43rd IEEE Conference on Decision and Control
December 14-17, 2004
Atlantis, Paradise Island, Bahamas

Estimation in Virtual Sensor-Actuator Arrays Using Reduced-Order Physical Models

Haotian Zhang, Bruce Krogh, José M. F. Moura and Wei Zhang

Abstract—**In many applications envisioned for wireless sensors, physical variables governed by continuous, distributed dynamics need to be monitored and controlled. Spatial and temporal irregularities in sampling available from the wireless nodes are usually inconsistent with the application requirements. In this paper we propose virtual sensor-actuator arrays to address this problem and consider the problem of estimating values of physical variables at points other than the wireless node locations. We use reduced-order models of the physical dynamics to generate these estimates. Model parameters are estimated using parametric system identification techniques. The concepts are illustrated for the problem of estimating temperatures based on circuit models of thermal dynamics. Results are presented from experiments using actual data collected by a wireless sensor network.**

I. Introduction

Large-scale wireless sensor/actuator arrays are envisioned as being useful in a variety of applications ranging from wide-area monitoring and surveillance to control of flexible space structures. A number of research programs are focusing on the development of lower-level protocols and middleware services that take care of network formation, timing synchronization, calibration and real-time quality-of-service. Even when these problems are solved, signal and information processing algorithms will be needed to deal with the temporal and spatial irregularities inherent in the information from these networks. We are developing information processing middleware that will make it possible for application-domain algorithms to be implemented without having to deal explicitly with the irregularities in the physical data and the physical device array. The goal is to make it possible for application algorithms to be written as if the sensing and actuating devices are located as desired in the application design model. We call this a *virtual sensor-actuator array* (VSAA).

This paper presents a first step toward realizing VSAAs. We use physical models to produce real-time estimates of the values of a distributed field at points where there are no sensors. These estimates are based on a subset of sensors surrounding each point of interest. Figure 1 illustrates our approach. We first construct for the real world a detailed parameterized physical model $\mathcal{M}(\theta)$ with unknown parameters θ. In this model $\mathcal{M}(\theta)$, we assign as inputs U the measurements at a subset of the sensors and as outputs Y the measurements at the remaining sensors. Then we estimate by system identification the values of the unknown parameters θ based on the measured inputs U and measured outputs Y. Based on the resulting model $\mathcal{M}(\hat{\theta})$, we can calculate the output $\hat{Y}'$ at the locations of interest, where no sensor is available. By utilizing the cut set concept and the Substitution Theorem, see p. 28 and pp. 129-130, [1], respectively, a reduced model $\mathcal{M}_r(\theta_r)$ with unknown parameters θ_r is derived from the detailed model $\mathcal{M}(\hat{\theta})$. We assign $U' = (U^C, Y^C)$ as the inputs to the reduced model, where U^C and Y^C are a subset of the inputs U and outputs Y of the real world, and $\hat{Y}'$ as its outputs. By system identification, we obtain again a reduced model $\mathcal{M}_r(\hat{\theta}_r)$ with estimated parameters $\hat{\theta}_r$ that can be used subsequently to estimate the outputs Y^N at the locations of interest for arbitrary inputs U^N.

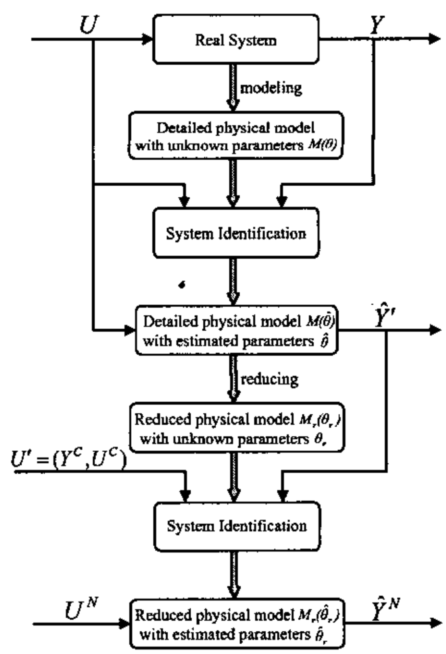

Fig. 1. Approach diagram.

The following section describes lumped-parameter models for a physical world of thermal capacities and flows. Section III presents our approach to identify model parameters for estimating temperatures at locations without

Research supported by NSF grant # ECS-0225449.

Haotian Zhang, Bruce Krogh, José M. F. Moura are with the Department of Electrical and Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213, USA. {haotian, krogh, moura}@ece.cmu.edu

Wei Zhang is with the Robotics Institute, Carnegie Mellon University, Pittsburgh, PA 15213, USA. weizhang@frc.ri.cmu.edu

sensors. The approach is illustrated in Section IV using data collected with a wireless sensor array in the Intelligent Workplace at Carnegie Mellon University. The concluding section discusses our current research directions.

II. Thermal Models

Monitoring and control in buildings is an important application for large arrays of wireless sensors and actuators [2], [3]. Heating and cooling strategies optimize the comfort of the people in a building subject to constraints on power and total energy consumption. The problem is that sensors must be distributed in a building in some way that is convenient for making sure they will not be tampered with and the HVAC actuators are fixed by the construction of the building. Consequently, the building control system often optimizes the comfort objectives at the sensor locations, rather than where the users are currently located. If large-arrays of wireless sensors are used, it may be possible to have much better knowledge of the temperature distribution in the building, but that still does not address the problem of matching the information to the control objectives.

In this paper we consider the problem of estimating the temperature at locations in a building different from where sensors are placed. Temperature dynamics are governed by partial differential equations (PDEs) characterizing the thermal effects of convection, conduction, and radiation. Electrical circuit equivalents for these dynamics are standard for simulating the thermal properties of buildings [4], [5]. More recently, there has been considerable research into methods for creating effective, parsimonious lumped parameter models of thermal dynamics using equivalent electrical circuits to characterize the thermal properties of IC and MEMS components [6], [7], [8]. These compact models are developed using a combination of physical insight and empirical data, often obtained from simulations of detailed CFD (computational fluid dynamics) models of the thermal properties of IC packages. Physical insight suggests the basic circuit structure that should be used, and the empirical data is used to estimate appropriate values for the parameters in the circuit model. Our approach is motivated by this approach.

To model thermal dynamics in a building using circuit components, voltage sources model constant temperature elements, such as the ambient external temperature, and current sources model sources of radiant heat. Capacitors model thermal capacity and resistors model thermal conduction effects. The actual values of these circuit elements are derived by system identification techniques rather than from first principles.

We explain our approach with respect to the rectangular room whose layout is shown in Figure 2. The heat transfers and conduction-convection effects are modeled by the electrical circuit shown in Figure 3. In this figure, the outside ambient temperature gives rise to 8 identical voltage sources labeled by T_o. Resistors $R_{ok}, k = 1, \ldots, 8$ represent the conduction effects across the walls, between the outside and the inside of the walls. Resistors R_{wij} and capacitors C_{wj}, where $i, j = 1, \ldots, 8$, and i is indices adjacent to j, represent the conduction effects and the thermal capacity along the walls. The resistors $R_{ij}, j = 1, \ldots, 8$ represent the combined conduction and convection effects between the walls and the air inside the room. Capacitors $C_j, j = 1, \ldots, 4$ represent the thermal capacity of each of the room divisions. Finally, the resistors R_{12}, R_{23}, R_{34}, and R_{41} represent the heat transfers by conduction and convection between the air in the four subdivisions of the room.

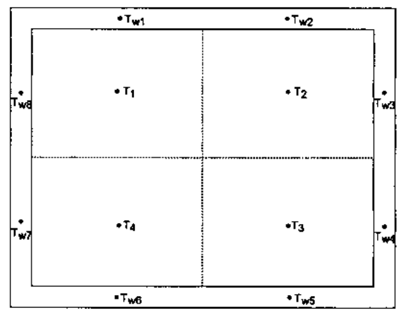

Fig. 2. Layout of studied room.

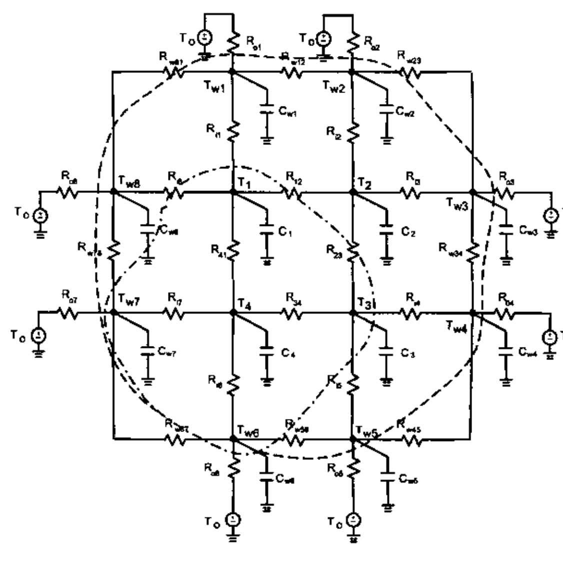

Fig. 3. RC network for studied room.

III. System Identification

We assume that temperature sensors are placed at nodes T_{w1} through T_{w8}, and also at nodes labeled T_1, T_2, and T_3, see Figure 2. No temperature sensor is placed at node T_4. Our goal is to predict the temperature at node T_4.

By the *substitution theorem* [1] in circuit theory, we know that the dynamical behavior of the circuit is not affected if we insert voltage sources in nodes T_{w1} through T_{w8} whose values are exactly the time series measured by the sensors placed at these nodes. Further, we note that the outer dashed line drawn over the circuit in Figure 3 is a cut set. From these considerations, if we are only interested in the dynamical behavior *inside* the cut set (dashed line), we can simplify the circuit in Figure 3 to that in Figure 4. In this circuit, we have eliminated the sources representing the outside ambient temperature T_o and the corresponding resistors $R_{oj}, j = 1, \ldots, 8$, the wall resistors R_{wj}, and the capacitors $C_{wj}, j = 1, \ldots, 8$.

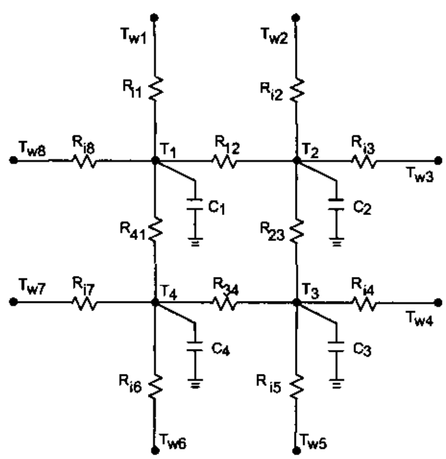

Fig. 4. Detailed model.

This is justified by the cut set concept and the Substitution Theorem [1] as follows. The RC network shown in Figure 3 is a nonplanar circuit. We redraw the RC network as a three-layer structure: the top layer is a resistor network, the bottom layer is just the ground, and the middle layer is composed of capacitors and voltage/current sources that are between the nodes in the top layer and the ground. Assume the node T_4 is of interest. We can choose a cut set in the top layer to make sure that if the branches in the cut set are removed the resulting inner subcircuit SC_1 includes the node T_4. Assume the selected cut set is the set of all the branches crossed by the outer (or inner) dashed line in Figure 3. In the resulting inner subcircuit SC_1, let N_1 be the set of all the nodes associated with the branches in the cut set. In this example, $N_1 = \{T_{wi} \mid i = 1, 2, \ldots, 8\}$. The RC network in Figure 3 is equivalent to the structure in Figure 5, where: the subcircuit SC_1 is the resulting inner subcircuit if the branches in the cut set are removed; subcircuit SC_2 is the outer subcircuit of the original circuit; u_1, u_2, ..., u_n are all the nodes associated with the branches in the cut set, i.e., $N_1 = \{u_1, u_2, \ldots, u_n\}$; and C_1, C_2, ..., C_n are the capacitors between these nodes and the ground. Suppose we know the voltages at all the nodes in N_1, i.e., $u_1(t)$, $u_2(t)$, ..., $u_n(t)$. Branches (u_k, ground) can be replaced by voltage sources of value $u_k(t)$, $k = 1, 2, \ldots, n$, leading to the circuit shown in Figure 6, where the subcircuits SC_1 and SC_2 are connected by a subcircuit of only the n voltage sources. If we duplicate these voltage sources and divide the whole circuit into two parts, as shown in Figure 7, we get two independent circuits. Subcircuits SC1 and SC2 have the same external sources in both Figure 6 and 7. Therefore, the voltages at the nodes inside SC_1 and SC_2 are not altered, in particular, the voltage at the node of interest (T_4) is not altered.

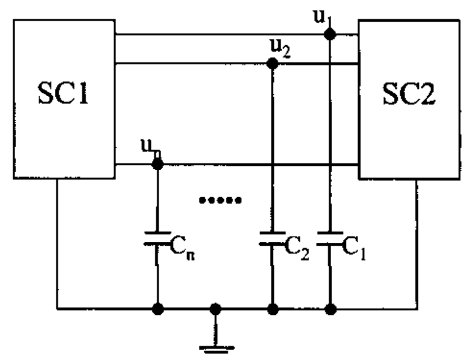

Fig. 5. Equivalent structure of the RC network.

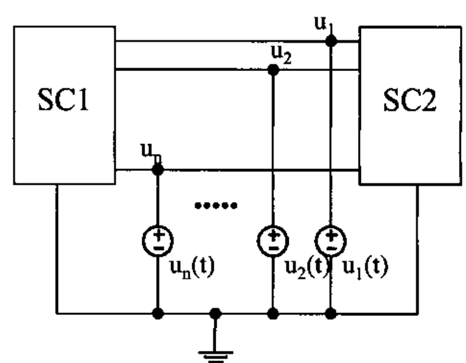

Fig. 6. Circuit after applying the Substitution Theorem.

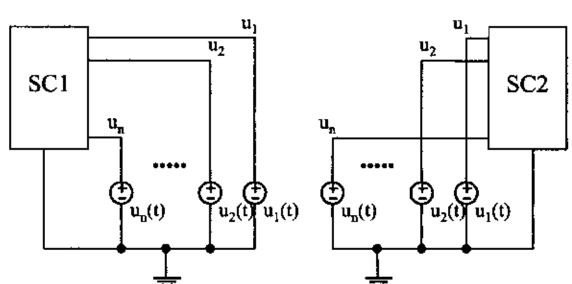

Fig. 7. Substitution Theorem: Two independent circuits.

We now work with the circuit in Figure 4. The goal is to estimate from the time series measured by the sensors in $T_{wj}, j = 1, \ldots, 8$, acting as inputs, and the time series measured by the sensors in T_1, T_2, and T_3, acting as outputs, all the unknown parameters in the circuit of Figure 4. We adopt a state-space model for the RC-circuit. The state space is represented by the four temperatures $T_k, k = 1, \ldots, 4$ (voltage at the capacitors $C_k, k = 1, \ldots, 4$)

$$\mathbf{x}_n = [T_{1n}, \ldots, T_{4n}]^T, \quad (1)$$

where time is indicated by the subscript n. The input vector in the state equation collects the measured temperatures:

$$\mathbf{u}_n = [T_{w1n}, \ldots, T_{w8n}]^T. \quad (2)$$

The output vector is

$$\mathbf{y}_n = [T_{1n}, T_{2n}, T_{3n}]^T, \quad (3)$$

since we assume as stated before that we place sensors at positions T_1, T_2, and T_3.

From standard Kirchoff Current and Voltage Laws [9], we can determine the state and output equations

$$\mathbf{x}_{n+1} = A\mathbf{x}_n + B\mathbf{u}_n, \qquad \mathbf{y}_n = C\mathbf{x}_n + \mathbf{w}_n \quad (4)$$

where A, B, C are matrices with appropriate dimensions, in our case A is 3×3, B is 3×8, and C is 3×4. We note that C is a known matrix given by

$$C = \begin{pmatrix} I_3 & \underline{0} \end{pmatrix}, \quad (5)$$

where I_3 is the 3×3 identity matrix. The entries in the model matrices A and B are either 0 or expressed in terms of the unknown resistors and capacitors in the network of Figure 4. We do not present the structure of the matrices here.

From the sensor measurements, with T_{wjn} adopted as inputs, $j = 1, \ldots, 8$, and $n = 1, \ldots, N$, where N is the total number of measurements collected by each sensor, and the sensor measurements $T_{jn}, j = 1, 2, 3, n = 1, \ldots, N$ acting as outputs, we can estimate the network parameters by standard techniques from identification theory, e.g., [10]. We use least squares and Guass-Newton algorithms implemented in MATLAB $m = pem(data, mi)$, where mi is the initial model with the user-defined model structure and the initial values of the parameters to estimate, $data$ is an iddata object that contains the input/output data, and m is returned as the best fitting model with estimated parameters in the model structure defined by mi [11].

Reduced Model Once the network R and C parameters have been identified, we can derive the detailed network of Figure 4 with the source $T_{wjn}, j = 1, \ldots, 8$, and $n = 1, \ldots, N$ and predict the temperature at location T_4 where there is no sensor. In a real-life application, the number of locations where it may be desirable to predict the temperature but where no sensors are placed may be large, and it may be impractical to run the detailed model in Figure 4 in real time. Instead we derive for each of the desired locations a reduced model. We illustrate the approach by developing for our running model of Figure 4 this reduced model for location T_4.

To predict the temperature at the node T_4 of the circuit of Figure 4, given the measurements at $T_{wj}, j = 1, \ldots, 8$ and $T_j, j = 1, 2, 3$, we derive a reduced model for the model in Figure 4. To do that, we observe that the dashed-point line in Figure 4 represents a cut set. Like before, we can replace the circuit of Figure 4 by the reduced circuit in Figure 8 where the sensor measurements T_1, T_2, T_3 and T_{w6} and T_{w7} are modeled now as known sources, and the temperature at node T_4 is modeled as the output of the circuit. We use the measured data $T_{1n}, T_{2n}, T_{3n}, T_{w6n}$, and T_{w7n}, and the temperature time series T_{4n} predicted at T_4 from the detailed model in Figure 4 to estimate the unknown parameters in the model of Figure 8. This is again an identification problem for a dynamical system for which we use the same MATLAB routine indicated previously to estimate the parameter values.

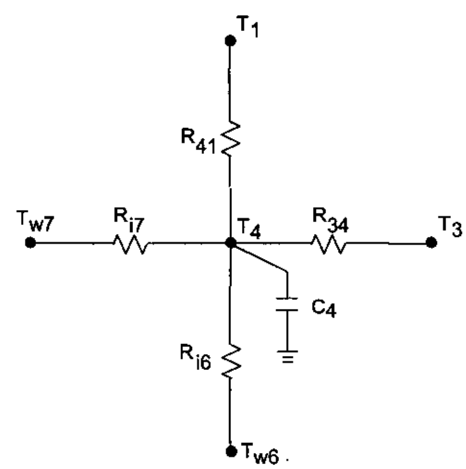

Fig. 8. Reduced model.

IV. VIRTUAL SENSING IN THE INTELLIGENCE WORKPLACE

We apply the approach in sections II and III to the Carnegie Mellon Unversity Intelligent Workplace (IW). The layout of the workplace is shown in Figure 9. We placed 10 Crossbow, Inc. wireless "motes" with temperature sensors at the locations labeled s_1 through s_{10} in Figure 9. Due to transmission errors between the motes and the base station, and other problems, about 6 percent of the readings were lost. These gaps in the time series were filled by linear interpolation.

Note that sensors s_1 through s_7 are placed at the periphery of the IW. We will use these to monitor and control the temperature inside the IW. Figure 10 indicates the detailed network model, playing the role of the network in Figure 4 for the IW. To identify the unknown parameters in this network model, we consider sensors s_8 and s_9 as output sensors in the first phase of the method presented in section III, while sensors s_1 through s_7 are treated as

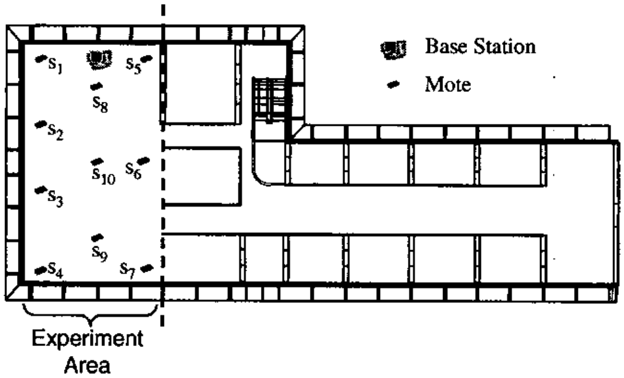

Fig. 9. Floor layout of the IW building and deployment of motes.

the inputs. Location s_{10} is where we want to predict the temperature, and, like in section III, it is not used in this phase of the study. The state and output equations for the model are as follows:

$$\mathbf{x}_{n+1} = A\mathbf{x}_n + B\mathbf{u}_n, \qquad \mathbf{y}_n = C\mathbf{x}_n + \mathbf{w}_n \tag{6}$$

where

$$A = \begin{pmatrix} -\frac{G_1}{C_1} & 0 & 0 \\ 0 & -\frac{G_2}{C_2} & 0 \\ \frac{1}{R_7C_3} & \frac{1}{R_7C_3} & -\frac{G_3}{C_3} \end{pmatrix}, \tag{7}$$

$$G_1 = \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5},$$
$$G_2 = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_4},$$
$$G_3 = \frac{2}{R_1} + \frac{1}{R_6} + \frac{2}{R_7}$$

$$B = \begin{pmatrix} \frac{1}{R_3C_1} & \frac{1}{R_2C_1} & 0 & 0 & \frac{1}{R_4C_1} & \frac{1}{R_5C_1} & 0 \\ 0 & 0 & \frac{1}{R_2C_2} & \frac{1}{R_3C_2} & 0 & \frac{1}{R_5C_2} & \frac{1}{R_4C_2} \\ 0 & \frac{1}{R_1C_3} & \frac{1}{R_1C_3} & 0 & 0 & \frac{1}{R_6C_3} & 0 \end{pmatrix}, \tag{8}$$

$$C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \tag{9}$$

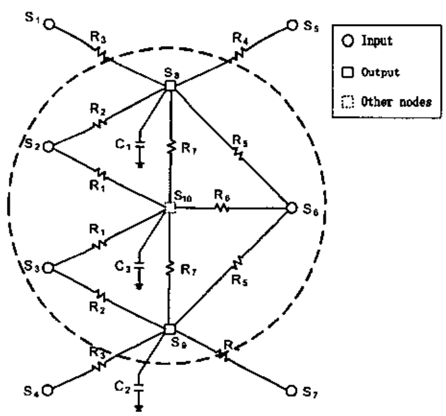

Fig. 10. Detailed RC network thermal model for the IW.

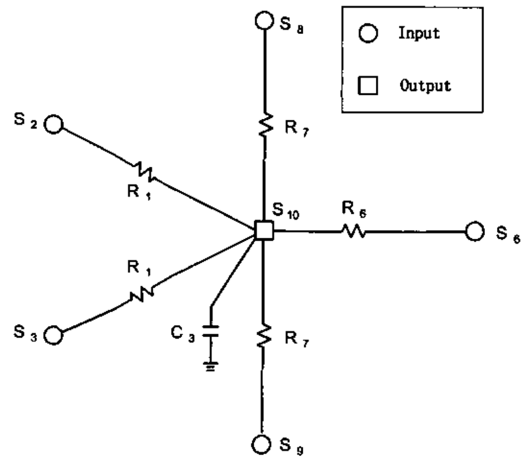

Fig. 11. Reduced RC network thermal model for the IW.

With the network in Figure 10 now identified, we use it to estimate the temperature at location T_{10}. We then derive for this location a reduced model. Again, this reduced model needs to be identified. We proceed as described in section III. First, we identify an appropriate cut set that encircles location s_{10}. This cut set is indicated by the dashed line in Figure 10. The resulting reduced model is indicated in Figure 11. In this reduced model the measurement in sensors s_2, s_3, s_6, s_8, and s_9 act as inputs, and the temperature time series predicted with the detailed model of Figure 10 at s_{10} act as outputs. We use these input and output time series to estimate the parameters of the reduced model of Figure 11.

Figures 12 and 13 compare the actual temperature measurements in sensor s_8 and s_9, with the prediction of these temperatures when we drive the detailed network model of Figure 10, after it has been identified, with the inputs s_1 through s_7. We note that, of the about 4,000 samples available, only the first half are used to identify the network parameters; the second half is used for testing. The plots show very good agreement between the actual and predicted temperatures at both locations s_8 and s_9.

Figure 14 shows similar plots for location s_{10}. We emphasize that the temperature readings by the sensor s_{10} placed at location s_{10} were NOT used to identify either of the models in Figure 10 or Figure 11. The agreement displayed in this Figure between the actual readings and their prediction across the 4000 samples (except for wild variations in the real measurements that are attributed to malfunctioning of the sensors between readings 2600 and 2800) demonstrates the methodology presented in this paper can successfully predict the temperature field at locations other than the ones where we have physically placed our sensors.

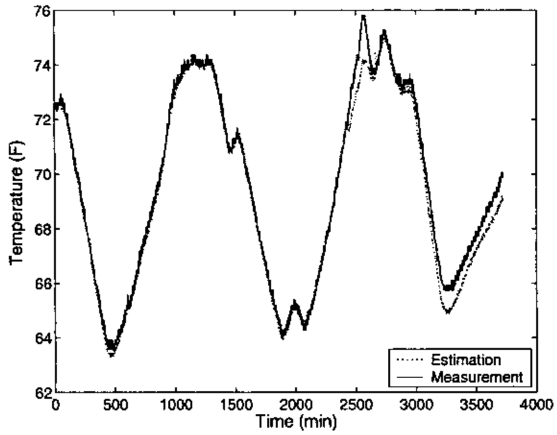

Fig. 12. Actual temperature and estimated temperature at location s_8 for the IW.

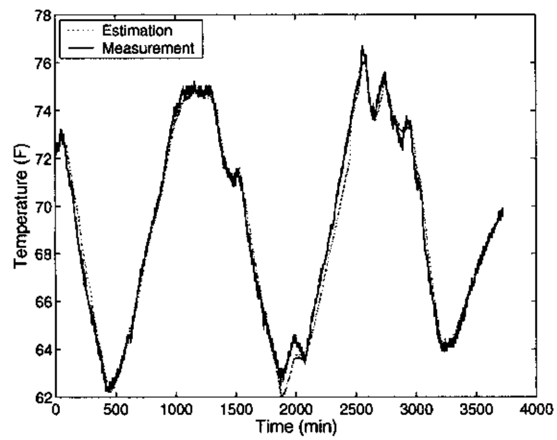

Fig. 13. Actual temperature and estimated temperature at location s_9 for the IW.

V. Conclusions

This paper presents a first step in realizing virtual sensor-actuator arrays in which middleware produces data from physical sensor-actuator arrays that is compatible spatially and temporally with the requirements of the application layer. In particular, we consider the problem of estimating the values of a physically distributed field at points where there are no sensors. Our approach is to construct parameterized models motivated by physical considerations and to use empirical data to estimate the model parameters. For the case of temperature estimation, we demonstrate the method using experimental data collected by a network of wireless sensors in the Intelligent Workplace at Carnegie Mellon University.

Our current research aims to extend the approach proposed in this paper to deal with more complex distributed environments and to consider temporal as well as spatial

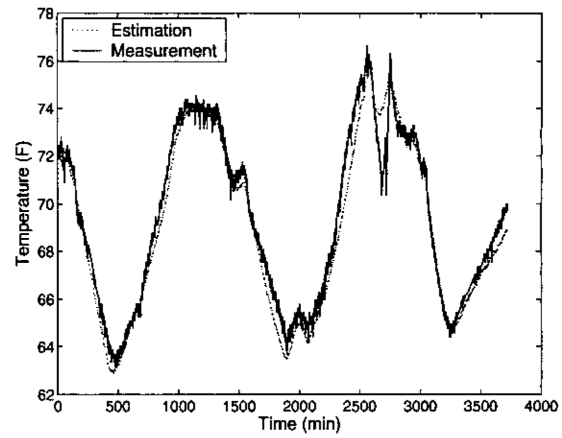

Fig. 14. Actual temperature and estimated temperature at location s_{10} for the IW.

irregularities. Key issues are how to combine the goals of obtaining effective models and effective estimates from the real-time data, and how to incorporate all available information (e.g., time of day and the state of environmental conditions). We are also investigating methods for distributing the computation of the estimates throughout the network.

References

[1] Stephen W. Director, *Circuit Theory - A Computational Approach*, John Wiley & Sons, Inc., 1975.

[2] J. Rabaey, E. Arens, C. Federspiel, A. Gadgil, D. Messerschmitt, W. Nazaroff, K. Pister, S. Oren, and P. Varaiya, "Smart energy distribution and consumption: Information technology as an enabling force," Tech. Rep., Center for Information Technology Research in the Interest of Society, University of California at Berkeley, 2001.

[3] Albert T. P. So, *Perspectives in Control Engineering*, chapter Building control and automation systems, pp. 393–416, IEEE Press, 2001.

[4] J. Kreider, *Handbook of Heating, Ventilation, and Air Conditioning*, Kreider & Associates, 2000.

[5] R. Yao, N. Baker, and M. McEvoy, "A simplified thermal resistance network model for building thermal simulation," in *Canadian Conference on Building Energy Simulation*, Sept 2000.

[6] M. Carmona, S. Marco, J. Palacin, and J. Smitier, "A time-domain method for the analysis of thermal impedance response preserving the convolution form," *IEEE Trans. on Components and Packaging Technology*, vol. 22, no. 2, pp. 238–244, June 1999.

[7] A. Aranyosi, A. Ortega, J. Evans, T. Tarter, J. Pursel, and J. Radhakrishnan, "Development of compact thermal models for advanced electronic packaging: Methodology and experimental validation for a single-chip CPGA package," in *Itherm 2000 Conference*, May 2000, pp. 225–232.

[8] M. Rencz, V. Szkely, Zs. Kohri, and B. Courtois, "A method for thermal model generation of MEMS packages," in *International Conference on Modeling and Simulation of Microsystems*, 2000, pp. 209–212.

[9] B. James Ley, Samuel G. Lutz, and Charles F. Rehberg, *Linear Circuit Analysis*, pp. 12–13, McGRAW-HILL Book Company, Inc., 1959.

[10] Lennart Ljung, *System identification: theory for the user*, Prentice Hall, 1999.

[11] Lennart Ljung, *System Identification Toolbox For Use with MATLAB*, The MathWorks Inc., Jul. 1991.