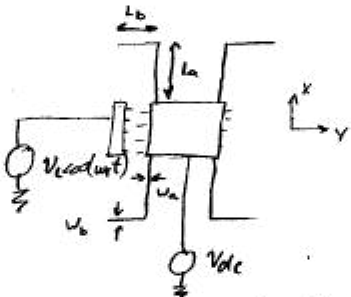
J. Ramsey
April 2002

Analysis of Crab-Leg Flexure



$$K_x = E \cdot h \left(\frac{w_b}{L_b}\right)^3 \frac{4L_b + \alpha L_a}{L_b + \alpha L_a}$$

$$K_y = E \cdot h \left(\frac{w_a}{L_a}\right)^3 \frac{L_b + 4\alpha L_a}{L_b + \alpha L_a} \qquad \alpha = \frac{I_b}{I_a}$$

assume $h = 5\,\mu m$, $E = 160\,GPa$

$L_b = 34\,\mu m$, $w_b = 2.1\,\mu m$ $\qquad I = \frac{h w^3}{12} \qquad \alpha = \frac{I_b}{I_a} = \frac{w_b^3}{w_a^3}$

$l_a = 90\,\mu m$, $w_a = 2.0\,\mu m$

$\alpha = 1.158$

$K_x = 327.6\ N/m$

$K_y = 28.64\ N/m$

comb capacitor $\qquad N = 20, \quad l_o = 9\,\mu m, \quad g = 1.2\,\mu m$

$$C_c(y) = \frac{2N\varepsilon (l_o - y) h}{g} = 1.476\times10^{-15}\ F/\mu m\ (9\,\mu m - y)$$

$$\frac{dC_c}{dy} = -\frac{2N\varepsilon_o h}{g}, \qquad F_y = \frac{1}{2}\left[v_i \cos(w_r t) - v_{dc}\right]^2 \frac{dC_c}{dy}$$

$$F_y = -\frac{N\varepsilon_o h}{g}\left[v_i \cos(w_r t) - v_{dc}\right]^2 = -7.378\times10^{-10}\ N/V^2 \left[v_i \cos(w_r t) - v_{dc}\right]^2$$

static force balance

$$F_y(w_r = 0) = F_{mech} = K_y y \rightarrow -\frac{N\varepsilon_o h}{g}\left[v_i - v_{dc}\right]^2 = K_x y$$

$$-7.378\times10^{-10}\ N/V^2 \left[v_i - v_{dc}\right]^2 = (28.64\ N/m)\, y$$

amplitude of electrostatic force at w_r

$$\left[v_i \cos(w_r t) - v_{dc}\right]^2 = v_i^2 \cos^2(w_r t) + v_{dc}^2 - 2 v_i v_{dc} \cos(w_r t)$$

$$F_o = \frac{N\varepsilon_o h}{g}(2 v_i v_{dc}) = (1.476\times10^{-9}\ N/V^2)\, v_i v_{dc}$$

Crab-Leg Flexure (continued)

amplitude of AC displacement

$k_E = 0$

$$x_0 = \frac{F_0}{K_x - \cancel{K_E}^{=0}} Q = \frac{F_0}{K_x} Q = \frac{(1.476\times10^{-9}\,N/V^2)(v_i\,V_{dc})}{28.64\,N/m} Q$$

$$= (5.153\times10^{-5}\,{}^{m}/_{V^2})(v_i\,V_{dc})\,Q$$

solving for DC displacement

$$y = -\frac{7.378\times10^{-10}\,N/V^2}{28.64\,N/m}[v_i - V_{dc}]^2 = (-2.576\times10^{-5}\,{}^{m}/_{V^2})[v_i - V_{dc}]^2$$

- additional calculations -

calculation of mass would lead to resonant frequency

$$\omega = 2\pi f = \sqrt{\frac{K}{m}}$$

note: m_{eff} - effective mass from springs "Rayleigh's Method"

calculation of quality factor, Q

$$\zeta = \text{damping factor} = \frac{B}{2\sqrt{Km}}$$

$$Q = \tfrac{1}{2}\zeta$$

B is the damping coefficient of the plate, comb fingers, etc.