IEEE 39th Asilomar Conference on Signals, Systems, and Computers,
Page(s):665 – 669, Pacific Grove, CA, October 30 – November 2, 2005.

Waveform Shaping For Time Reversal Interference Cancellation: A Time Domain Approach

José M.F. Moura, Yuanwei Jin, Jian-Gang Zhu, Yi Jiang, Dan Stancil, Ahmet Cepni and Ben Henty
Department of Electrical and Computer Engineering
Carnegie Mellon University
Pittsburgh, PA 15213 *

Abstract

In this paper, we present a time domain interference cancellation scheme using time reversal. In time reversal, a signal, for example, transmitted by a weak source through a dispersive medium, is received by an array, time reversed, power normalized, and retransmitted through the same medium. If the channel is reciprocal, and the clutter is sufficiently rich, the retransmitted waveform refocuses on the original source. This focusing effect implies that a significant portion of the received signal energy concentrates in a very small time/space support compared with the rest of the signal components. In this paper, we consider the opposite problem: rather than refocusing, we reshape the retransmitted waveform to reduce the returned echo, i.e., to mitigate the clutter returns. Proof of concept is provided using experimental electromagnetic data and wave propagation simulations.

1 Introduction

The problem of designing clutter resistant radar waveforms has been studied extensively (see, e.g., [1, 2] and the references therein). The general problem considered in these references is that of designing radar waveforms and receivers that are optimum for detecting a target masked by a background of clutter returns and thermal noise. In this paper, we develop a clutter mitigation scheme using time reversal.

Time reversal is an emerging technique that utilizes to advantage a rich scattering environment to detect weak targets buried in the clutter. In time reversal, a wideband signal is: (1) transmitted by a weak source through a dispersive medium; (2) received by an array, recorded, time reversed, power normalized; and (3) retransmitted back to the same medium. If the channel is reciprocal, and the clutter is sufficiently rich, the retransmitted waveform refocuses on the original source. This distinct feature of time reversal is called focusing [3, 4]. For interference cancellation, we consider the opposite problem; rather than focusing, we use time reversal to mitigate the returns from the clutter by reshaping the retransmitted waveform. We have explored this viewpoint for interference cancellation in the frequency domain, [5, 6]. In this work, a stepped frequency wideband signal illuminates the scattering field, and the recorded clutter returns are time reversed and passed through a whitening filter before retransmission. The whitened clutter are then "subtracted" out. In this paper, we propose a time reversal based waveform reshaping scheme in the time domain.

The basic idea and procedure can be illustrated as follows: when *focusing* with time reversal, a wideband signal $s_1(t)$ transmitted by a source is received after propagation through a dispersive clutter channel with impulse response $c(t)$. The received signal, excluding noise for the moment, is $r(t) \quad c(t) \star s_1(t)$, where $\star$ stands for convolution. Let $_{cc}(t) \quad c(T-t) * c(t)$ be the autocorrelation function of the clutter returns and $r(T-t)$ be the time-reversed signal, where T is the time frame length and is sufficiently long so that $r(T-t)$ contains all the significant multipath components. Assuming a reciprocal medium, the echo scattered from the retransmitted time reversal signal is

$$s_1(T-t) \star c(T-t) \star c(t) \quad s_1(T-t) \star \;_{cc}(t). \quad (1)$$

This signal is symmetric if $s_1(t)$ is symmetric, and a large portion of its energy concentrates in a very short time support in the center of the time window due to clutter focusing. Rather than focusing, our goal is to null the clutter returns. To achieve this, we reshape the retransmitted waveform to force the received signal to reduce to a pre-determined signal level. Unlike focusing, where a time-reversed signal $r(T-t)$ is re-transmitted directly, we reshape the waveform $r(T-t)$ prior to retransmission by multiplying $r(T-t)$ with a reshaping signal $g(t)$. This signal $g(t)$ is designed so that after retransmitting the reshaped waveform $g(t)r(T-t)$, the returned signal at the original source position is nulled, or cancelled, rather than focused. This is useful when detecting weak sources in highly cluttered environments.

The remainging of the paper is organized as follows. Section 2 provides the signal model and nulling strategy. Section 3 describes the experimental setup. Section 4 exam-

*This work is funded by the Defence Advanced Research Projects Agency through the Army Research Office under grant no. W911NF-04-1-0031

ines the performance of the proposed algorithm using experimental data and electromagnetic wave simulation. We show that the nulling scheme herein proposed significantly reduces the returned energy from the clutter, and time reversal provides a target return with higher energy than conventional detection. We leave the noise analysis of clutter nulling and target detection for future research.

2 Signal Model and the Nulling Scheme

For simplicity, we assume that the transmitted signal $s_1(t) \quad \delta(t)$ is a Dirac function, and noise is ignored for the moment. The shaping filter $g(t)$ will be taken to be a uniform burst of waveforms with arbitrary amplitudes and phases on the subpulses. The reshaping pulse train $g(t)$ is defined by the equation

$$g(t) \quad \sum_{i=0}^{\frac{K}{L}-1} u_i p(t - i \quad), \tag{2}$$

where: u_i is a complex number that gives the amplitude and phase of the ith subpulse; $p(t)$ is the subpulse shape; denotes the interpulse spacing and also the period of $p(t)$; $K \quad T/T_s$ and $L \quad /T_s$, where $/T_s$ is the sampling rate; and T is the total time duration of the returned signal. For simplicity, $p(t)$ is chosen to be a square wave as follows:

$$p(t) \quad \begin{cases} + \ , & \leq t \leq \frac{\Delta}{2} \\ - \ , & \frac{\Delta}{2} < t \leq \quad , \\ \ , & \text{otherwise.} \end{cases} \tag{3}$$

Note that we choose to be the time support of the original waveform so that there is no bandwidth expansion in the processing. Let

$$\mathbf{u} \quad u_0, \cdots, u_{\frac{K}{L}-1} \ ^T \tag{4}$$

be the weight vector; Our goal is to design the weight vector $\mathbf{u}$ by forcing the target return echo to have a pre-specified shape $\mathbf{y}_0$ that is defined in (14) and (16) below. We will show later that the optimal $\mathbf{u}^*$ is

$$\mathbf{u}^* \quad \arg \ _{\mathbf{u}} \ \|\mathbf{C}_p\mathbf{u} - \mathbf{y}_0\|^2, \tag{5}$$

where the system matrix $\mathbf{C}_p$ is defined in (13).

The reshaped time-reversed waveform $x(t)$ is, for $\leq t \leq T$,

$$\begin{aligned} x(t) \quad & c(T-t)g(t), \\ & \textstyle\sum_{i=0}^{\frac{K}{L}-1} u_i p(t - i \quad)c(T-t), \\ & \textstyle\sum_{i=0}^{\frac{K}{L}-1} u_i p(t - iLT_s)c(KT_s - t). \end{aligned} \tag{6}$$

A discrete time representation of Eqn. (6) is

$$x \ nT_s \quad \sum_{i=0}^{\frac{K}{L}-1} u_i p \ nT_s - iLT_s \ c \ KT_s - nT_s \ , \tag{7}$$

where $t \quad nT_s, n \quad , \cdots, K$. Equivalently, dropping T_s,

$$x \ n \quad \sum_{i=0}^{\frac{K}{L}-1} u_i p \ n - iL \ c \ K - n \ . \tag{8}$$

The received echo is obtained by convolving $x(t)$ with the clutter channel response $c(t)$, i.e., $y(t) \quad x(t) \star c(t)$. Written in discrete form,

$$y \ k \quad \sum_{m=0}^{2K} x \ m \ c \ k - m \quad \sum_{i=0}^{\frac{K}{L}-1} u_i c_p \ k, i \ , \tag{9}$$

where

$$c_p \ k, i \quad \sum_{m=0}^{2K} p \ m - iL \ c \ K - m \ c \ k - m \ . \tag{10}$$

Note that $p \ m$ is only non-zero for $m \quad , \cdots, L$. We write Eqn. (9) in a compact form. Let

$$\mathbf{y} \quad y \quad , \cdots, y \ k \ , \cdots, y \quad K \ ^T. \tag{11}$$

Then Eqn. (9) leads to

$$\mathbf{C}_p\mathbf{u} \quad \mathbf{y}, \tag{12}$$

where

$$\mathbf{C}_p \quad \begin{bmatrix} c_p \ , & c_p \ , & \cdots & c_p \ , \frac{K}{L} - \\ \vdots & \vdots & \vdots & \vdots \\ c_p \ k, & c_p \ k, & \cdots & c_p \ k, \frac{K}{L} - \\ \vdots & \vdots & \vdots & \vdots \\ c_p \quad K, & c_p \quad K, & \cdots & c_p \quad K, \frac{K}{L} - \end{bmatrix}. \tag{13}$$

For $iL \leq k \leq (i + \quad)L$, we have

$$c_p \ k, i \quad \begin{bmatrix} p \\ \vdots \\ \vdots \\ p \ L \end{bmatrix}^T \begin{bmatrix} c \ K - iL \ c \ k - iL \\ \vdots \\ c \ K - k \ c \\ \left\{ \vdots \right\} (i + \quad)L - k \end{bmatrix}$$

for $(i + \quad)L + \quad \leq k \leq K + iL$, we have

$$c_p \ k, i \quad \begin{bmatrix} p \\ \vdots \\ \vdots \\ p \ L \end{bmatrix}^T \begin{bmatrix} c \ K - iL \ c \ k - iL \\ \vdots \\ c \ K - (i + \quad)L \ c \ k - (i + \quad)L \end{bmatrix}$$

and for $K + iL + \quad \leq k \leq K + (i + \quad)L$, we have

$$c_p \; k, i \quad \begin{bmatrix} p \\ \vdots \\ \vdots \\ p \; L \end{bmatrix}^T \begin{bmatrix} \left\{ \begin{matrix} \vdots \end{matrix} \right\} k - K - iL \\ c \quad K - k \; c \; K \\ \vdots \\ c \; K - (i + \quad)L \; c \; k - (i + \quad)L \end{bmatrix} .$$

Let $\mathbf{y}_0$ be a reference vector. This $\mathbf{y}_0$ is used to calculate the weight vector $\mathbf{u}$, where

$$\mathbf{y}_0 \qquad y_0 \quad , \cdots, y_0 \; k \;, \cdots, y_0 \quad K \quad ^T. \tag{14}$$

Recall from (1) that $\quad _{cc}(t)$, the autocorrelation of the clutter returns, is the time reversal clutter response when the transmitted signal is a delta function. A discrete representation of $\quad _{cc}(t)$ is

$$\overline{\psi} \qquad _{cc} \quad , \cdots, \quad _{cc} \quad K \quad , \tag{15}$$

with a peak value at $\quad _{cc} \; K$. Since our goal is to null the clutter return, we choose the elements of $\mathbf{y}_0$ to be

$$y_0 \; k \qquad \begin{cases} \mathrm{sgn}(\quad _{cc} \; k \;)\alpha, & \text{if } | \quad _{cc} \; k \; | > \alpha \\ \quad _{cc} \; k \; , & \text{if elsewhere,} \end{cases} \tag{16}$$

where $\alpha > \quad$ is a predefined threshold, $\mathrm{sgn}(z)$ retains the sign of the scalar z. Therefore, the estimate of $\mathbf{u}$ can be obtained by replacing $\mathbf{y}$ with $\mathbf{y}_0$ as follows,

$$\mathbf{C}_p \widehat{\mathbf{u}} \quad \mathbf{y}_0. \tag{17}$$

$\mathbf{C}_p$ is a rank deficient matrix, we use

$$\widehat{\mathbf{u}} \quad (\mathbf{C}_p^T \mathbf{C}_p + \beta \mathbf{I})^{-1} \mathbf{C}_p^T \mathbf{y}_0, \tag{18}$$

as the estimate of $\widehat{\mathbf{u}}$, and β is a regularization parameter. Hence, the resulting returned waveform is given by

$$\widehat{\mathbf{y}}_0 \quad \mathbf{C}_p \widehat{\mathbf{u}}. \tag{19}$$

To evaluate the performance of the estimate, we define $\quad \|\mathbf{y}_0 - \widehat{\mathbf{y}}_0\|$ as the performance metric.

2.1 Time reversal detection

We describe in this section a general iterative transmission strategy of time reversal clutter nulling and target detection. We introduce the two channel responses: the clutter response $c(t)$ is the channel response from the scatterers only; the target channel response $h(t)$ is the difference between the channel response when a target is present and the channel response when a target is not present. In general $h(t)$ contains secondary scattering, i.e., returns from the target due to scattering from the surrounding scatterers. The transmission strategy is summarized in the following four steps: Steps 1-2 are for clutter nulling, un-doing the clutter focusing by reshaping the signal as explained above; Steps 3-4 are for target detection by using time reversal.

1—clutter probing: we illuminate the scattering medium with a wideband signal $s_1(t)$. The received signal, excluding noise, is

$$r_1(t) \quad s_1(t) \star c(t). \tag{20}$$

From (20), we can recover the clutter channel response $c(t)$.

2—time reversal nulling: the received signal $r_1(t)$ is time reversed, reshaped by $g(t)$ and power normalized. The retransmitted waveform takes the form:

$$s_2(t) \quad k_1 r_1(T_1 - t) g(t), \tag{21}$$

where k_1 is the normalization factor and T_1 is the length of the time frame. We expect that this step yields a significant reduction in returned clutter energy.

3—target monitoring: the received echo is recorded as

$$s_2(t) \star (c(t) + h(t)), \tag{22}$$

where $h(t)$ is the effective target response. Because $c(t)$ is known, the clutter response can be subtracted out, which leads to

$$r_2(t) \quad s_2(t) \star h(t). \tag{23}$$

The resulting signal is time-reversed again for retransmission, i.e.,

$$s_3(t) \quad k_2 r_2(T_2 - t), \tag{24}$$

where k_2 is the normalization factor, and T_2 is the length of the time frame.

4—time reversal target focusing: finally, the signal $s_3(t)$ is retransmitted through the medium. The received signal is

$$s_3(t) \star (c(t) + h(t)). \tag{25}$$

Again, the clutter response is subtracted out, and the resulting signal becomes

$$r_3(t) \qquad s_3(t) \star h(t) \tag{26}$$

$$\qquad k_2 s_2(T_2 - t) \star \; h(T_2 - t) \star h(t) \; . \tag{27}$$

The total energy of the received signal is:

$$E_{\mathrm{TR}} \quad \int |r_3(t)|^2 dt. \tag{28}$$

Direct subtraction, also called change detection (CD), is used for benchmarking purposes. For change detection, we have Steps 1 and 3 only. From Step 3, the clutter is subtracted out directly; we obtain

$$f(t) \quad s_1(t) \star h(t), \tag{29}$$

which is the difference between the received signal with a target and the signal without a target. The residue signal energy is:

$$E_{\text{CD}} \quad \int |f(t)|^2 dt. \tag{30}$$

We then define the gain of the time reversal nulling scheme over change detection by

$$\text{Gain} \quad * \log_{10} \left(\frac{E_{\text{TR}}}{E_{\text{CD}}} \right). \tag{31}$$

3 Experimental Setup and Measurements

A set of channel responses with and without a target is measured and recorded in a highly cluttered laboratory environment. We use a pair of horn antennas with operational bandwidth of GHz. A total of Q frequency samples across $-$ GHz are collected. The center frequency is GHz (or wavelength λ_c cm). The scattering environment is created by placing dielectric pipes with outer diameters . cm in a wood platform in front of an absorber wall. A wideband signal of $-$ GHz is generated by an Agilent network analyzer. The in-phase (I channel) and quadrature phase (Q channel) data streams are recorded. We calculate the real valued time domain waveform by the inverse Fourier transform of the frequency samples.

We demonstrate the impact of time reversal with scatterers and the target hiding among them. A bi-static array configuration is chosen with a pair of transmit/receive antennas. The coordinates of antenna A and antenna B are $(x \quad , y \quad + \quad . \quad \text{cm})$ and $(x \quad , y \quad - \quad . \quad \text{cm})$, respectively, where x is the range and y is the cross range.

4 Results

Next, we examine the proposed time reversal nulling and detection scheme using measured experimental electromagnetic data and wave propagation simulations.

First we examine the clutter nulling algorithm using measured data. Fig. 1 depicts the returned echo from the clutter before and after the nulling algorithm is applied. The top plot shows a focused sharp pulse that represents the returned clutter waveform $\overline{\psi}$ using time reversal without waveform reshaping; the middle plot depicts the returned reference waveform $\mathbf{y}_0$ by placing a threshold, a pre-determined value α that is chosen to be $-$ dB down relative to the maximum of $\overline{\psi}$, on the focused waveform $\overline{\psi}$; the bottom plot depicts the nulled waveform after the nulling algorithm is applied. This plot is the resulting clutter waveform $\mathbf{y}_0$ whose $\mathbf{u}$ is calculated in Eqn. (9). We observe a significant signal echo reduction between the focused waveform and the nulled, returned echo. Furthermore, there is a distortion between the middle plot and the bottom plot. This is due to the ill conditioned matrix $\mathbf{C}_p$ in (18).

After the clutter is nulled, we follow the transmission strategy described in section 2.1 for target detection, where the target is masked by the scatterers. Compared with conventional change detection (CD), time reversal yields an energy gain $\frac{E_{\text{TR}}}{E_{\text{CD}}} \quad .$ dB.

Next, we provide physical insight on the proposed time domain nulling scheme. We use the Finite Difference Time Domain (FDTD) method to mimic the experimental setup and to simulate the radiated energy pattern distribution of the electromagnetic scattered field. FDTD is a well known electromagnetic modelling technique for solving Maxwel's equations given boundary conditions and initial conditions. Simulation parameters are described as follows: A free space region of m in cross range and m in range is chosen. An cm thick perfect matched layer is assumed for all four boundaries of the simulation region. Again, scatterers, assumed to be perfect conductors, are placed inside the simulation region. A Gaussian pulse $s_1(t)$ of width ns with GHz bandwidth is generated at baseband and mixed up to GHz frequency. A pair of omni-directional antennas A and B, which can both switch between transmit mode and receive mode, are placed . cm apart, which corresponds to a little over wavelength at GHz.

Fig. 2 depicts the simulated energy pattern in the scattering field before and after the nulling algorithm is applied. The antennas A and B are on the left edge of the region. The brighter the color, the greater the clutter electrical field energy level. The top figure of Fig. 2 shows that there is a large amount of reflected wave energy from the scatterers coming back towards the receive antenna (brighter half on the left of the figure). This is the pre-nulling energy pattern. Applying the proposed nulling scheme to the clutter, we observe, in the bottom figure of Fig. 2, a significant reduction of the energy distribution in the field (dimmer half on the left of the figure). This is the result of the Steps 1-2 in section 2.1.

Next, for target detection, we follow Steps 3-4. Fig. 3 depicts the returned waveforms from the target. The upper plot shows a strong target returned echo $r_3(t)$ when the clutter nulling and target focusing scheme is employed. The lower plot shows the returned echo $f(t)$ using the conventional detection (direct subtraction) scheme. The energy gain of time reversal over conventional detection is $\frac{E_{\text{TR}}}{E_{\text{CD}}} \quad .$ dB.

References

[1] M. R. Bell, "Information theory and radar waveform design," *IEEE Transactions on Information Theory*, vol. 39, no. 5, pp. 1578–1597, September 1993.

[2] D. A. Garren, A. C. Odom, M. K. Osborn, J. S. Goldstein, S. U. Pillai, and J. R. Guerci, "Ehanced target

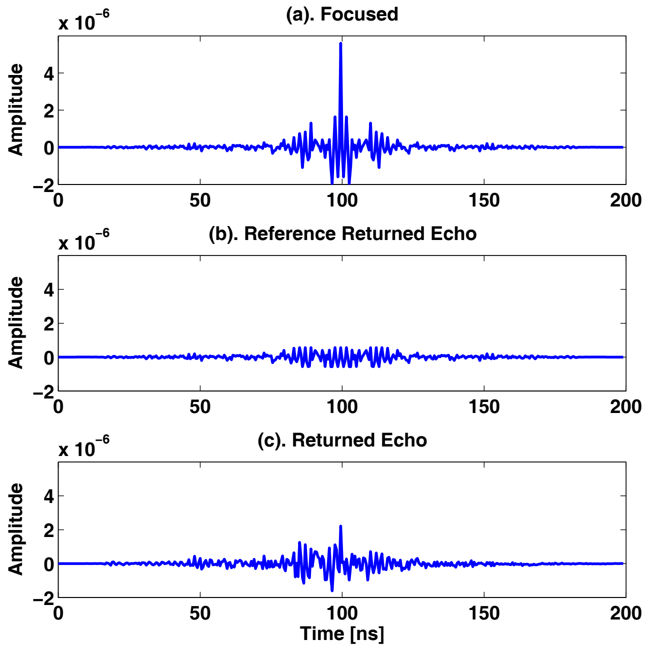

Figure 1. Focusing vs. nulling. (a): focused sharp pulse (before nulling); (b): reference echo (after nulling); (c): returned echo (after nulling).

detection and identification via optimised radar transmission pulse shape," *IEE Proc. Radar, Sonar Navig.*, vol. 148, no. 3, pp. 130–138, June 2001.

[3] M. Fink, "Time reversed acoustics," *Physics Today*, pp. 34–40, March 1997.

[4] B. E. Henty and D. D. Stancil, "Multipath enabled super-resolution for RF/microwave communication using phase-conjugate arrays," *Physical Review Letters*, 93, 243904 (2004).

[5] J. M. F. Moura, Y. Jin, D. Stancil, J. Zhu, A. Cepni, Y. Jiang, and B. Henty, "Single antenna time reversal adaptive interference cancelation," in *ICASSP'05, IEEE International Conference on Signal Processing*, vol. IV. Philadelphia, PA: March 2005, pp. 1121–1124.

[6] J. M. F. Moura, Y. Jin, D. Stancil, J. G. Zhu, A. Cepni, Y. Jiang, and B. Henty, "Time reversal adaptive interference cancellation and detection," in *the 13th Annual Workshop on Adaptive Sensor Array Processing*. Lexington, MA: M.I.T. Lincoln Laboratory, June 2005.

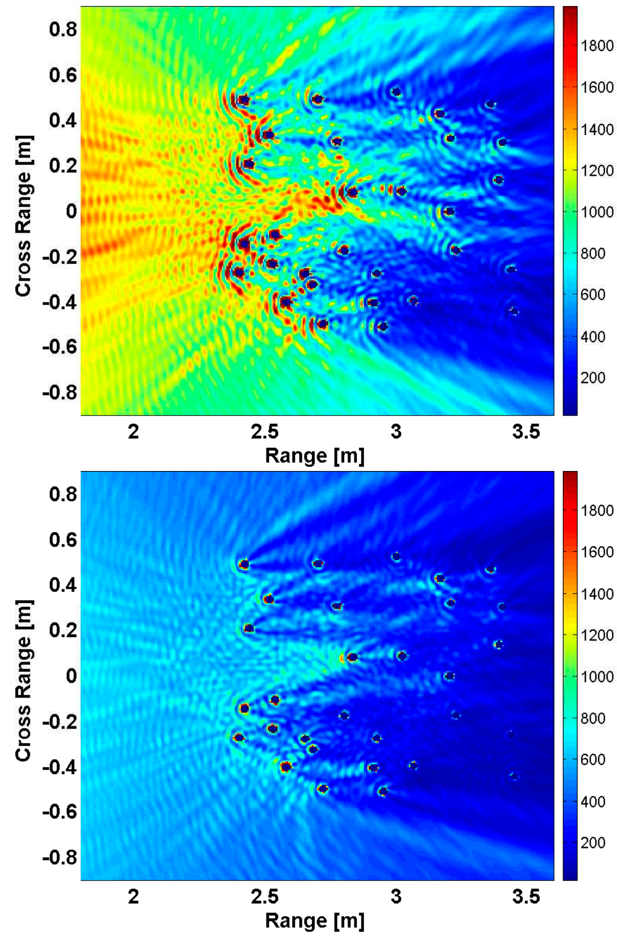

Figure 2. Scattering field energy pattern. Top: before nulling; bottom: after nulling.

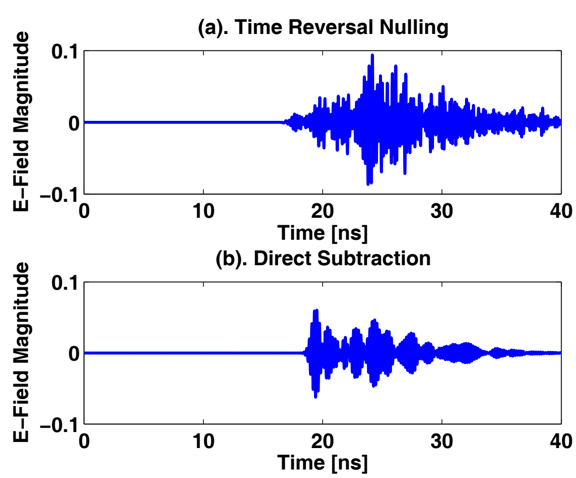

Figure 3. Returned target echo: (a). clutter nulling and target focusing $r_3(t)$; (b). change detection $f(t)$.