p,q)$. Thus (p_0,q_0) is obtained as the model ARMA(p,q) belonging to $\mathcal{L}(p,q)$ and with the smallest number of parameters.

$$(p_0,q_0) = \min_{(p,q)}\{p+q,\; (p,q) \in \mathcal{L}(p,q)\}. \quad (41)$$

When the order is decided, the algorithm simultaneously provides the corresponding AR (vector ${}^{1}\gamma$) and MA (vector ${}^{1}\delta$) components.

4. SIMULATION RESULTS

In the presence of a finite sample of lenght T of the observation process, the exact values of the prediction and innovation filter coefficients are replaced by its estimates obtained from the data using the Burg technique, [7].

Due to estimation errors on these cofficients, $P1 - P6$ do not exactly hold. The selection algorithm is implemented replacing the zero in (39) by a positive constant. The intersection point of an orthogonal pattern is obtained searching the functional $d(N,p,q)$ by increasing diagonals, i.e., $p+q = k$, $1 \leq k < N$ and picking the first pair (p,q) for which $d(N,p,q)$ is small and much smaller than the other values in the same diagonal.

For the ARMA(1,1) model

$$y_n - 0.6y_{n-1} = e_n + 0.6e_{n-1} \quad (42)$$

with $\sigma^2 = 1$, and $N = 10$, we perform 100 independent Monte-Carlo runs. Table 1 displays the number of correct estimates of the orders p_0 and q_0 for several values of T.

	T=100	T=250	T=500
Number of correct estimates	62	87	99

Table 1.

The same kind of simulation results were obtained for the ARMA(2,1) model

$$y_n - 1.2y_{n-1} + 0.36y_{n-2} = e_n + 0.9e_{n-1} \quad (43)$$

with $\sigma^2 = 1$, $N = 15$ and 100 independent Monte-Carlo runs. Table 2 displays the number of runs leading to the correct order selection for several values of T.

	T=500	T=1000	T=5000
Number of correct estimates	52	82	100

Table 2.

5. CONCLUSIONS

A joint order and parameter estimation algorithm for ARMA processes was presented, based on the reflection coefficient sequence associated with the process. The order selection is obtained through a functional minimization. The AR and MA coefficients are the solution of systems of linear equations. Some simulation examples display the order selection performance. In a future work we will compare it with known order techniques.

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