Linear Estimation of ARMA Processes

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Abstract

In this paper we propose an identification algorithm for ARMA processes. Given a finite lenght sample drawn from an ARMA(p_0, q_0) model, the technique provides the estimated values of the orders p_0 and q_0, as well as the AR and the MA coefficients. They are obtained from the reflection coefficient sequence estimated directly from the data. The order selection scheme is based upon the minimization of a functional that measures the mismatch of the data to any ARMA(p,q) assumed as its model.

Introduction

The estimation of autoregressive moving-average (ARMA) processes has been an area of increasing interest in the last few years. In contrast with the linearity displayed by the AR processes, the ARMA estimation is a nonlinear problem. Several optimization techniques based on the Maximum Likelihood method have been developped for the simultaneous evaluation of the AR and the MA coefficients, [2], [9]. These techniques require a large amount of computation power and are not guaranteed to converge.

Several nonoptimal linear techniques have been presented in a three step sequential procedure based on the Modified Yule-Walker (MYW) algorithm, (see e.g.; [6], [7], [9], [11]). Prior to the AR estimation, obtained as the solution of a system of linear equations, the data is processed by an autocorrelation estimator. On the third step of the algorithm, the MA coefficients are evaluated as a function of the previously estimated AR component.

A common feature of these approaches is that the estimated values of the MA coefficients depend on the AR component estimation. Techniques exploring the opposite dependence are presented in [3] [8], [10], where, implicitly or explicitly, the sequential estimation procedure starts with the MA component.

A dual algorithm for ARMA parameter estimation has been proposed in [12], [13], [14]. The AR and the MA components are obtained in dual, independent procedures, as the solution of two systems of linear equations. These procedures depart from the reflection coefficient sequence estimated from the data using the Burg technique, [5]. This linear ARMA estimation algorithm does not rely on any sample autocorrelation estimator.

In any identification scheme going from data to a model, the structure of the model (e.g., the number of poles and zeros) is to be determined prior or within the estimation procedure. Two standard criterions for model order selection of ARMA processes, the AIC, [1] and the BIC, [2], [16], obtain the number of poles and zeros as the pair (p,q) that minimizes a functional that accounts for the residual power and for the overparametrization.

In this paper we present an alternate order selection scheme for ARMA processes. It is based on the minimization of a functional, d, that measures the mismatch between the data and the ARMA(p,q) that is assumed to model it. For each pair (p,q), d is evaluated from the solution of two systems of linear equations and two sets of linear algebraic relations. Both pairs are constructed from the reflection coefficient sequence estimated directly from the data using the Burg technique, [5]. When the orders have been decided, the algorithm simultaneously provides an estimate of the AR and the MA coefficients.

The paper outline is the following. In section 2 we briefly recall the main points of the dual ARMA estimation algorithm presented in [12], [13], [14] for known orders, $p = p_0$ and $q = q_0$. The order selection algorithm is presented in section 3. For each pair (p,q), we assume an ARMA(p,q) model for the data and implement the set of relations derived in section 2 as if this was the correct model. In section 4 we present simulation results for the two cases

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where the model orders are known and not known a priori. Finally, section 5 concludes the paper.

Estimation Algorithm

Let $\{y_n\}$ be a scalar, Gaussian process satisfying the recursion

$$y_n + \sum_{i=1}^{p_0} a_i y_{n-i} = e_n + \sum_{i=1}^{q_0} b_i e_{n-i}, \qquad (1)$$

where $\{e_n\}$ is a Gaussian, zero mean, white noise with variance σ^2. The polynomials,

$$B(z) = \sum_{i=0}^{q_0} b_i z^{-i}, \quad b_0 = 1 \qquad (2)$$

$$A(z) = \sum_{i=0}^{p_0} a_i z^{-i}, \quad a_0 = 1 \qquad (3)$$

are assumed to be stable and having no common roots.

Throughout this section we will assume that the number of poles, p_0, and the number of zeros, q_0, of the ARMA(p_0, q_0) process are known. Define

$$\mathbf{a} = [a_1 \; a_2 \; \ldots \; a_{p_0}]^T \qquad (4)$$

$$\mathbf{b} = [b_1 \; b_2 \; \ldots \; b_{q_0}]^T \qquad (5)$$

as the AR and the MA components of the ARMA process.

The ARMA estimation algorithm presented in [12] [13], [14] and briefly reviewed on this section obtains the AR and the MA components as the solution of systems of linear equations built from the coefficients of the prediction and the innovation filters associated with the process.

Let $\{v_n\}$ be the innovation sequence associated with the ARMA(p_0, q_0) process,

$$v_n = y_n - E[y_n \mid y_0, \, y_1, \, \ldots, \, y_{n-1}] \qquad (6)$$

and define the coefficients of the prediction error filter of order n, $\{a_i^n, \; 1 \le i \le n\}$ as

$$v_n = \sum_{i=0}^{n} a_i^n y_{n-i}, \quad a_0^n = 1. \qquad (7)$$

From the definition of the innovation sequence, it follows that [4],

$$v_n = y_n - E[y_n \mid v_0, \, v_1, \, \ldots, \, v_{n-1}], \qquad (8)$$

leading to the innovation representation of $\{y_n\}$

$$y_n = \sum_{i=0}^{n} W_i^n v_{n-i}, \quad W_0^n = 1. \qquad (9)$$

The set $\{W_i^n, \; 1 \le i \le n\}$ will be designated as the coefficients of the innovation filter of order n.

Note that $a_n^n = c_n$ is the reflection coefficient of order n associated with $\{y_n\}$. The prediction and the innovation filter coefficients of increasing orders are computed from the sequence of reflection coefficients $\{c_1, c_2, \; \ldots \; c_N\}$. In fact, the lines of the matrix

$$\mathbf{W}_N^{-1} = \begin{bmatrix} 1 & & & & \\ a_1^1 & 1 & & & \\ a_2^2 & a_1^2 & 1 & & \\ & & & \ddots & \\ a_N^N & a_{N-1}^N & \cdots & a_1^N & 1 \end{bmatrix}. \qquad (10)$$

are obtained from $\{c_1, c_2, \ldots, c_N\}$ through the Levinson algorithm and, a recursive inversion of (10) yields, [4]

$$\mathbf{W}_N = \begin{bmatrix} 1 & & & & \\ W_1^1 & 1 & & & \\ W_2^2 & W_1^2 & 1 & & \\ & & & \ddots & \\ W_N^N & W_{N-1}^N & \cdots & W_1^N & 1 \end{bmatrix}. \qquad (11)$$

The following results relate the AR and the MA components of the process with the increasing order innovation and prediction error filter coefficients, assuming the exact knowledge of those coefficients. They are the basis for the dual estimation procedure and for the order selection scheme proposed in section 3.

We assume that,

$$a_j^k = W_j^k = 0, \quad \text{for } j < 0, \text{ or } j > k. \qquad (12)$$

Result 1: For all $k \in [p_0 + q_0, N]$, the AR component satisfy the system of $k - q_0$ linear equations,

$$\begin{bmatrix} W_{k-p_0}^{k-p_0} & \cdots & W_{q_0-p_0+1}^{k-p_0} \\ \vdots & & \vdots \\ W_{k-2}^{k-2} & \cdots & W_{q_0-1}^{k-2} \\ W_{k-1}^{k-1} & \cdots & W_{q_0}^{k-1} \end{bmatrix}^T \cdot \begin{bmatrix} a_{p_0} \\ \vdots \\ a_1 \end{bmatrix} = -\left[W_k^k \; W_{k-1}^k \ldots W_{q_0+1}^k\right]^T. \qquad (13)$$

Proof: in [13], [14].

□

Result 2: For each $k \in [p_0 + q_0, N]$, there exists a vector $\mathbf{\Omega}(k) = [\Omega_1(k), \ldots, \Omega_{q_0}(k)] \in R^{q_0}$ that satisfy the system of $k - p_0$ linear equations,

$$\begin{bmatrix} a_{k-q_0}^{k-q_0} & \cdots & a_{p_0-q_0+1}^{k-q_0} \\ \vdots & & \vdots \\ a_{k-2}^{k-2} & \cdots & a_{p_0-1}^{k-2} \\ a_{k-1}^{k-1} & \cdots & a_{p_0}^{k-1} \end{bmatrix}^T \cdot \begin{bmatrix} \Omega_{q_0}(k) \\ \vdots \\ \Omega_1(k) \end{bmatrix} = -\left[a_k^k \; a_{k-1}^k \; \ldots \; a_{p_0+1}^k\right]^T, \qquad (14)$$

and

$$\lim_{k\to\infty} \mathbf{\Omega}(k) = \mathbf{b} \qquad (15)$$

Proof: in [13], [14].

□

For the sake of compactness, let us rewrite the systems of equations (13) and (14) as

$$\mathbf{M}_{21}^T(k)\mathbf{J}_{p_0}\mathbf{a} = -\mathbf{m}_1(k), \ \forall k, p_0+q_0 \le k \le N \quad (16)$$
$$\mathbf{N}_{21}^T(k)\mathbf{J}_{q_0}\mathbf{\Omega}(k) = -\mathbf{n}_1(k), \quad p_0+q_0 \le k \le N \quad (17)$$

where $\mathbf{J}_k$ is the circular permutation matrix of order k and $\mathbf{M}_{21}$, $\mathbf{m}_1^T$ and $\mathbf{N}_{21}$, $\mathbf{n}_1^T$ are defined according to the Results 1 and 2. Note that these matrices are blocks of $\mathbf{W}_N^{-1}$ and $\mathbf{W}_N$ respectively.

The systems (13) and (14) display a dual and independent behavior of the AR component, $\mathbf{a}$, and of the vector $\mathbf{\Omega}(k)$, that converges to the MA component. However, as the solution in (16) does not depend on k, we may write it as

$$\mathcal{M}^T(N)\mathbf{J}_{p_0}\mathbf{a} = -m(N) \qquad (18)$$

$$\mathcal{M}(N) = [\mathbf{M}_{21}(p_0+q_0) \mid \dots \mid \mathbf{M}_{21}(N)] \qquad (19)$$
$$m^T(N) = \left[\mathbf{m}_1^T(p_0+q_0) \mid \dots \mid \mathbf{m}_1^T(N)\right]^T. \qquad (20)$$

When the prediction and the innovation filter coefficients are exactly known, $\mathbf{a}$ and $\mathbf{\Omega}(k)$ may be obtained as the solution of any set of p_0 and q_0 linear equations arbitrarily chosen among (18) and (17), respectively. However, when those coefficients are estimated from the data, the use of overdetermined systems of equations has the statistical advantages reported in [6].

If p_0 and q_0 are known a priori, the dual estimation algorithm is based on the Results 1 and 2, or equivalently on (18) and (17), with the matrices $\mathcal{M}^T, m, \mathbf{M}_{21}^T$ and $\mathbf{m}_1$ replaced by suitable estimates. When p_0 and q_0 are not known, the order selection scheme uses the set of coupled algebraic relations expressed in Result 3.

Result 3: For each value of $k \in [p_0+q_0, N]$, the vectors $\mathbf{\Omega}(k)$ and $\mathbf{a}$ satisfy the linear algebraic relations,

$$\begin{bmatrix} W_{q_0-p_0}^{k-p_0} & & & \\ \vdots & \vdots & \ddots & \mathbf{0} \\ W_{q_0-2}^{k-2} & \dots & 1 & \\ W_{q_0-1}^{k-1} & \dots & W_1^{k-1} & 1 \end{bmatrix}^T \cdot \begin{bmatrix} a_{p_0} \\ \vdots \\ a_2 \\ a_1 \end{bmatrix} + \left[W_{q_0}^k \dots W_2^k\, W_1^k\right]^T = [\Omega_{q_0}(k) \dots \Omega_1(k)]^T, \qquad (21)$$
$$p_0+q_0 \le k \le N,$$

and

$$\begin{bmatrix} a_{p_0-q_0}^{k-q_0-1} & & \\ \vdots & \ddots & \mathbf{0} \\ a_{p_0-2}^{k-2} & \dots & 1 \\ a_{p_0-1}^{k-1} & \dots\ a_1^{k-1} & 1 \end{bmatrix}^T \cdot \begin{bmatrix} \Omega_{q_0}(k) \\ \vdots \\ \Omega_2(k) \\ \Omega_1(k) \end{bmatrix} + \left[a_{p_0}^k \dots a_2^k\, a_1^k\right]^T = [a_{p_0} \dots a_2\, a_1]^T, \qquad (22)$$
$$p_0+q_0 \le k \le N.$$

Proof: in [13], [14].

□

Again we will represent (21) and (22) using a compact matrix notation,

$$\mathbf{M}_{22}^T(k)\mathbf{J}_{p_0}\mathbf{a} + \mathbf{m}_2(k) = \mathbf{J}_{q_0}\mathbf{\Omega}(k), \qquad (23)$$
$$\mathbf{N}_{22}^T(k)\mathbf{J}_{q_0}\mathbf{\Omega}(k) + \mathbf{n}_2(k) = \mathbf{J}_{p_0}\mathbf{a}, \qquad (24)$$
$$p_0+q_0 \le k \le N$$

where $\mathbf{M}_{22}, \mathbf{m}_2^T$ and $\mathbf{N}_{22}, \mathbf{n}_2^T$ are block matrices of $\mathbf{W}_N^{-1}$ and $\mathbf{W}_N$ defined according to (21) and (22).

Order Selection

In this section, we propose an order estimation algorithm for ARMA processes. Given a finite sample drawn from an ARMA(p_0, q_0) model, this method obtains the estimated values, $\hat{p}_0$ and $\hat{q}_0$, from the reflection coefficient sequence estimated from the data. When the orders are decided the algorithm simultaneously provides the AR and the MA component estimates.

For each pair $(p,q) \in N \times N$, we assume an ARMA(p,q) model for the data and test the validity of this hypothesis. The validation relies on a functional that provides a measure of the mismatch of the assumed model to the data. The pair $(\hat{p}_0, \hat{q}_0)$ minimizes this functional.

Through the section we will consider the exact knowledge of a second order characterization of $\{y_n\}$ given by the sequence of reflection coefficients, $\{c_1, c_2, \dots, c_N\}$. We will discuss how to relax this assumption in section 4 in the context of several simulation examples.

For each pair (p,q) and value of N, the functional $d(N,p,q)$ is introduced by Definitions 1 to 5. Let

$$\mathbf{N}_{21}(k,p,q), \mathbf{N}_{22}(k,p,q), \mathbf{n}_1(k,p,q), \mathbf{n}_2(k,p,q)$$
$$\mathbf{M}_{21}(k,p,q), \mathcal{M}(N,p,q), \mathbf{M}_{22}(k,p,q), \mathbf{m}_1(k,p,q),$$
$$m(N,p,q), \mathbf{m}_2(k,p,q)$$

be defined for the pair (p,q) as the corresponding matrices for (p_0, q_0) (see section 2).

Definition 1 ${}^1\delta(k,p,q) \in R^q$ is the minimum norm vector x that minimizes,

$$\| \mathbf{N}_{21}^T(k,p,q)\mathbf{J}_q\mathbf{x} + \mathbf{n}_1(k,p,q) \|_2, \quad p+q \leq k \leq N \tag{25}$$

□

Definition 2 ${}^1\gamma(N,p,q) \in R^p$ is the minimum norm vector y that minimizes,

$$\| \mathcal{M}^T(N,p,q)\mathbf{J}_p\mathbf{y} + m(N,p,q) \|_2, N \geq p+q \tag{26}$$

□

For the model ARMA(p,q), (25) and (26) coincide with (17) and (18) if this were the correct model. In fact,

$$\begin{aligned} {}^1\delta(k,p_0,q_0) &= \Omega(k), p_0+q_0 \leq k \leq N \quad (27)\\ {}^1\gamma(N,p_0,q_0) &= \mathbf{a}, N \geq p_0+q_0. \quad (28) \end{aligned}$$

In the general case, the minimum errors associated with the Definitions 1 and 2 are not zero. We will represent them as,

$${}^{2.k}\mathbf{e}_{AR}(k,p,q) = \mathbf{N}_{21}^T(k,p,q)\mathbf{J}_q\,{}^1\delta(k,p,q) + \mathbf{n}_1(k,p,q) \tag{29}$$

and

$$\mathbf{e}_{MA}(N,p,q) = \left[{}^{2.p+q}\mathbf{e}_{MA}^T(N,p,q) \mid \ldots \mid {}^{2.N}\,\mathbf{e}_{MA}^T(N,p,q)\right]^T$$

where,

$${}^{2.k}\mathbf{e}_{MA}(N,p,q) = \mathbf{M}_{21}^T(k,p,q)\mathbf{J}_p\,{}^1\gamma(N,p,q) + \mathbf{m}_1(k,p,q) \tag{30}$$

due to a block partition of $\mathcal{M}(N,p,q)$ and $m(N,p,q)$ similar to (19)-(20). For the correct model, these errors are zero, i.e.,

$$\begin{aligned} {}^{2.k}\mathbf{e}_{AR}(k,p_0,q_0) &= \mathbf{0}, \; p_0+q_0 \leq k \leq N \quad (31)\\ {}^{2.k}\mathbf{e}_{MA}(N,p_0,q_0) &= \mathbf{0}, \quad N \geq p_0+q_0. \quad (32) \end{aligned}$$

For the ARMA(p,q) model and the corresponding vectors ${}^1\delta$ and ${}^1\gamma$, the next definitions establish two sets of linear algebraic relations that coincide with (22) and (21) if the assumed model was the correct one.

Definition 3: ${}^{2.k}\gamma(k,p,q) \in R^p$ is given by

$$\begin{aligned} \mathbf{J}_p\,{}^{2.k}\gamma(k,p,q) = \;& \mathbf{N}_{22}^T(k,p,q)\mathbf{J}_q\,{}^1\delta(k,p,q) + \\ & \mathbf{n}_2(k,p,q), \quad p+q \leq k \leq N. \quad (33) \end{aligned}$$

□

Definition 4: ${}^{2.k}\delta(N,p,q) \in R^q$ is given by,

$$\begin{aligned} \mathbf{J}_q\,{}^{2.k}\delta(N,p,q) = \;& \mathbf{M}_{22}^T(k,p,q)\mathbf{J}_p\,{}^1\gamma(N,p,q) + \\ & + \mathbf{m}_2(k,p,q) \quad p+q \leq k \leq N. \quad (34) \end{aligned}$$

□

For $p = p_0$ and $q = q_0$, replacing (27) and (28 in (33) and (34), and comparing with the linear algebraic relations obtained for the correct model in section 2, yields,

$$\begin{aligned} {}^{2.k}\gamma(k,p_0,q_0) &= \mathbf{a}, \quad (35)\\ {}^{2.k}\delta(N,p_0,q_0) &= \Omega(k), \quad (36)\\ & p_0+q_0 \leq k \leq N. \end{aligned}$$

We finally define the functional $d(N,p,q)$.

Definition 5: The functional $d(N,p,q)$ is given by

$$d(N,p,q) = d_{AR}(N,p,q) + d_{MA}(N,p,q) \tag{37}$$

with,

$$\begin{aligned} d_{AR}(N,p,q) &= \textstyle\sum_{k=p+q}^{N} \|{}^1\,\gamma(N,p,q) - {}^{2.k}\,\gamma(k,p,q)\|_2^2 + \\ & + \|{}^{2.k}\,\mathbf{e}_{AR}(k,p,q)\|_2^2, \quad (38) \end{aligned}$$

$$\begin{aligned} d_{MA}(N,p,q) &= \textstyle\sum_{k=p+q}^{N} \|{}^1\,\delta(k,p,q) - {}^{2.k}\,\delta(N,p,q)\|_2^2 + \\ & + \|{}^{2.k}\,\mathbf{e}_{MA}(N,p,q)\|_2^2\,. \quad (39) \end{aligned}$$

□

The order selection scheme is based upon the values of $d(N,p,q)$ for each pair (p,q) and increasing values of N. The following properties, proved in [14], show what some of those values are.

Properties

P1. $d(N,p,q) \neq 0$ for $p < p_0,\; q < q_0,\; N \geq p_0+q_0$.
P2. $d(N,p,q) \neq 0$ for $p \geq p_0,\; q < q_0,\; N \geq p+q_0$.
P3. $d(N,p,q) \neq 0$ for $p < p_0,\; q \geq q_0,\; N \geq p_0+q$.
P4. $d(N,p,q_0) = 0$ for $p \geq p_0,\; N \geq p+q_0$.
P5. $d(N,p_0,q) = 0$ for $q \geq q_0,\; N \geq p_0+q$.

The values of the functional d referred in P1-P5 are represented in Fig. 1 for a fixed value of N ($N \geq p_0+q_0$) and for all the pairs (p,q) with $p+q \leq N$. The symbol //// accounts for a nonnull functional, whereas **0** represent the locus of $d=0$.

On Fig. 1 we identify an orthogonal pattern $\mathcal{L}(p,q)$ with $d=0$,

$$\mathcal{L}(p,q) = \{(p,q) : (p = p_0, q \geq q_0) \vee (p \geq p_0, q = q_0)\} \tag{40}$$

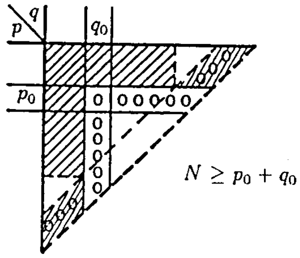

Figure 1: Locus considered in P1-P5

The following result gives close expressions for the six vectors in (38) and (39) for $(p,q) \in \mathcal{L}(p,q)$.

Result 4: For $p \geq p_0$ and $q = q_0$,

$$^{1}\gamma(N,p,q_0) = {}^{2.k}\gamma(k,p,q_0) = \begin{bmatrix} \mathbf{a} \\ \hline \mathbf{0}_{p-p_0} \end{bmatrix}, \quad (41)$$

$$^{1}\delta(k,p,q_0) = {}^{2.k}\delta(k,p,q_0) = \mathbf{\Omega}(k), \quad (42)$$

$$^{2.k}\mathbf{e}_{AR}(k,p,q_0) = \mathbf{0}, \quad (43)$$

$$^{2.k}\mathbf{e}_{MA}(N,p,q_0) = \mathbf{0}, \quad (44)$$

$$p + q_0 \leq k \leq N.$$

Proof: in [14].

□

From Result 4, we conclude that for $p \geq p_0$, each assumed ARMA(p,q_0) model has p_0 poles at the same locations as the poles of (1), the remaining $p - p_0$ poles being at the origin. From (42) the two models have the same zeros. A dual conclusion holds for $p = p_0$ and $q \geq q_0$. Thus, all the models corresponding to pairs (p,q) at the orthogonal pattern in Fig. 1 have the same spectrum. Among them, the order selection algorithm chooses the one with the smallest number of parameters.

Selection

Define

$$\mathcal{I}(N,p,q) = \{(p,q) : d(N,p,q) = 0 \vee N < p+q\}. \quad (45)$$

For an interpretation of this set, see [14] or [15].

From P1 to P5, the orthogonal pattern $\mathcal{L}(p,q)$ is given by

$$\mathcal{L}(p,q) = \bigcap_{N \geq 1} \mathcal{I}(N,p,q), \quad (46)$$

where $\bigcap$ stands for set intersection.

The selection scheme chooses (p_0, q_0) as,

$$(p_0,q_0) = \arg \min_{(p,q)\in\mathcal{L}(p,q)} \{p+q\}, \quad (47)$$

When the order is decided, the algorithm simultaneously provides the corresponding AR (vector $^{1}\gamma$) and MA (vector $^{1}\delta$) components.

Simulation Results

In this section we present simulated results concerning both the order selection scheme and the dual ARMA estimation algorithm.

The examples displayed refer to two distinct ARMA(4,2) processes, with pole zero locations shown in Table 1 and $\sigma^2 = 1$. Case 1 was presented in [10] for the performace evaluation of an ARMA estimator considering known orders.

ARMA(4,2)	Zeros	Poles
Case 1	$0.7exp\{\pm j0.5\pi\}$	$0.9exp\{\pm j0.3\pi\}$
Case 2	$0.9exp\{\pm j0.5\pi\}$	$0.95exp\{\pm j0.7\pi\}$

Table 1

For the processes in Table 1, we perform 100 Monte-Carlo independent experiments, generating a sample function of lenght T for each run. The order estimation was implemented based on (45)-(47) with the zero in (45) replaced by a small constant, $\varepsilon = 0.05$ and using the first 15 reflection coefficients estimated from the data.

For three values of T, the Table 2 represents the number of correct ($\hat{p}_0 = p_0$, $\hat{q}_0 = q_0$) estimates over the 100 independent runs, showing the influence of the zero location on the performance of the order selection algorithm.

Number of correct estimates	T=500	T=1000	T=1500
Case 1	30	82	96
Case 2	22	77	94

Table 2

In Fig.2, we represent the estimated pole zero location for Case 2, T=500, and 10 runs for which the selection scheme chooses the corrrect orders.

In Fig.3, we represent the estimated pole zero locations for Case 2, T=500 and the 6 runs when an ARMA(4,3) model was chosen by the order selection algorithm, i.e., an extra zero was estimated.

From Fig.3, we conclude that the extra zero is close to the origin, as stated in Section 3.

Additional order selection simulation results for processes with a smaller number of poles and zeros are presented in [14] and [15].

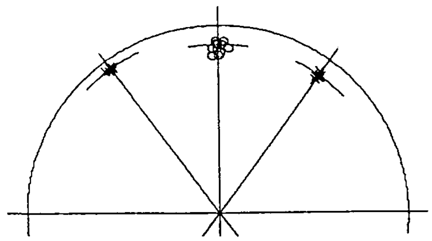

Figure 2: Estimated pole zero location for correct order selection. Case 2, T=500

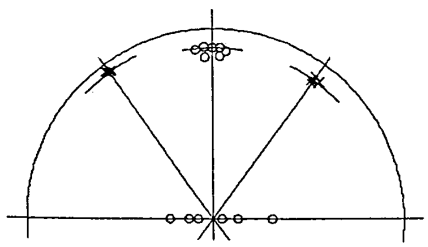

Figure 3: Estimated pole zero location for $\hat{p}_0 = 4$, $\hat{q}_0 = 3$. Case 2, T=500.

Conclusions

This paper presents an identification method for ARMA processes. The number of poles and zeros and the AR and the MA components are estimated from data through a **linear** algorithm that **dualizes** the roles of both components. The estimation procedures are easy to implement and have good performance.

The full development of the algorithm, its statistical analysis and extended examples are presented in [14]. Additional results on the order selection algorithm will be presented elsewhere.

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